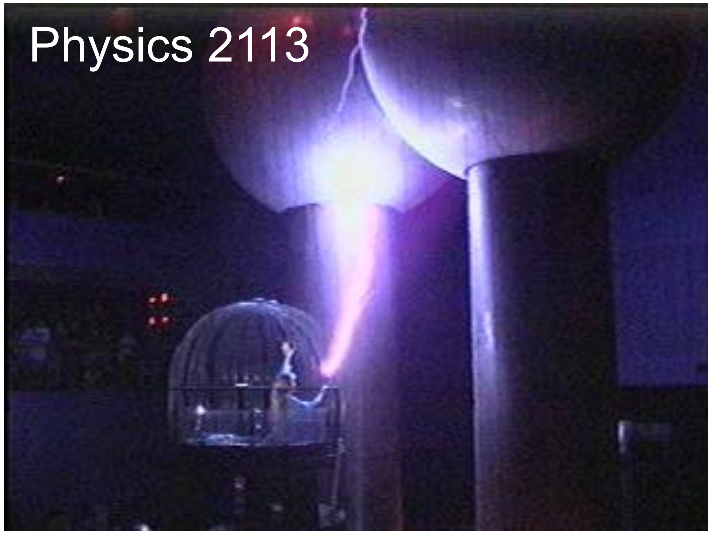
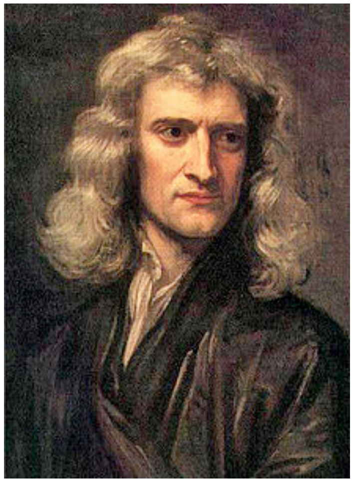




Isaac Newton
(1642–1727)

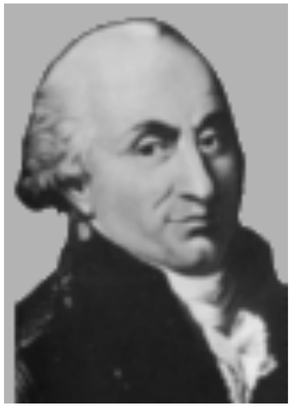
Physics 2113
Lecture 05: FRI 5 OCT

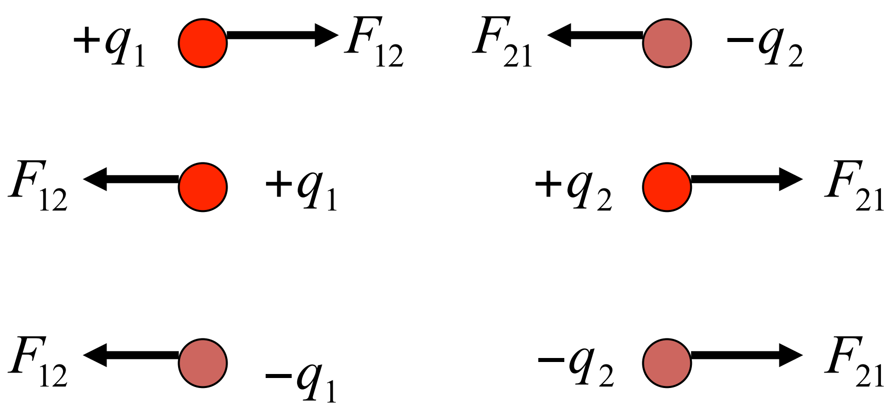
CH21: Electric Charge

Michael Faraday
(1791–1867)

Charles-Augustin
de Coulomb
(1736-1806)

Force between pairs of point charges: Coulomb's law



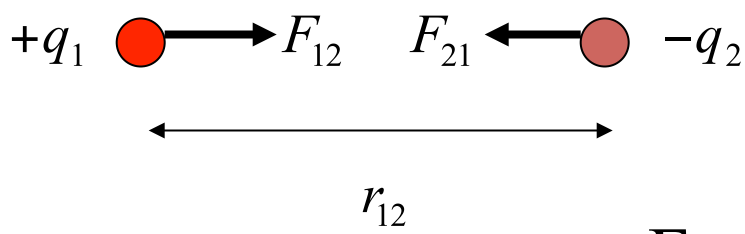


or

Coulomb's law -- the force between point charges:

- Lies along the line connecting the charges.
- Is proportional to the magnitude of each charge.
- Is inversely proportional to the distance squared.
- Note that Newton's third law says $|F_{12}| = |F_{21}|$!!

Coulomb's law



$$|F_{12}| = \frac{k\,|q_1||q_2|}{r_{12}^2}$$

For charges in a VACUUM

$$\mathrm{k} = 8.99\times10^{9}\ \frac{N\,m^2}{C^2}$$

Often, we write *k* as:

$$k = \frac{1}{4\pi\varepsilon_0} \text{ with } \varepsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{N\,m^2}$$

Example:

The charges and coordinates of two charged particles held fixed in the x-y plane are,

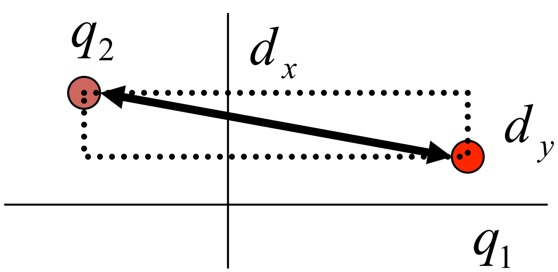
$$q_1 = 3\mu C, \quad x_1 = 3.5cm, \quad y_1 = 0.5cm$$
$$q_2 = -3\mu C, \quad x_2 = -2cm, \quad y_2 = 1.5cm$$

Find the magnitude and direction of the force on q2.

Magnitude: proportional to product of charges divided by distance squared.

Distance: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{5.5^2 + 1^2}\,\text{cm} = 5.59\text{cm}$

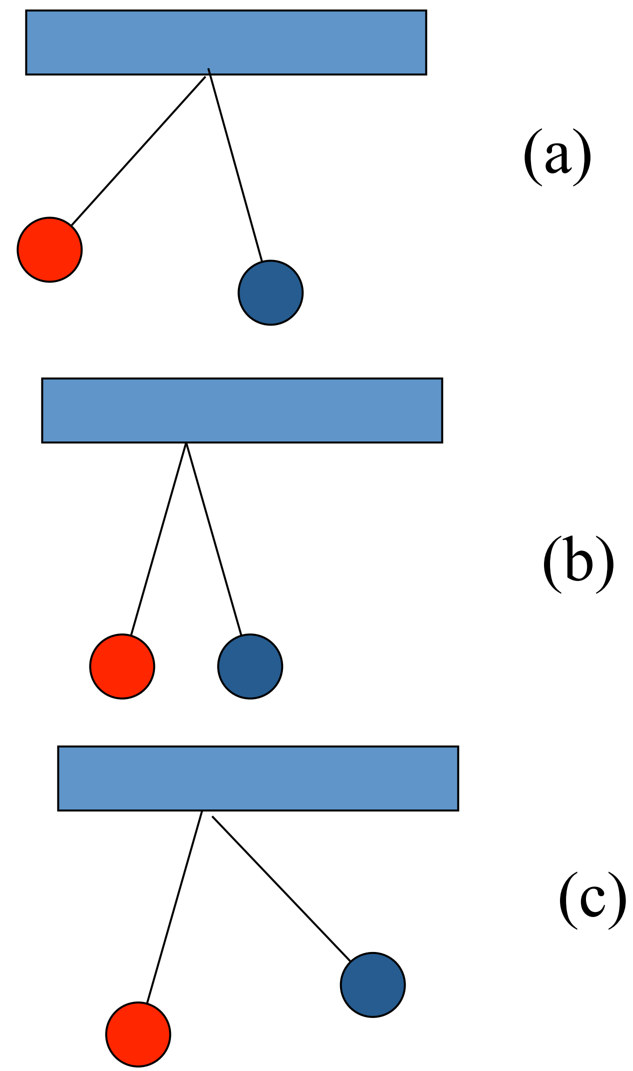
$$F = \frac{k\,q_1 q_2}{d^2} = 9\times10^9 \frac{Nm^2}{C^2}\frac{3\times10^{-6}C\times4\times10^{-6}C}{(5.59\times10^{-2}m)^2} = 35N$$



$$\alpha = \arctan\left(\frac{d_y}{d_x}\right) = \arctan\left(\frac{1}{5.5}\right) = -10.30^o$$

Example

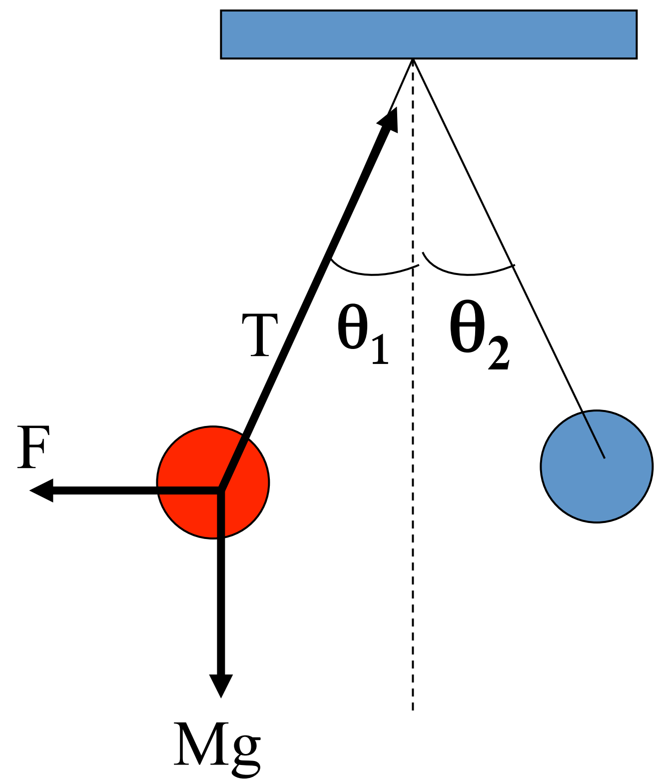
- Two small uniformly charged spheres of equal mass M are suspended by string of equal length.
- Left sphere: +5 C
- Right sphere: +10 C
- Assume all angles and dimensions are SMALL
- **Which picture correctly represents equilibrium?**



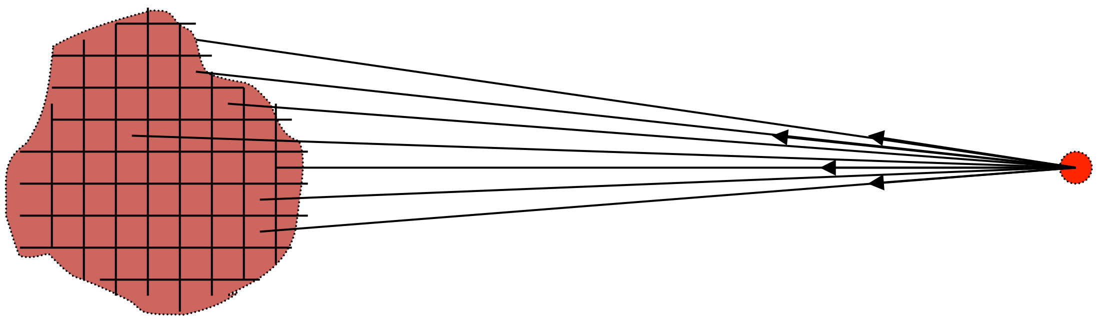
Solution to Example

- F -- Coulomb force
- T -- Tension in string
- $T \sin\theta_1 = F$
- $T \cos\theta_1 = Mg$
- $\tan \theta_1 = F/Mg$
- Newton's 3rd Law: spheres exert equal & opposite forces on each other

SO: $\theta_1 = \theta_2$!!



- What if the object is not a point particle?

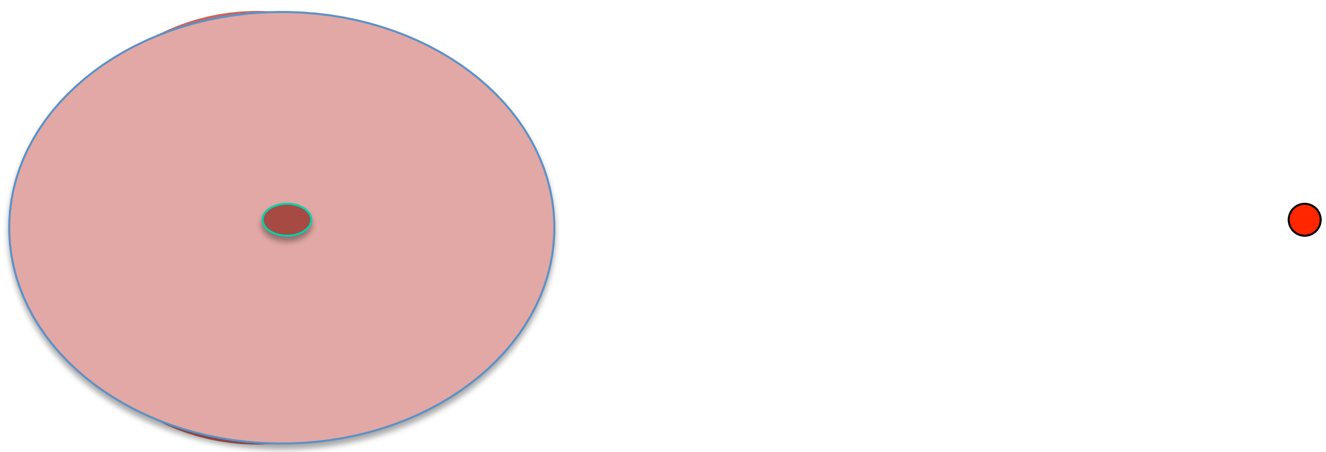


You break it up into small pieces,

treat each little piece like a point charge, and add up the resulting force (as a vector, of course!).

This can be messy to do! Typically one considers infinitesimal pieces and ends up with a 3D integral, usually evaluated numerically by a computer

An exceptional case (because it is so simple) is the the case of a sphere.



A (uniformly) charged sphere behaves like a point charge at the center of the sphere with the same total charge.

A (uniformly) charged shell behaves like a point charge at the center of the shell. Point charges placed inside the shell do not feel force (just like in gravity).

Notice that the previous result is only true if the sphere is uniformly charged.

If the sphere is made of an insulating material, one simply distributes the charge uniformly when the sphere is charged.

For a conducting sphere, the charges will automatically become uniformly distributed as long as there are no other charged objects nearby. Otherwise induction will change the distribution and the sphere cannot be treated as a point anymore.

sec. 21-4 Coulomb's Law

•1 SSM ILW Of the charge Q initially on a tiny sphere, a portion q is to be transferred to a second, nearby sphere. Both spheres can be treated as particles. For what value of q/Q will the electrostatic force between the two spheres be maximized?

1. **THINK** After the transfer, the charges on the two spheres are $Q-q$ and q.

EXPRESS The magnitude of the electrostatic force between two charges q_1 and q_2 separated by a distance r is given by the Coulomb's law (see Eq. 21-1):

$$F = k\frac{q_1 q_2}{r^2},$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$. In our case, $q_1 = Q-q$ and $q_2 = q$, so the magnitude of the force of either of the charges on the other is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}.$$

We want the value of q that maximizes the function $f(q) = q(Q-q)$.

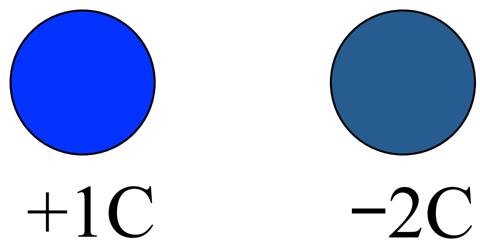
ANALYZE Setting the derivative df/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$. Thus, $q/Q = 0.500$.

LEARN The force between the two spheres is a maximum when charges are distributed evenly between them.

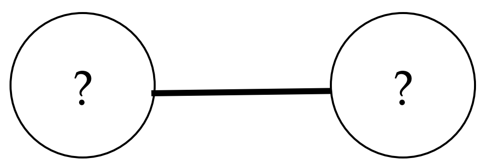
Conservation of Charge

Total amount of charge in an isolated system is fixed ("conserved")

Example: 2 identical metal spheres have charges +1C and –2C.



You connect these together with a metal wire; what is the final charge distribution?



When charged conductors are brought into contact, charge smears across them. In general the way it gets smeared is complicated, depending on the geometry of the conductors. It is easier if the geometries are simple. For instance if one has two identical spheres, One with charge q and the other uncharged and one brings them into contact, each will get q/2 of charge.

“Grounding” is a process in which a charged object is connected to the “Earth”. This leads it to lose its charge. In practice this can be accomplished by connecting it to a faucet, which connects it to the piping and eventually to the Earth itself.

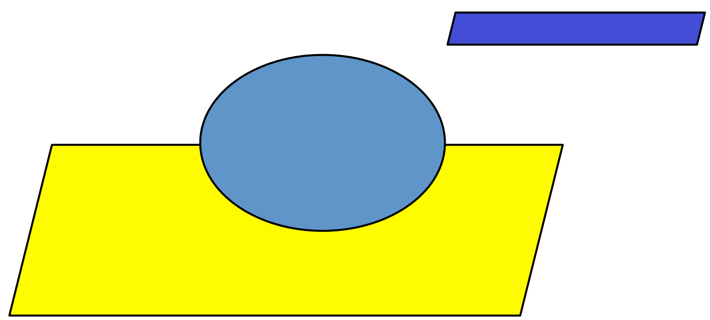
Situations involving grounding and induction can get tricky.

An initially electrically neutral conducting sphere is placed on an insulating stand. A negatively-charged glass rod is brought near, but does not touch the sphere. Without moving the rod, a wire is then attached to the sphere that connects it to earth ground. The rod and wire are then removed simultaneously. What is the final charge on the sphere?

a) negative

b) positive

c) neutral



d) It has a fifty percent chance of having a positive charge and a fifty percent chance of having a negative charge.

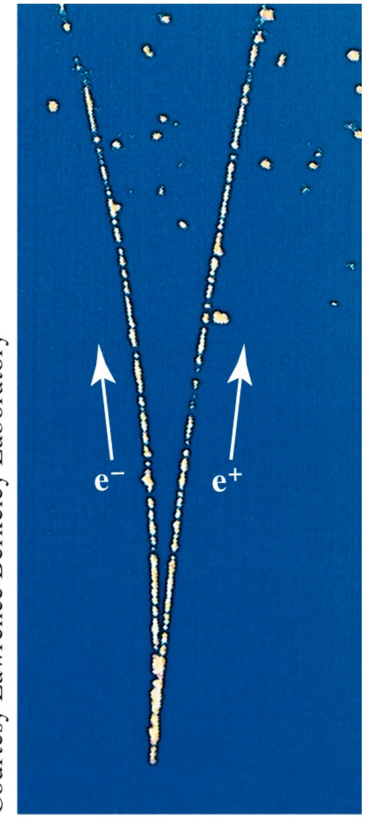
The net electric charge of any isolated system is always conserved.

If two charged particles undergo an annihilation process, they have equal and opposite signs of charge.

If two charged particles appear as a result of a pair production process, they have equal and opposite signs of charge.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

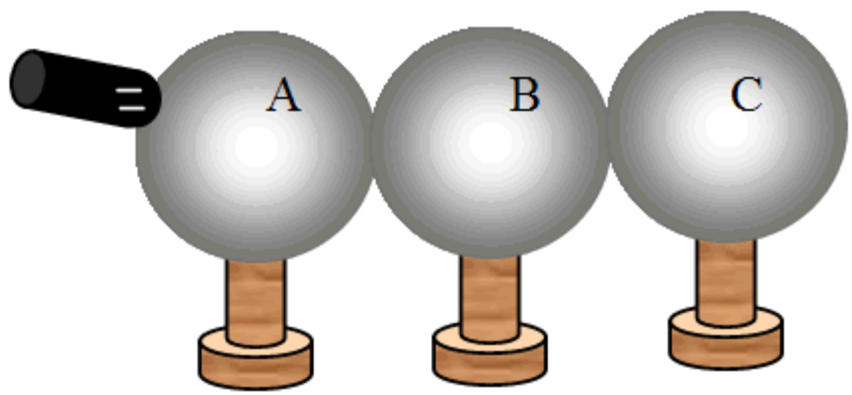
$$\gamma \rightarrow e^- + e^+$$



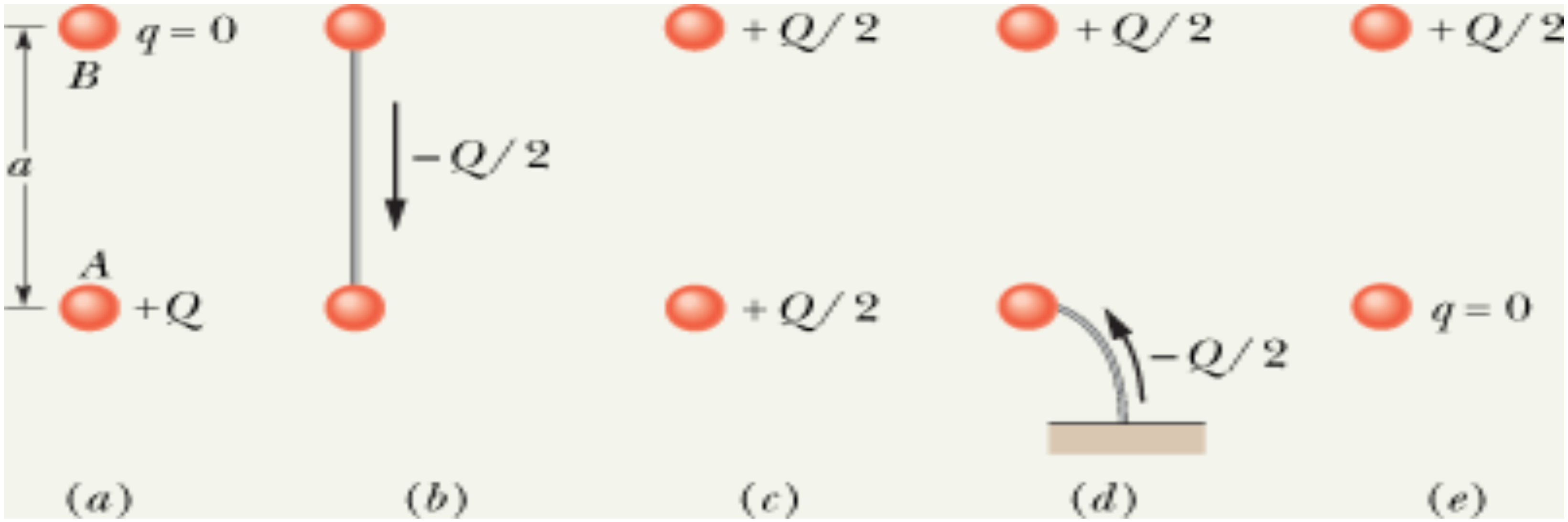
A photograph of trails of bubbles left in a bubble chamber by an electron and a positron. The pair of particles was produced by a gamma ray that entered the chamber directly from the bottom. Being electrically neutral, the gamma ray did not generate a telltale trail of bubbles along its path, as the electron and positron did.

Three identical conducting spheres on individual insulating stands are initially electrically neutral. The three spheres are arranged so that they are in a line and touching as shown. A negatively-charged conducting rod is brought into contact with sphere A. Subsequently, someone takes sphere C away. Then, someone takes sphere B away. Finally, the rod is taken away. What is the sign of the final charge, if any, of the three spheres?

	A	B	C
a)	+	+	-
b)	+	-	+
c)	+	0	-
d)	-	+	0
e)	-	-	-



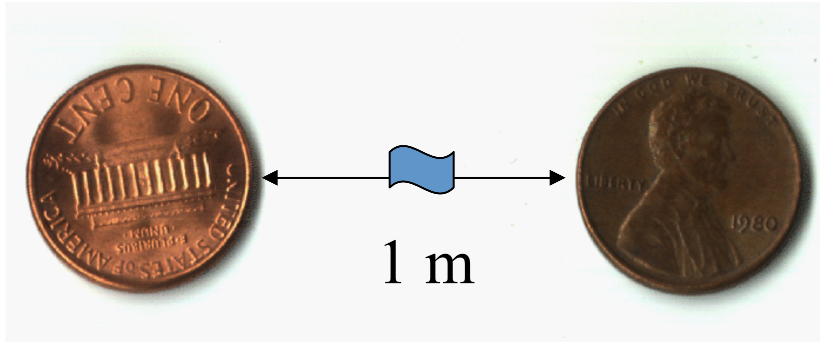
Transfer of charge between two conductors



Quantization of Charge

- Charge is always found in **INTEGER** multiples of the charge on an electron/proton ([[*why*?]])
- Electron charge = e = −1.6 x 10^{-19} Coulombs
- Proton charge = p = +1.6 x 10^{-19} Coulombs
- Unit of charge: Coulomb (C) in MKS units
- One cannot ISOLATE FRACTIONAL CHARGE (e.g. −0.8 x 10^{-19} C, +1.9 x 10^{-19} C, etc.) [[but what about quarks…?]]

Example:



Assume that the positive and negative charges within each penny are not exactly canceling each other, but there is an excess of 0.0001% of one of them. What force would two such pennies exert on each other if they are 1m apart?

One penny = 3g copper (approx.)

Molar mass of Cu 63.5g (from chemistry book)

Therefore 1 penny = 3/63.5 mols of Cu = 2.84×10^{22} Cu atoms.

One Cu atom = 29 electrons, therefore in one penny 8.25×10^{23} electrons.

Then, one penny = 132000 Coulombs. 0.0001% of that is 0.132 C.

Now, 0.132C at 1m of distance gives F = 157,000,000 N = 17,662 Tons !

So when we say that macroscopic objects are very approximately neutral, we mean it!

Summary

• **Electric charges** come with two signs: **positive and negative**.

• Like charges repel, opposite charges attract, with a magnitude calculated from **Coulomb's law**: $F=kq_1q_2/r^2$

• Electric forces are added as **vectors.**

•**Charge is quantized.**

•**Charge is conserved.**