Physics 2113

# Physics 2113 Lecture 39: WED 3 DEC <br> CH32: Electromagnetic waves 



Problems

1 If the magnetic field of a light wave oscillates parallel to a $y$ axis and is given by $B_{y}=B_{m} \sin (k z-\omega t)$, (a) in what direction does the wave travel and (b) parallel to which axis does the associated electric field oscillate?

We know E and B are perpendicular to each other and to the direction of propagation. In this case $B$ is in the $y$ direction and propagation is in the z direction. So E has to oscillate in the x direction.

Wave propagates in the positive z direction.

-4 About how far apart must you hold your hands for them to be separated by 1.0 nano-light-second (the distance light travels in 1.0 ns )?
4. In air, light travels at roughly $c=3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$. Therefore, for $t=1.0 \mathrm{~ns}$, we have a distance of

$$
d=c t=\left(3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)\left(1.0 \times 10^{-9} \mathrm{~s}\right)=0.30 \mathrm{~m}
$$

-5 SSM What inductance must be connected to a 17 pF capacitor in an oscillator capable of generating 550 nm (i.e., visible) electromagnetic waves? Comment on your answer.
5. THINK The frequency of oscillation of the current in the $L C$ circuit of the generator is $f=1 / 2 \pi \sqrt{L C}$, where $C$ is the capacitance and $L$ is the inductance. This frequency is the same as the frequency of an electromagnetic wave.

EXPRESS If $f$ is the frequency and $\lambda$ is the wavelength of an electromagnetic wave, then $f \lambda=c$. Thus,

$$
\frac{\lambda}{2 \pi \sqrt{L C}}=c .
$$

ANALYZE The solution for $L$ is

$$
L=\frac{\lambda^{2}}{4 \pi^{2} C c^{2}}=\frac{\left(550 \times 10^{-9} \mathrm{~m}\right)^{2}}{4 \pi^{2}\left(17 \times 10^{-12} \mathrm{~F}\right)\left(2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)^{2}}=5.00 \times 10^{-21} \mathrm{H} .
$$

This is exceedingly small.
LEARN The frequency is

$$
f=\frac{c}{\lambda}=\frac{3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}}{550 \times 10^{-9} \mathrm{~m}}=5.45 \times 10^{14} \mathrm{~Hz}
$$

The EM wave is in the visible spectrum.
-21 ILW What is the radiation pressure 1.5 m away from a 500 W lightbulb? Assume that the surface on which the pressure is exerted faces the bulb and is perfectly absorbing and that the bulb radiates uniformly in all directions.
21. Since the surface is perfectly absorbing, the radiation pressure is given by $p_{r}=I / c$, where $I$ is the intensity. Since the bulb radiates uniformly in all directions, the intensity a distance $r$ from it is given by $I=P / 4 \pi r^{2}$, where $P$ is the power of the bulb. Thus

$$
p_{r}=\frac{P}{4 \pi r^{2} c}=\frac{500 \mathrm{~W}}{4 \pi(1.5 \mathrm{~m})^{2}\left(2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)}=5.9 \times 10^{-8} \mathrm{~Pa} .
$$

-12 In a plane radio wave the maximum value of the electric field component is $5.00 \mathrm{~V} / \mathrm{m}$. Calculate (a) the maximum value of the magnetic field component and (b) the wave intensity.
12. (a) The amplitude of the magnetic field in the wave is

$$
B_{m}=\frac{E_{m}}{c}=\frac{5.00 \mathrm{~V} / \mathrm{m}}{2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}}=1.67 \times 10^{-8} \mathrm{~T} .
$$

(b) The intensity is the average of the Poynting vector:

$$
I=S_{\mathrm{avg}}=\frac{E_{m}^{2}}{2 \mu_{0} c}=\frac{(5.00 \mathrm{~V} / \mathrm{m})^{2}}{2\left(4 \pi \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}\right)\left(2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)}=3.31 \times 10^{-2} \mathrm{~W} / \mathrm{m}^{2}
$$

-32 In Fig. 33-40, initially unpolarized light is sent into a system of three polarizing sheets whose polarizing directions make angles of $\theta_{1}=\theta_{2}=\theta_{3}=50^{\circ}$ with the direction of the $y$ axis. What percentage of the initial intensity is transmitted by the system? (Hint: Be careful with the angles.)


$$
\begin{gathered}
E_{y}=E \cos (\theta) \\
I=I_{0} \cos ^{2} \theta
\end{gathered}
$$

32. After passing through the first polarizer the initial intensity $I_{0}$ reduces by a factor of $1 / 2$. After passing through the second one it is further reduced by a factor of $\cos ^{2}(\pi-$ $\left.\theta_{1}-\theta_{2}\right)=\cos ^{2}\left(\theta_{1}+\theta_{2}\right)$. Finally, after passing through the third one it is again reduced by a factor of $\cos ^{2}\left(\pi-\theta_{2}-\theta_{3}\right)=\cos ^{2}\left(\theta_{2}+\theta_{3}\right)$. Therefore,

$$
\begin{aligned}
\frac{I_{f}}{I_{0}} & =\frac{1}{2} \cos ^{2}\left(\theta_{1}+\theta_{2}\right) \cos ^{2}\left(\theta_{2}+\theta_{3}\right)=\frac{1}{2} \cos ^{2}\left(50^{\circ}+50^{\circ}\right) \cos ^{2}\left(50^{\circ}+50^{\circ}\right) \\
& =4.5 \times 10^{-4} .
\end{aligned}
$$

Thus, $0.045 \%$ of the light's initial intensity is transmitted.
-49 Figure 33-49 shows light reflecting from two perpendicular reflecting surfaces $A$ and $B$. Find the angle between the incoming ray $i$ and the outgoing ray $r^{\prime}$.


Fig. 33-49 Problem 49.

Idea: reflecting surfaces have incidence and reflected angles equal.
49. The angle of incidence for the light ray on mirror $B$ is $90^{\circ}-\theta$. So the outgoing ray $r^{\prime}$ makes an angle $90^{\circ}-\left(90^{\circ}-\theta\right)=\theta$ with the vertical direction, and is antiparallel to the incoming one. The angle between $i$ and $r^{\prime}$ is therefore $180^{\circ}$.
*050 In Fig. 33-50a, a beam of light in material 1 is incident on a boundary at an angle $\theta_{1}=40^{\circ}$. Some of the light travels through material 2, and then some of it emerges into material 3. The two boundaries between the three materials are parallel. The final direction of the beam depends, in part, on the index of refraction $n_{3}$ of the third material. Figure $33-50 b$ gives the angle of refraction $\theta_{3}$ in that material versus $n_{3}$ for a range of possible $n_{3}$ values. The vertical axis scale is set by $\theta_{3 a}=30.0^{\circ}$ and $\theta_{3 b}=50.0^{\circ}$. (a) What is the index of refraction of material 1 , or is the index impossible to calculate without more information? (b) What is the index of refraction of material 2, or is the index impossible to calculate without more information? (c) If $\theta_{1}$ is changed to $70^{\circ}$ and the index of refraction of material 3 is 2.4 , what is $\theta_{3}$ ?

## Snell's law



Fig. 33-50 Problem 50.

$$
n_{2} \sin \theta_{2}=n_{1} \sin \theta_{1}
$$

50. (a) From $n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$ and $n_{2} \sin \theta_{2}=n_{3} \sin \theta_{3}$, we find $n_{1} \sin \theta_{1}=n_{3} \sin \theta_{3}$. This has a simple implication: that $\theta_{1}=\theta_{3}$ when $n_{1}=n_{3}$. Since we are given $\theta_{1}=40^{\circ}$ in Fig. 3350(a), then we look for a point in Fig. 33-50(b) where $\theta_{3}=40^{\circ}$. This seems to occur at $n_{3}$ $=1.6$, so we infer that $n_{1}=1.6$.
(b) Our first step in our solution to part (a) shows that information concerning $n_{2}$ disappears (cancels) in the manipulation. Thus, we cannot tell; we need more information.
(c) From $1.6 \sin 70^{\circ}=2.4 \sin \theta_{3}$ we obtain $\theta_{3}=39^{\circ}$.
-57 A point source of light is 80.0 cm below the surface of a body of water. Find the diameter of the circle at the surface through which light emerges from the water.

## Idea: total internal reflection.

57. Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point $a$ to point $f$ in that figure) is related to the tangent of the angle of incidence. Thus, the diameter $D$ of the circle in question is

$$
D=2 h \tan \theta_{c}=2 h \tan \left[\sin ^{-1}\left(\frac{1}{n_{w}}\right)\right]=2(80.0 \mathrm{~cm}) \tan \left[\sin ^{-1}\left(\frac{1}{1.33}\right)\right]=182 \mathrm{~cm} .
$$


(a)

(b)

## Summary

## Electromagnetic Waves

- An electromagnetic wave consists of oscillating electric and magnetic fields as given by,

$$
\begin{aligned}
& E=E_{m} \sin (k x-\omega t) \\
& B=B_{m} \sin (k x-\omega t)
\end{aligned}
$$

- The speed of any electromagnetic wave in vacuum is $c$, which can be written as

$$
c=\frac{E}{B}=\frac{1}{\sqrt{\mu_{0} \varepsilon_{0}}}
$$

## Energy Flow

- The rate per unit area at which energy is trans- ported via an electromagnetic wave is given by the Poynting vector $\boldsymbol{S}$ :

$$
\vec{S}=\frac{1}{\mu_{0}} \vec{E} \times \vec{B} .
$$

- The intensity $I$ of the wave is:

$$
I=\frac{1}{c \mu_{0}} E_{\mathrm{rms}}^{2}
$$

- The intensity of the waves at distance $r$ from a point source of power Ps is

$$
I=\frac{P_{s}}{4 \pi r^{2}} .
$$

## Radiation Pressure

- If the radiation is totally absorbed by the surface, the force is

$$
F=\frac{I A}{c}
$$

- If the raurauon is totally absorbed by the surface, the force is

$$
F=\frac{2 I A}{c}
$$

## Summary

## Radiation Pressure

- The radiation pressure $p_{r}$ is the force per unit area.
- For total absorption

$$
p_{r}=\frac{I}{c}
$$

- For total reflection back along path,

$$
p_{r}=\frac{2 I}{c}
$$

## Polarization

- Electromagnetic waves are polarized if their electric field vectors are all in a single plane, called the plane of oscillation.
- If the original light is initially unpolarized, the transmitted intensity I is

$$
I=\frac{1}{2} I_{0} .
$$

- If the original light is initially polarized, the transmitted intensity depends on the angle $u$ between the polarization direction of the original light (the axis along which the fields oscillate) and the polarizing direction of the sheet:

$$
I=I_{0} \cos ^{2} \theta
$$

## Reflection and Refraction

- The angle of reflection is equal to the angle of incidence, and the angle of refraction is related to the angle of incidence by Snell's law,

$$
n_{2} \sin \theta_{2}=n_{1} \sin \theta_{1}
$$

## Summary

## Total Internal Reflection

- A wave encountering a boundary across which the index of refraction decreases will experience total internal reflection if the angle of incidence exceeds a critical angle,

$$
\theta_{c}=\sin ^{-1} \frac{n_{2}}{n_{1}}
$$

## Polarization by Reflection

- A reflected wave will be fully polarized, if the incident, unpolarized wave strikes a boundary at the Brewster angle

$$
\theta_{\mathrm{B}}=\tan ^{-1} \frac{n_{2}}{n_{1}}
$$

