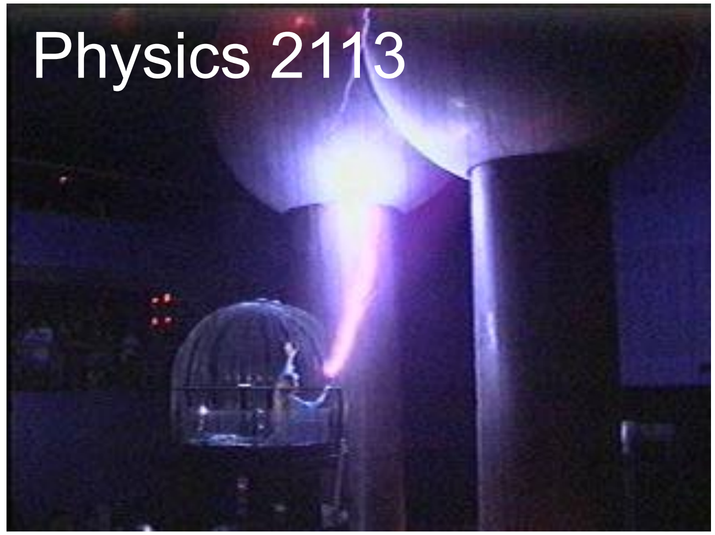


Physics 2113
Lecture 37: MON 24 NOV

CH32: Electromagnetic waves

When we discussed Maxwell's equations, we noticed that they admitted a solution that was had the form of a wave. More precisely, one had electric and magnetic fields of the form,

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$

$\frac{\omega}{k} = c,$ speed of propagation.

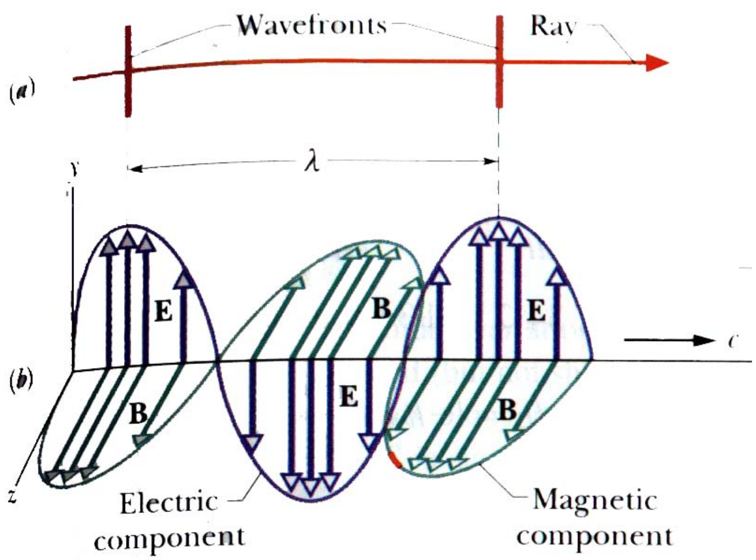
ω is the (angular frequency), k is the wave vector

$\mathrm{k} = \frac{1}{\lambda},$ with λ the wavelength.

And we noted that

$$c = \frac{E_m}{B_m} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

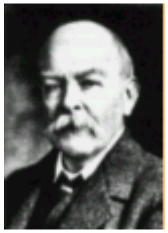
$$= 299{,}462{,}954 \frac{m}{s} = 187{,}163 mph$$



And light, infrared, ultraviolet, radio waves, X rays, Gamma rays are all electromagnetic waves.

Electromagnetic waves are able to transport energy from transmitter to receiver (example: from the Sun to our skin).

The amount of power transported by the wave and its direction is quantified by a vector called Poynting vector.



John Henry Poynting (1852-1914)

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$|S| = \frac{1}{\mu_0}EB = \frac{1}{c\mu_0}E^2$$

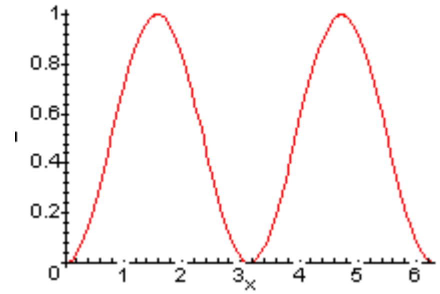
For a wave since E is perpendicular to B

The units are power per unit area, i.e. Watt/m^2

In a wave, the fields change with time in a fixed way. Therefore the Poynting vector changes too. A better measure of the amount of energy is obtained by averaginng the Poynting vector over one wave cycle. The resulting quantity is called intensity

$$I = \bar{S} = \frac{1}{c\mu_0}\overline{E^2} = \frac{1}{c\mu_0}E_m{}^2\,\overline{\sin^2(kx-\omega t)}$$

The average of $\sin^2$ over one cycle is 1/2.



$$I = \frac{1}{2c\mu_0}E_m{}^2$$

Engineers commonly use the term "root mean square" value of a quantity,

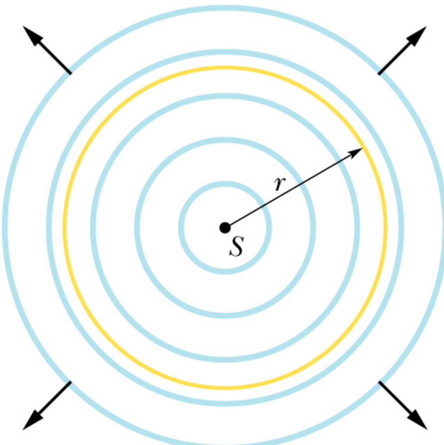
$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{\sqrt{2}}{2}E_m \cong 1.44E_m$$

$$I = \frac{1}{c\mu_0}E_{rms}{}^2$$

Both fields have the same energy density

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (cB)^2 = \frac{1}{2}\varepsilon_0\frac{B^2}{\varepsilon_0\mu_0} = u_B$$

The intensity of a wave is power per unit area. If one has a source that emits isotropically (equally in all directions) the power emitted by the source pierces a larger and larger sphere as the wave travels outwards. Therefore,



$$I = \frac{P_s}{4\pi r^2}$$

So the power per unit area decreases as the inverse of distance squared.

Waves not only carry energy but also momentum. The effect is very small (we don't ordinarily feel pressure from light). If light is completely absorbed during an interval Δt, the momentum transferred is given by,

$$\Delta p = \frac{\Delta u}{c}$$

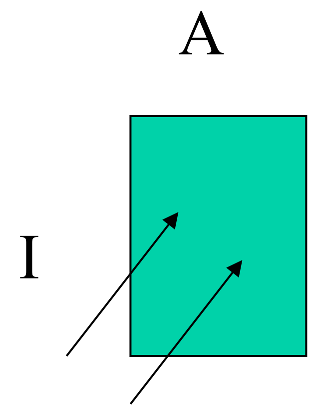
And if light is reflected, one gets double this amount.

This led William Crookes in 1875 to propose a “light mill”, a mechanical device that could garner rotational energy from a light beam. His paper was published after Maxwell accepted his explanation. Unfortunately, the explanation is wrong. Light pressure is too small to overcome friction. The device works due to certain temperature differences and how they affect molecules of the residual gas in the bottle. A full explanation is in the course website.

Radiation pressure:

From Newton's law, one can relate the change in momentum with the force

$$F = \frac{\Delta p}{\Delta t}$$



Now, supposing one has a wave that hits a surface of area A (perpendicularly), the amount of energy transferred to that surface in time Δt will be

$$\Delta U = IA\Delta t$$ therefore $$\Delta p = \frac{IA\Delta t}{c}$$

Combining this with Newton's law, $$F = \frac{IA}{c}$$

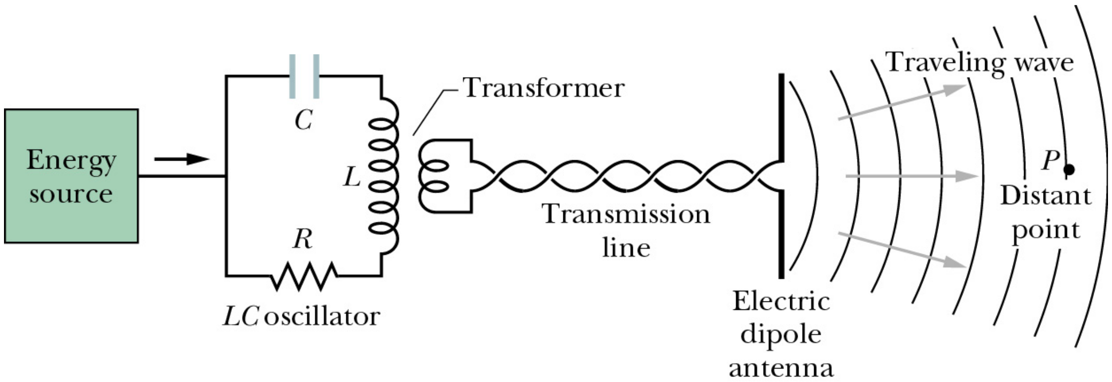
(if the surface absorbs entirely the wave, if it reflects it we get double the amount)

In terms of the pressure (force per unit area) one has

$$p_r = \frac{I}{c} \text{ (total absorption)}, \quad p_r = \frac{2I}{c} \text{ (total reflection)}$$

Polarization:

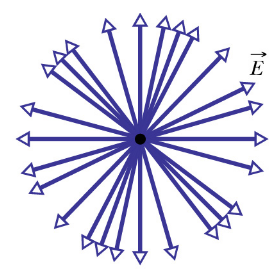
Let us consider how a radio transmitter works:



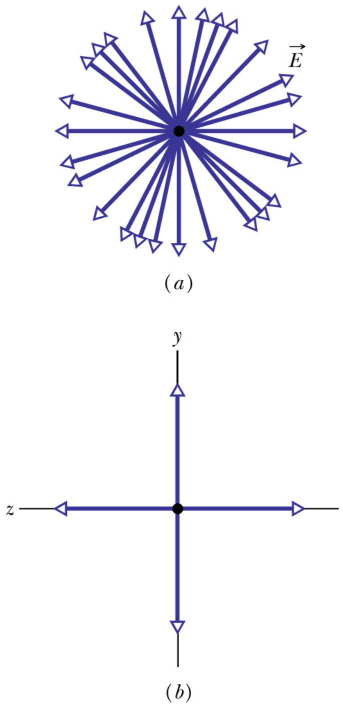
If the dipole antenna is vertical, so will be the electric fields. The magnetic field will be horizontal.

The radio wave generated is said to be "polarized".

When one has a light source (say, the Sun or a lightbulb filament), the situation is in general more complicated. The electric fields that constitute the light are produced by charges in individual molecules, each executing a random, violent motion. The resulting wave is therefore a superposition of the wave produced by each molecule. The electric field does not point in any defined direction. This is an "unpolarized wave"

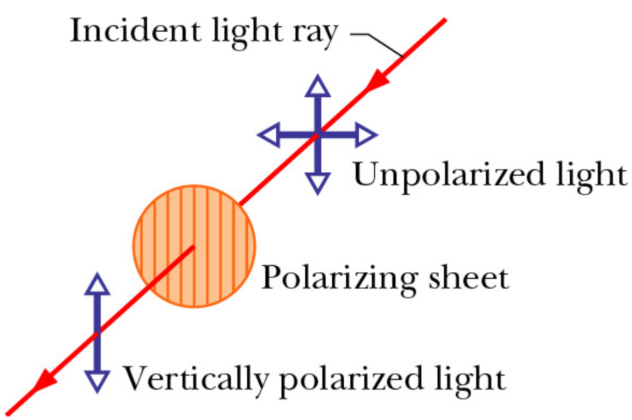


(a)



The resulting electric field from the superposition of all the above waves will point in some direction. We can decompose that direction into vertical and horizontal components. That is, an unpolarized wave can always be decomposed into the sum of a horizontally and a vertically polarized wave.

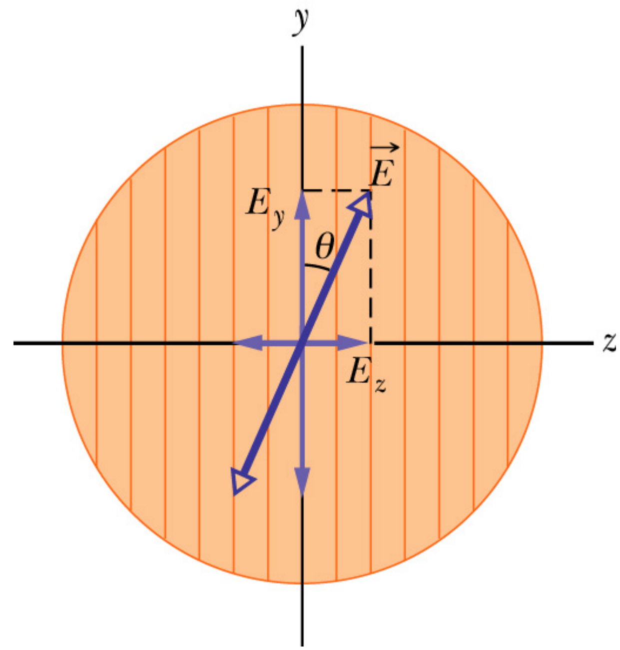
One can convert unpolarized light into polarized light by reflecting it on certain surfaces. Another way is to use specially designed plastic filters called "Polaroids" (they were invented by Edwin Land 1932, which also patented the Polaroid picture camera).



Completely unpolarized light will have equal components in horizontal and vertical directions. Therefore running the light through a polarizer will cut the intensity in half

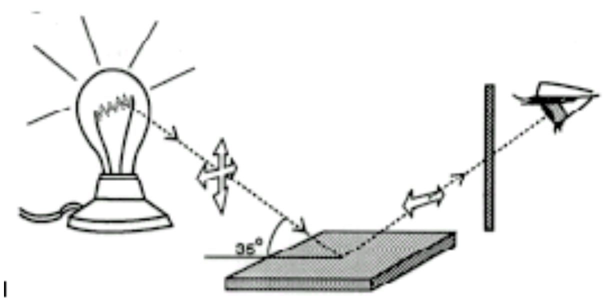
$$I = \frac{1}{2} I_0$$

When polarized light hits a polarizing sheet, only the component of the field aligned with the sheet will get through.



$$E_y = E\cos(\theta)$$

And therefore: $$I = I_0 \cos^2\theta$$



Polarized sunglasses operate on this formula. They cut the horizontally polarized light from glare (reflections on roads, cars, etc).

Summary

- Electromagnetic waves can carry energy and momentum, which in turn implies they can exert pressure.
- Waves can be polarized if the fields are aligned in a plane and polarizations can be chosen with polaroid sheets.