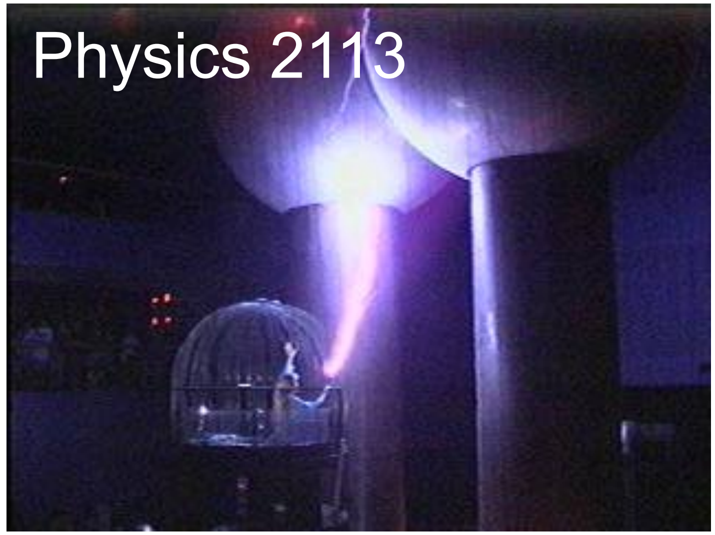



James Clerk Maxwell (1831-1879)

# Physics 2113
# Lecture 34: MON 17 NOV

## CH32: Maxwell's equations. The dawn of the 20th century revolution in physics

Up to now in this class, we have introduced a series of facts about electricity and magnetism. Roughly speaking, our knowledge is similar to that of cutting-edge physicists of the 1850's.

A rather unappealing fact of our knowledge is that it does not form a coherent body. We have one law for electricity, another for magnetism, another for magnetic forces, etc.

In 1860 Maxwell addressed this problem. He laid out a consistent compact formulation of the laws needed to describe all electromagnetic phenomena.

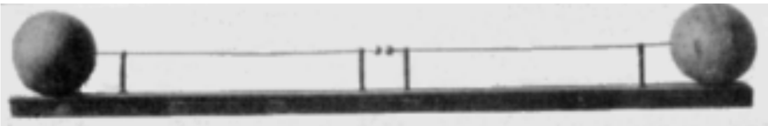
James Clerk Maxwell
(1831-1879)

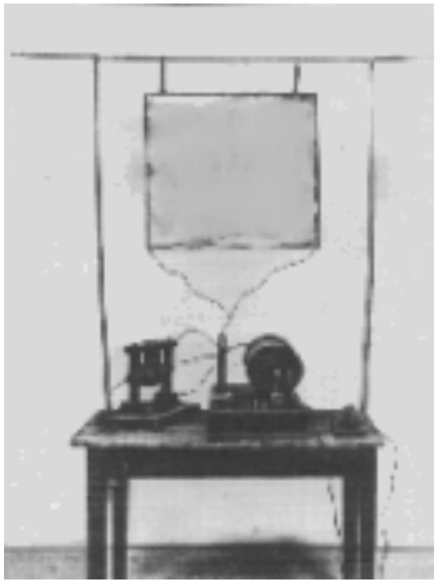
While doing so, he noticed that one of the laws in question (Ampere's law) had a missing term. It was obvious one needed to add it for things to be consistent.

The resulting theory (Maxwell's theory of electromagnetism) ended up being far more profound than simply rewriting existing knowledge.

The equations Maxwell laid out contained within them the prediction that electromagnetic waves could be used to carry energy and information. Less than ten years later this was experimentally confirmed by Hertz in the lab, and a few years later Marconi put it to practical use over the sea!

Heinrich Rudolf Hertz (1847-1894)



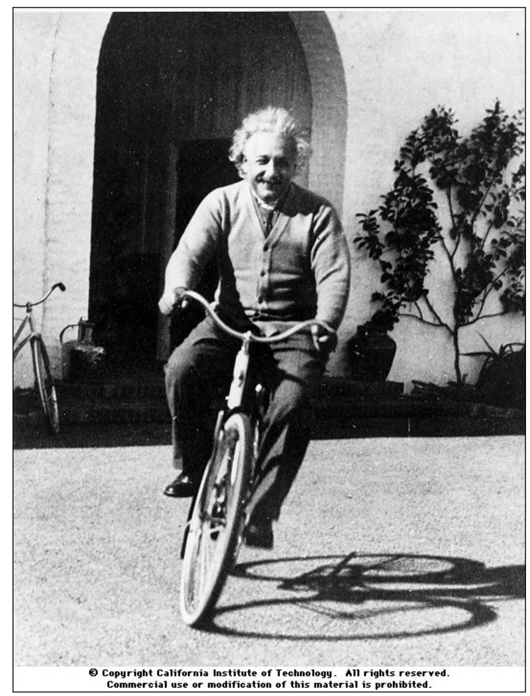


Guglielmo Marconi (1874-1937)

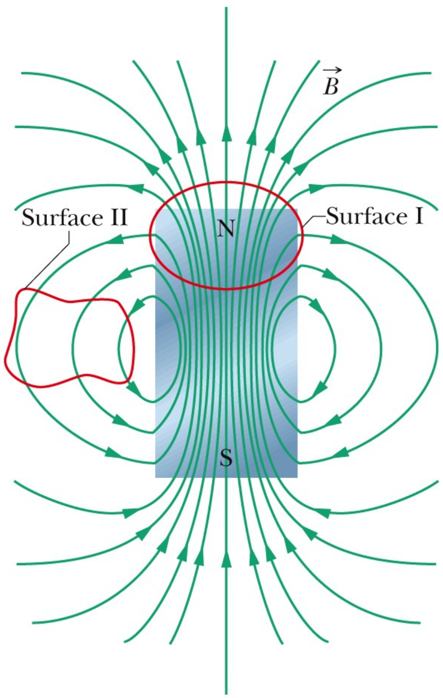
As if this were not enough, Maxwell's formulation implied that light waves and radio waves were different manifestations of the same phenomenon. The debate over the nature of light had been going on since the 1600's when Newton argued that light was made of corpuscles rather than waves.

To top it off, without realizing it, the additional term that Maxwell added to the equations implied that they were consistent with Einstein's theory of relativity, more than 30 years before the latter was introduced! Properties of Maxwell's equations and some inconsistencies they showed with Newton's mechanics were the key factors that led Einstein to reformulate space-time and our understanding of matter and energy.

Albert Einstein
(1879-1955)



# Gauss’ Law for magnetism





The field lines for the magnetic field ***B*** of a short bar magnet. The red curves represent cross sections of closed, three-dimensional Gaussian surfaces.

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).

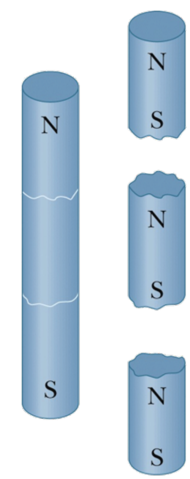
Gauss’ law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. The law asserts that the net magnetic flux $\Phi_B$ through <u>any</u> closed Gaussian surface is zero:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Contrast this with Gauss’ law for electric fields,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

**Gauss’ law for magnetic fields** says that there can be no net magnetic flux through the surface because there can be no net “magnetic charge” (individual magnetic poles) enclosed by the surface.





If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.

## Maxwell's theory:

Gauss' law (electricity): $\oint_S \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$

Gauss' law (magnetism): $\oint_S \vec{B} \cdot d\vec{A} = 0$ No isolated poles.

Faraday's law of induction: $\oint_S \vec{E} \cdot d\vec{S} = -\frac{d\Phi_B}{dt}$

Ampere's law: $\oint_S \vec{B} \cdot d\vec{S} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i$

New term

Lorentz' force law: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

This is all you should ever need to solve a problem in electromagnetism (at least in vacuum).

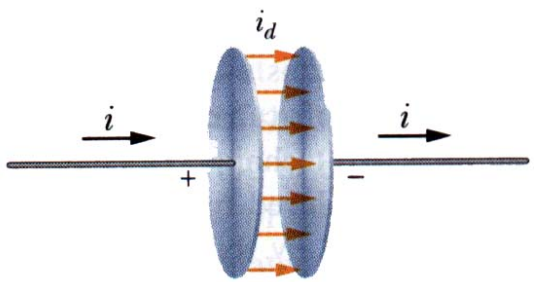
## The new term in Ampere's law (Ampere-Faraday),

$$\oint_S \vec{B}\cdot d\vec{S} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i$$

$$\oint_S \vec{B}\cdot d\vec{S} = \mu_0 (i + i_d)$$

Faraday's law told us that a changing magnetic flux produces an electric field. This law tells us that a changing electric flux produces a magnetic field.

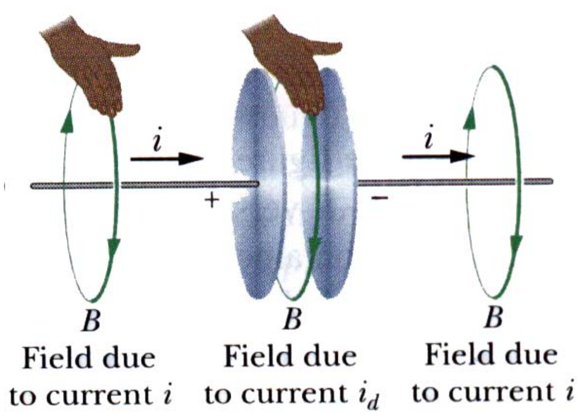
Where $i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$, "displacement current"




Example: a charging capacitor,

$$q = \varepsilon_0 AE \qquad \frac{dq}{dt} = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$i = \varepsilon_0 \frac{d\Phi_E}{dt} = i_d$$



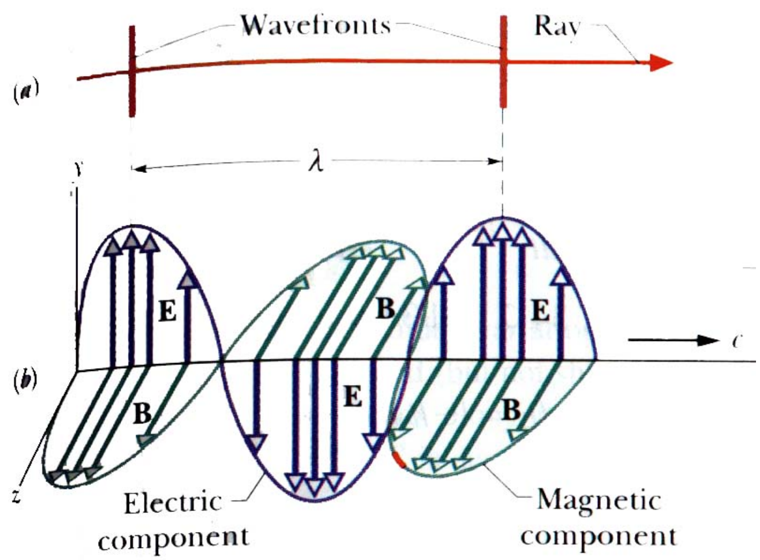
**Waves:** Maxwell noted that his equations admitted wave solutions, and that the waves travelled at the speed of light.

$$E = E_m \sin(kx - \omega t)$$

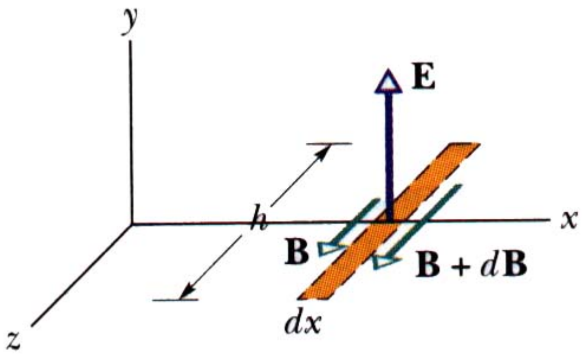
$$B = B_m \sin(kx - \omega t)$$

$$\frac{\omega}{k} = c, \quad \text{speed of propagation.}$$

Faraday's law $\Rightarrow \quad \mathrm{E} \perp \mathrm{B}$



Consider a closed loop in the direction of B, apply Ampere-Faraday,




$$\oint_S \vec{B} \cdot d\vec{S} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$-(B + dB)h + Bh = \mu_0 \varepsilon_0 \frac{d(Ehdx)}{dt}$$

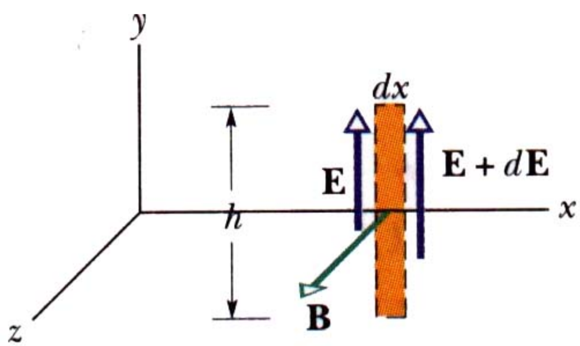
$$-\frac{dB}{dx} = \mu_0 \varepsilon_0 \frac{dE}{dt}$$

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$

$$\boxed{B_m k = \mu_0 \varepsilon_0 E_m \omega}$$

Same story with loop in direction of E, apply Faraday,




$$\oint_S \vec{E} \cdot d\vec{S} = -\frac{d\Phi_B}{dt}$$

$$(E + dE)h - Eh = -\frac{d(Bhdx)}{dt}$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

$$\boxed{B_m \omega = E_m k}$$

Therefore,

$$\boxed{B_m \omega = E_m k}$$

$$\boxed{B_m k = \mu_0 \varepsilon_0 E_m \omega}$$

$$\left(\frac{\omega}{k}\right)^2 = \frac{1}{\mu_0 \varepsilon_0} = c^2$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299{,}462{,}954 \frac{m}{s} = 187{,}163 mph}$$

Speed of light!

Light and electromagnetic waves are the same thing. Difference in effects is due to difference in frequency.

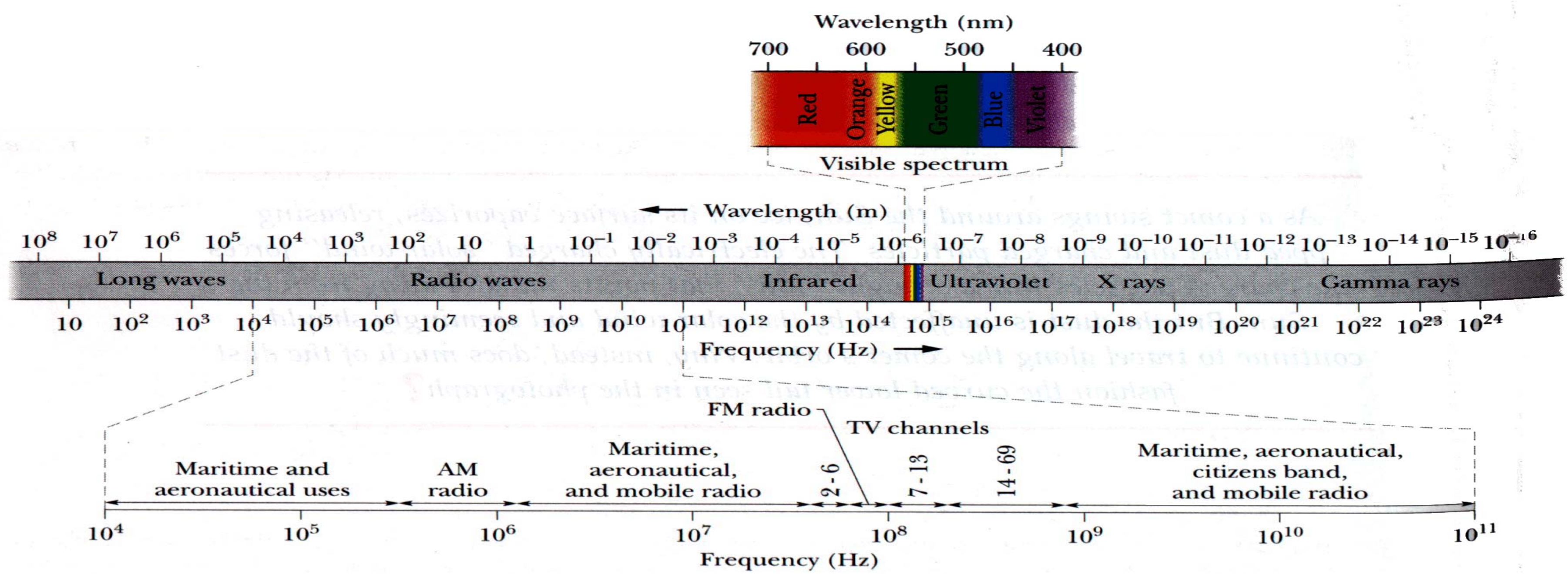



**FIGURE 34-1** The electromagnetic spectrum.

# Summary:

- Maxwell formulated electromagnetism in a set of five concise laws. Their form suggested modifications that had tremendous impact.
- Electromagnetic waves were predicted, and shown to include visible light as a particular case.