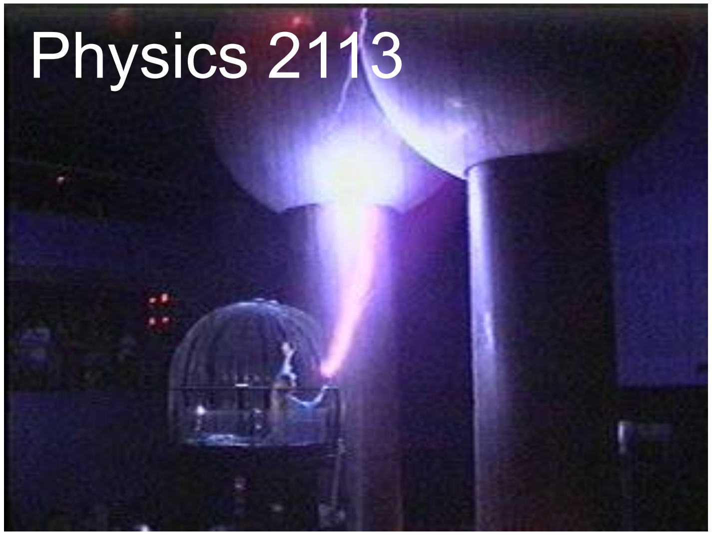


Physics 2113
Lecture 33: FRI 14 NOV

CH31: Oscillations and AC

Last class we discussed how alternating current (AC) was useful to keep electrical oscillators going. We also argued AC was easy to manipulate, amplify, decrease, etc. Today we will actually study these points in detail.

In preparation for this last class we studied how various components reacted to AC. We found that in coils and capacitors the current and the emf were out of phase. In capacitors the current was ahead of the emf, in coils it trailed. We also introduced the concept of "reactance", a sort of "effective resistance" that various devices presented to AC's.

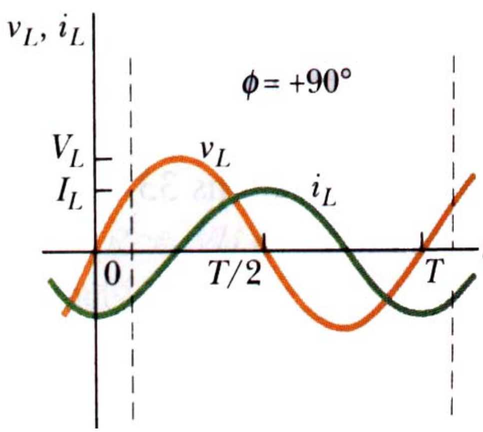
Reactances:

Resistors: $X = R$

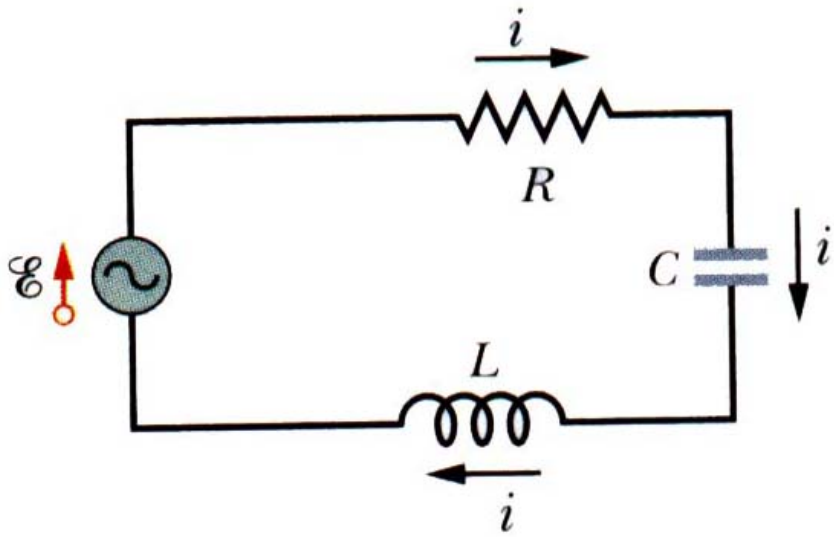
Inductors: $X = \omega_d L$

Capacitors: $X = 1/(\omega_d L)$

In a coil:



RLC circuit driven by AC:



Series circuit: current is the same in all devices.

"Taking a walk" we see that the emfs in the various devices should add up to that of the AC generator. Yet these emfs are out of phase with each other. How to keep track of this?

The math for this is complicated. We will not cover it in this course, you can see a glimpse in the book.

In a complex circuit involving coils, capacitors and resistors, the current will have a phase difference with the applied emf which we call ϕ.

In the case of the RLC series circuit we have:

If $X_L > X_C$, then circuit is "more inductive", i trails E, $\phi > 0$

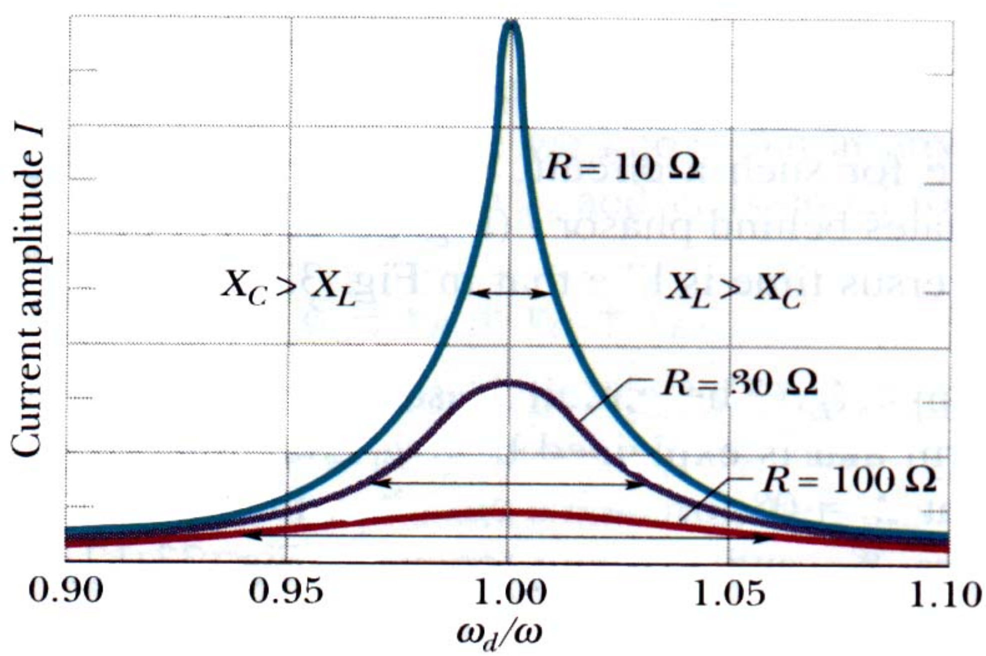
If $X_C > X_L$, then circuit is "more capacitive", i leads E, $\phi < 0$

If $X_C = X_L$, we have RESONANCE.

$$X_L = X_C \Rightarrow \omega_d L = \frac{1}{\omega_d C} \Rightarrow \omega_d = \sqrt{\frac{1}{LC}}$$

Circuit is being driven with the same frequency as the "natural" frequency of the circuit. Like if you push a swing exactly when the person is swinging away from you.

Plotting maximum current vs frequency for fixed L, R, C,

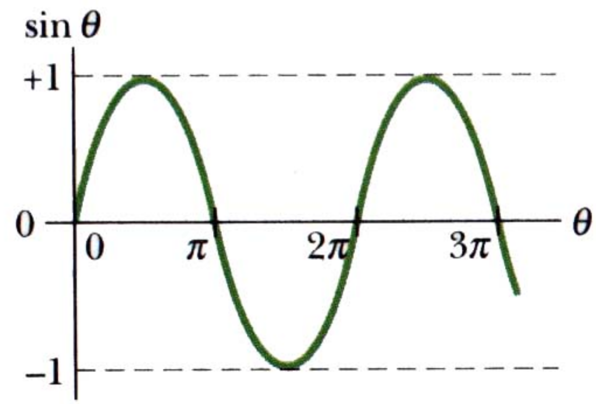


$$L = 100mH, \; C = 100pF$$

Size of amplitude is maximum at resonance. How sharp is the peak depends on how damped the circuit is. Circuits with low damping have sharp resonant behavior.

Power in AC circuits:

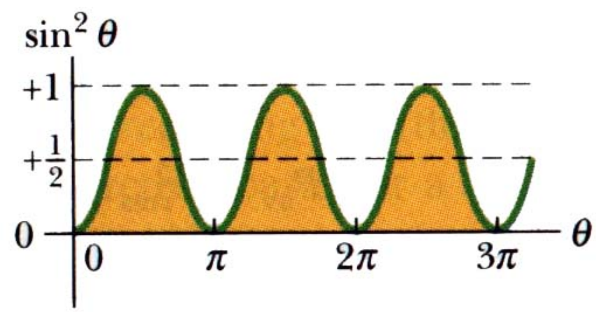
$$\text{Power} = i^2 R = i_m{}^2 R \sin^2(\omega_d t - \phi)$$



(a)

The average power dissipated in a resistor is given by the average of $\sin^2 x$, which equals 1/2.

$$\text{Average Power} = \frac{i_m{}^2 R}{2} = \left(\frac{i_m}{\sqrt{2}}\right)^2 R$$



(b)

$$P_{\text{average}} = i_{\text{RMS}}{}^2 R \qquad i_{RMS} = \frac{i_m}{\sqrt{2}}$$

Similar definition for root-mean-square voltages, emf˝ s, etc.
RMS definitions are commonly used in stereo systems.

Since i_{RMS} and i_{m} are proportional, $\quad i_{RMS} = \dfrac{E_{RMS}}{Z}$

We can rewrite the expression for the average power as,

$$P_{\text{average}} = i_{RMS}{}^{2}R = \frac{E_{RMS}}{Z} i_{RMS} R = E_{RMS}\, i_{RMS} \frac{R}{Z}$$

If Z = R, (circuit is purely resistive) then we get the usual expression.

$$\cos\varphi = \frac{R}{Z}$$

$$P_{\text{average}} = E_{RMS}\, i_{RMS} \cos\varphi$$

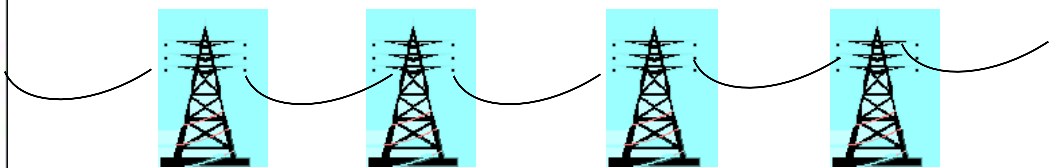
Purely resistive power, $\varphi = 0$

Maximize power supplied, maximize the cosine of phi. Power companies look at this at the time of figuring out bills for large consumers.

Transformers:

The problem

Tennessee Valley authority



Generator 735KV

Distance 1000 Km

Resistance 0.22Ω/Km

Current 500A

Baton Rouge

$$\text{Power generated} = 735{,}000 \times 500 = 368\text{MW}$$

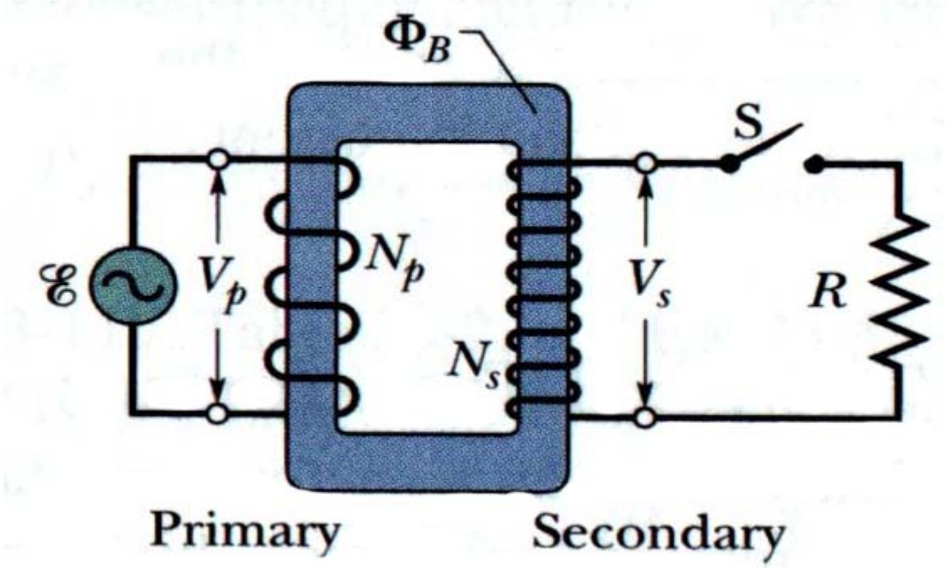
$$\text{Power lost in wires} = (500)^2 220 = 55MW$$ 15% of production

Proposal: double the voltage, cut current in half. This means you cut losses by a factor of four!

Bottomline: to distribute power efficiently you want the highest possible voltage. Yet, high voltages are very dangerous. What would you do at the point of entry at a home?

The solution: transformers

Two coils ("primary and secondary") sharing the same magnetic flux.



Lenz' law:

$$\frac{d\Phi}{dt} = \text{emf per turn} \qquad = \frac{V_P}{N_P} = \frac{V_S}{N_S} \qquad \boxed{V_S = \frac{N_S}{N_P} V_P}$$

You can get any voltage you wish just playing with the number of turns. For instance, the coil in the ignition system of a car goes from 12V to thousands of volts. Or the transformers in most consumer electronics go from 110V to 6 or 12 V.

Energy is conserved : $i_S = \frac{N_P}{N_S} i_P$

What you gain (lose) in voltage you lose (gain) in current.

A resistance of a given value connected to the secondary appears to the primary as a resistance of a different value.

Given: $i_S = \frac{N_P}{N_S} i_P$ and substituting $i_S = \frac{\mathrm{V}_S}{\mathrm{R}_S}$ and $\boxed{V_S = \frac{N_S}{N_P} V_P}$

Therefore: $i_p = \frac{1}{R}\left(\frac{N_P}{N_S}\right)^2 V_P$ and

$$R_{\mathrm{eq}} = \left(\frac{N_p}{N_s}\right)^2 R. \tag{31-82}$$

This R_{eq} is the value of the load resistance as "seen" by the generator; the generator produces the current I_p and voltage V_p as if the generator were connected to a resistance R_{eq}.

•64 Figure 31-36 shows an "autotransformer." It consists of a single coil (with an iron core). Three taps T_i are provided. Between taps T_1 and T_2 there are 200 turns, and between taps T_2 and T_3 there are 800 turns. Any two taps can be chosen as the primary terminals, and any two taps can be chosen as the secondary terminals. For choices producing a step-up transformer, what are the (a) smallest, (b) second smallest, and (c) largest values of the ratio V_s/V_p? For a step-down transformer, what are the (d) smallest, (e) second smallest, and (f) largest values of V_s/V_p?

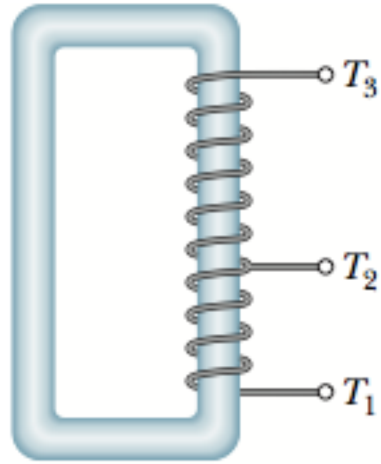


Fig. 31-36 Problem 64.

(a) The smallest value of the ratio V_s/V_p is achieved by using T_2T_3 as primary and T_1T_3 as secondary coil: $V_{13}/V_{23} = (800 + 200)/800 = 1.25$.

(b) The second smallest value of the ratio V_s/V_p is achieved by using T_1T_2 as primary and T_2T_3 as secondary coil: $V_{23}/V_{13} = 800/200 = 4.00$.

(c) The largest value of the ratio V_s/V_p is achieved by using T_1T_2 as primary and T_1T_3 as secondary coil: $V_{13}/V_{12} = (800 + 200)/200 = 5.00$.

For the step-down transformer, we simply exchange the primary and secondary coils in each of the three cases above.

(d) The smallest value of the ratio V_s / V_p is 1/5.00 = 0.200.

(e) The second smallest value of the ratio V_s / V_p is 1/4.00 = 0.250.

(f) The largest value of the ratio V_s / V_p is 1/1.25 = 0.800.

Transformer: turns ratio, average power, rms currents

A transformer on a utility pole operates at $V_p = 8.5$ kV on the primary side and supplies electrical energy to a number of nearby houses at $V_s = 120$ V, both quantities being rms values. Assume an ideal step-down transformer, a purely resistive load, and a power factor of unity.

(a) What is the turns ratio N_p/N_s of the transformer?

KEY IDEA

The turns ratio N_p/N_s is related to the (given) rms primary and secondary voltages via Eq. 31-79 ($V_s = V_p N_s/N_p$).

Calculation: We can write Eq. 31-79 as

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}. \qquad (31\text{-}83)$$

(Note that the right side of this equation is the *inverse* of the turns ratio.) Inverting both sides of Eq. 31-83 gives us

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{8.5 \times 10^3 \text{ V}}{120 \text{ V}} = 70.83 \approx 71. \qquad \text{(Answer)}$$

(b) The average rate of energy consumption (or dissipation) in the houses served by the transformer is 78 kW. What are the rms currents in the primary and secondary of the transformer?

KEY IDEA

For a purely resistive load, the power factor $\cos\phi$ is unity; thus, the average rate at which energy is supplied and dissipated is given by Eq. 31-77 ($P_{avg} = \mathscr{E}I = IV$).

Calculations: In the primary circuit, with $V_p = 8.5$ kV, Eq. 31-77 yields

$$I_p = \frac{P_{avg}}{V_p} = \frac{78 \times 10^3 \text{ W}}{8.5 \times 10^3 \text{ V}} = 9.176 \text{ A} \approx 9.2 \text{ A}. \qquad \text{(Answer)}$$

Similarly, in the secondary circuit,

$$I_s = \frac{P_{avg}}{V_s} = \frac{78 \times 10^3 \text{ W}}{120 \text{ V}} = 650 \text{ A}. \qquad \text{(Answer)}$$

You can check that $I_s = I_p(N_p/N_s)$ as required by Eq. 31-80.

(c) What is the resistive load R_s in the secondary circuit? What is the corresponding resistive load R_p in the primary circuit?

One way: We can use $V = IR$ to relate the resistive load to the rms voltage and current. For the secondary circuit, we find

$$R_s = \frac{V_s}{I_s} = \frac{120 \text{ V}}{650 \text{ A}} = 0.1846\ \Omega \approx 0.18\ \Omega. \qquad \text{(Answer)}$$

Similarly, for the primary circuit we find

$$R_p = \frac{V_p}{I_p} = \frac{8.5 \times 10^3 \text{ V}}{9.176 \text{ A}} = 926\ \Omega \approx 930\ \Omega. \qquad \text{(Answer)}$$

Second way: We use the fact that R_p equals the equivalent resistive load "seen" from the primary side of the transformer, which is a resistance modified by the turns ratio and given by Eq. 31-82 ($R_{eq} = (N_p/N_s)^2 R$). If we substitute R_p for R_{eq} and R_s for R, that equation yields

$$R_p = \left(\frac{N_p}{N_s}\right)^2 R_s = (70.83)^2(0.1846\ \Omega)$$
$$= 926\ \Omega \approx 930\ \Omega. \qquad \text{(Answer)}$$

Summary:

LC Energy Transfer

- In an oscillating LC circuit, instantaneous values of the two forms of energy are

$$U_E = \frac{q^2}{2C} \quad \text{and} \quad U_B = \frac{Li^2}{2}$$

LC Charge and Current Oscillations

- The principle of conservation of energy leads to

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

- The solution is

$$q = Q\cos(\omega t + \phi)$$

- the angular frequency v of the oscillations is

$$\omega = \frac{1}{\sqrt{LC}}.$$

Damped Oscillations

- Oscillations in an LC circuit are damped when a dissipative element R is also present in the circuit. Then

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

- The solution of this differential equation is

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi),$$

Alternating Currents; Forced Oscillations

- A series RLC circuit may be set into forced oscillation at a driving angular frequency by an external alternating emf $\mathscr{E} = \mathscr{E}_m \sin \omega_d t.$

- The current driven in the circuit is

$$i = I\sin(\omega_d t - \phi)$$

Summary:

Power

- In a series RLC circuit, the average power of the generator is,

$$P_{avg} = I_{rms}^2 R = \mathscr{E}_{rms} I_{rms} \cos \phi.$$

Transformers

- Primary and secondary voltage in a transformer is related by

$$V_s = V_p \frac{N_s}{N_p}$$

- The currents through the coils,

$$I_s = I_p \frac{N_p}{N_s}$$

- The equivalent resistance of the secondary circuit, as seen by the generator, is

$$R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R,$$