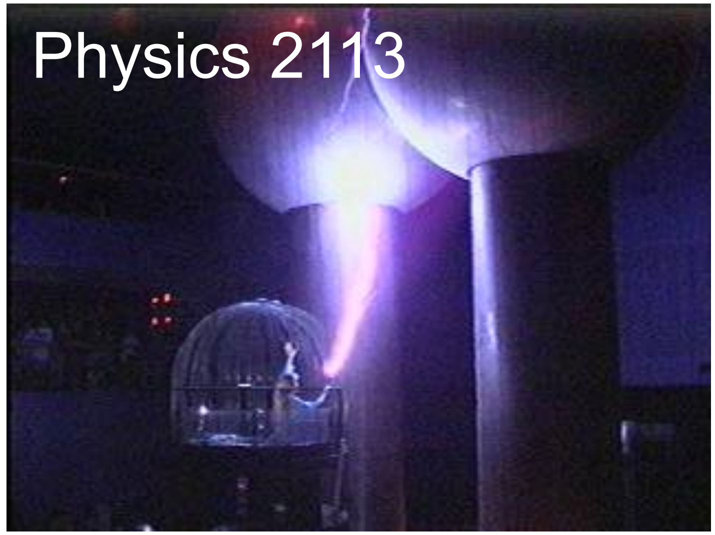
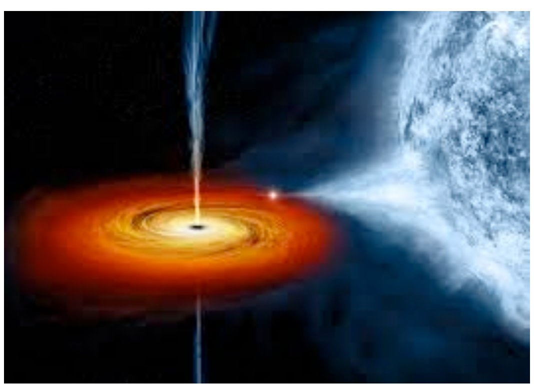



# Physics 2113
# Lecture 31: MON 10 NOV

## CH31: Oscillations and AC



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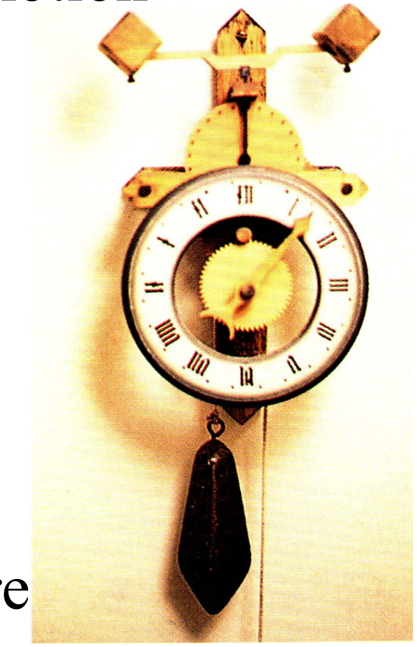
31-4 *LC* Oscillations, Quantitatively 830

In previous courses of physics, you have seen the concept of a mechanical oscillator. These are systems such that the motion returns periodically to the same point.



Oscillators are very useful in practical applications, for instance, to keep time, or to focus energy in a system.

All oscillators operate along the same principle: they are systems that can store energy in more than one way and exchange it back and forth between the different storage possibilities

For instance, in pendula one exchanges energy between kinetic and potential form.

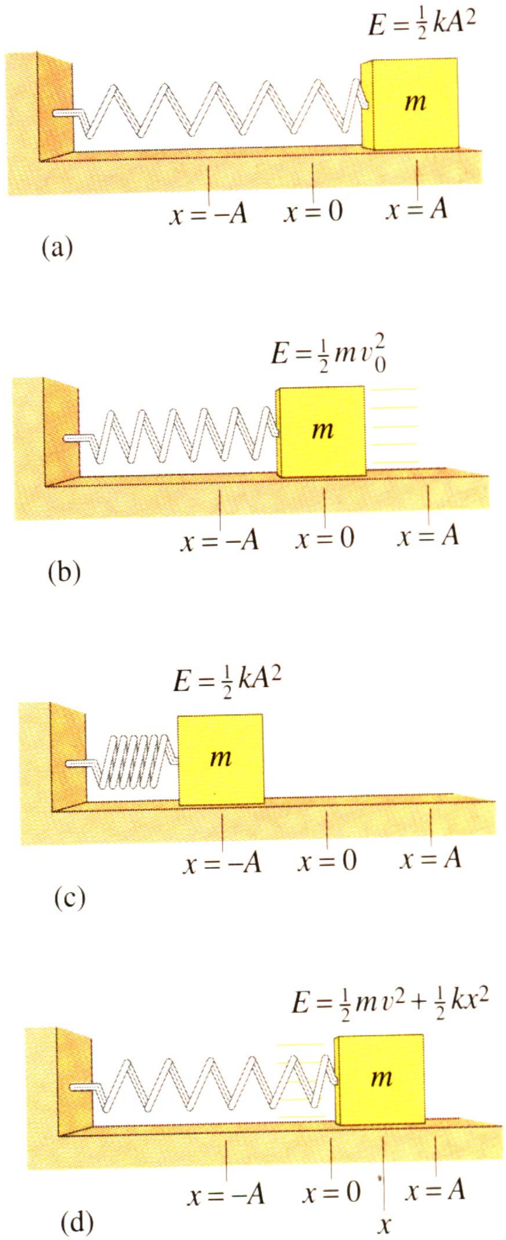
In this course we have studied that coils and capacitors are devices that can store electromagnetic energy. In one case it is stored in a magnetic field, in the other in an electric field.

It is therefore not surprising that one can construct electrical oscillators using coils and capacitors.

Electrical oscillators are crucial to technology. Radios, TVs and all communications operate on the basis of waves produced by oscillators. So do most electronic clocks, including those that pace the operations of all computers.

The math describing electrical oscillators is exactly the same as the math describing mechanical oscillators, so we will start by drawing an analogy with them.

# Mechanical oscillator: a spring with a mass



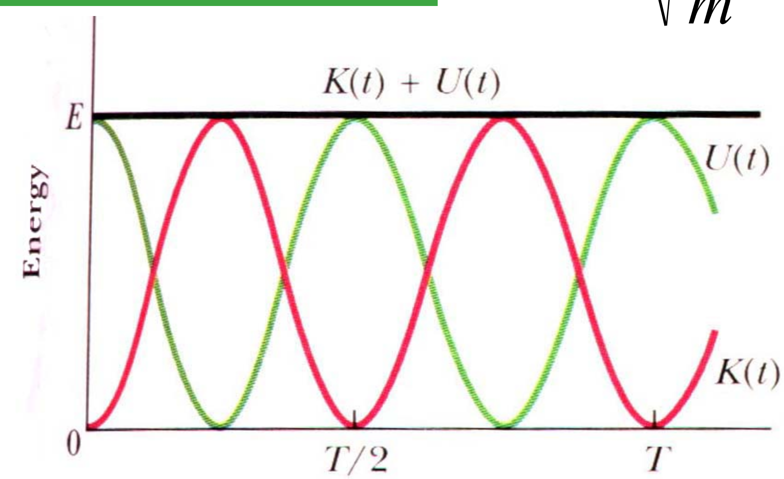

$$E_{tot} = E_{kin} + E_{pot} \qquad E_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\frac{dE_{tot}}{dt} = 0 = \frac{1}{2}m\left(2v\frac{dv}{dt}\right) + \frac{1}{2}k\left(2x\frac{dx}{dt}\right) \qquad v = \frac{dx}{dt}$$

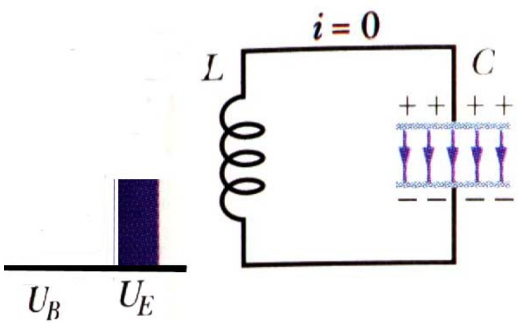
$$m\frac{dv}{dt} + kx = 0 \qquad \boxed{m\frac{d^2x}{dt^2} + kx = 0}$$

$$\boxed{\text{Solution:} \quad x(t) = x_0\cos(\omega t + \phi_0)} \qquad \omega = \sqrt{\frac{k}{m}}$$

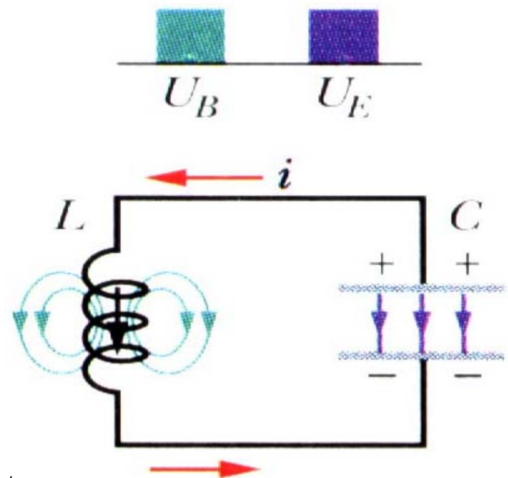
| | |
|---|---|
| $x_0$ : | amplitude |
| $\omega$ : | frequency |
| $\phi_0$ : | phase |



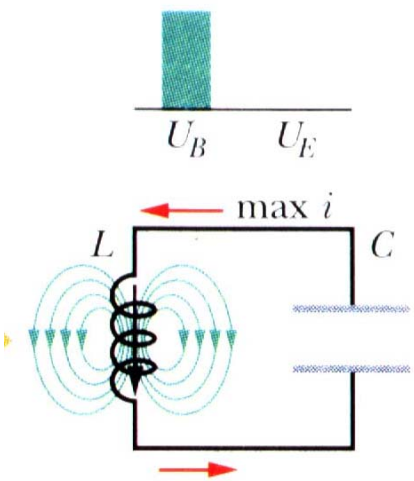
# Electrical oscillator: a coil and a capacitor



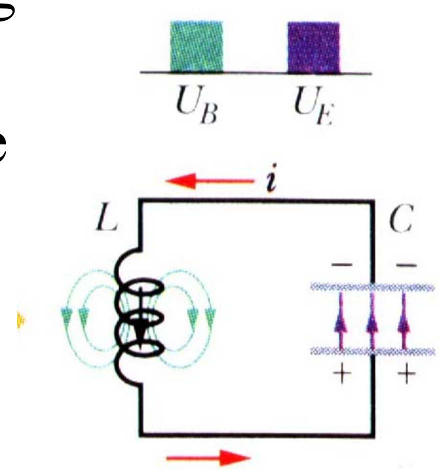

Capacitor initially charged, we hook it up. Initially, current is zero (coil opposes emf), energy is all stored in the capacitor.




A current gets going, energy gets split between the capacitor and the coil.

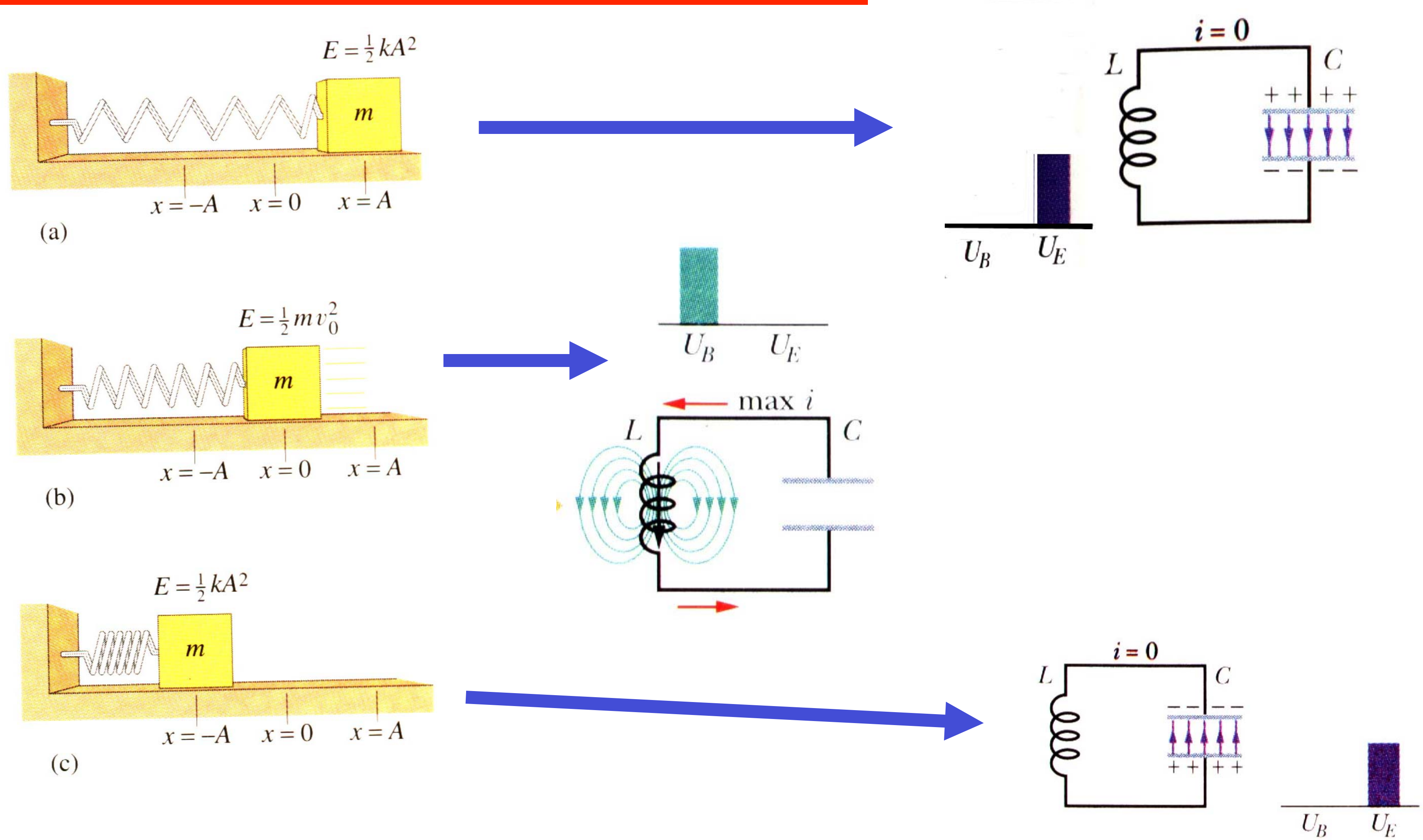



Capacitor discharges completely, yet current keeps going, and will keep going the magnetic field on the coil starts to collapse, which will start to recharge the capacitor (with opposite polarity).

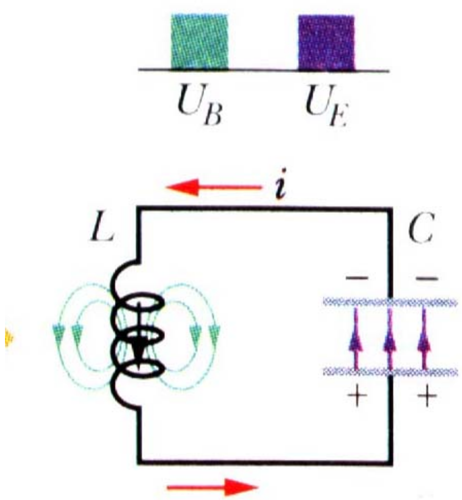



Finally, we reach the same state we started with (with opposite polarity) and the cycle restarts.

# Electric and mechanical parallels: energy



# Electric oscillations: math




$$E_{tot} = E_{mag} + E_{elec} \qquad E_{tot} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$$

$$\frac{dE_{tot}}{dt} = 0 = \frac{1}{2}L\left(2i\frac{di}{dt}\right) + \frac{1}{2C}\left(2q\frac{dq}{dt}\right) \qquad i = \frac{dq}{dt}$$

$$0 = L\left(i\frac{di}{dt}\right) + \frac{1}{C}(qi)$$

$$\boxed{0 = L\frac{d^2q}{dt^2} + \frac{q}{C}}$$ Compare with: $$\boxed{M\frac{d^2x}{dt^2} + kx = 0}$$

So the math is exactly the same as in the case of mechanical oscillators if one makes the substitutions:

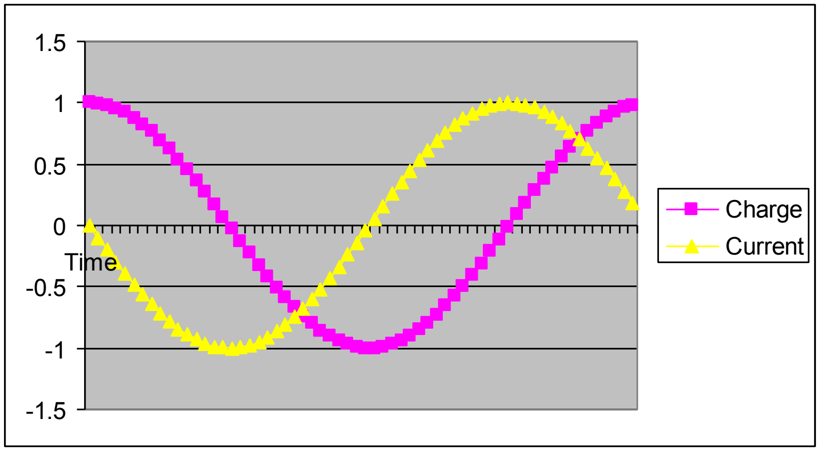
$$q \rightarrow x \qquad 1/C \rightarrow k$$

$$i \rightarrow v \qquad L \rightarrow M$$

$$q = q_0 \cos(\omega t + \phi_0)$$

$$\boxed{\omega = \sqrt{\frac{1}{LC}}}$$

# Electric oscillations: graphs and energy

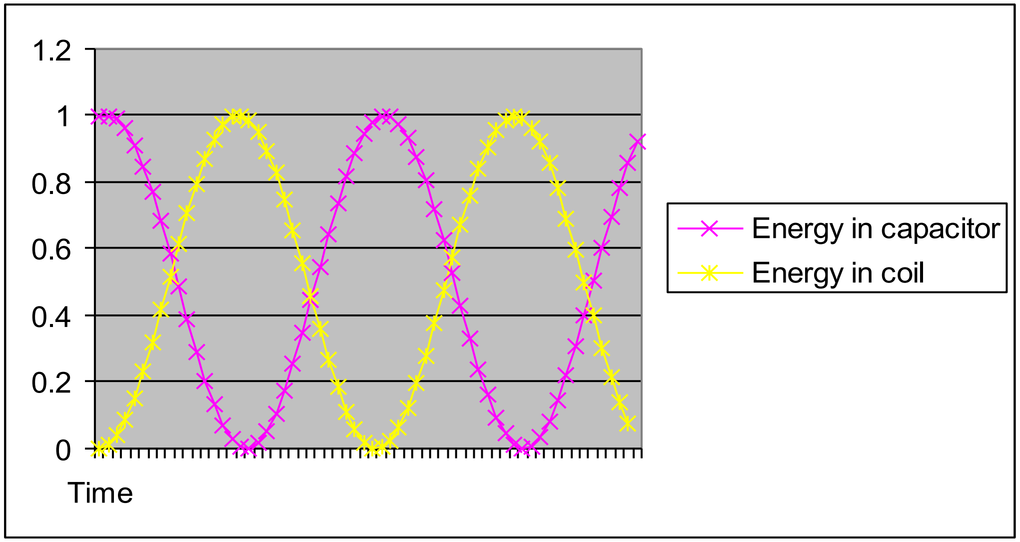



$$q = q_0 \cos(\omega t + \phi_0)$$

$$i = -\omega q_0 \sin(\omega t + \phi_0)$$

$$E_{mag} = \frac{1}{2} L i^2 = \frac{1}{2} L \omega^2 q_0^{\ 2} \sin^2(\omega t + \phi_0)$$

$$E_{ele} = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2C} q_0^{\ 2} \cos^2(\omega t + \phi_0)$$



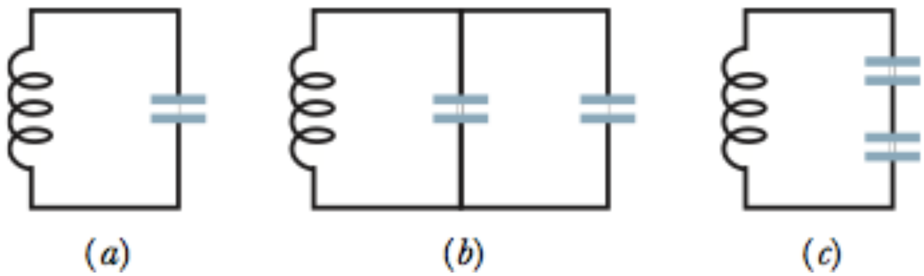

And remembering that,

$$\cos^2 x + \sin^2 x = 1, \text{ and } \omega = \sqrt{\frac{1}{LC}}$$

$$\boxed{E_{tot} = E_{mag} + E_{ele} = \frac{1}{2C} q_0^{\ 2}}$$

The energy is constant and equal to what we started with.

**1** Figure 31-19 shows three oscillating *LC* circuits with identical inductors and capacitors. Rank the circuits according to the time taken to fully discharge the capacitors during the oscillations, greatest first.




$$\omega = \sqrt{\frac{1}{LC}}, \qquad T = 2\pi\sqrt{LC}$$

**•3** In a certain oscillating $LC$ circuit, the total energy is converted from electrical energy in the capacitor to magnetic energy in the inductor in 1.50 $\mu$s. What are (a) the period of oscillation and (b) the frequency of oscillation? (c) How long after the magnetic energy is a maximum will it be a maximum again?

3. (a) The period is $T = 4(1.50\ \mu\text{s}) = 6.00\ \mu\text{s}$.

(b) The frequency is the reciprocal of the period: $f = \dfrac{1}{T} = \dfrac{1}{6.00\,\mu\text{s}} = 1.67 \times 10^5$ Hz.

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or 3.00 $\mu$s.

••13 In an oscillating $LC$ circuit, $L = 3.00$ mH and $C = 2.70$ $\mu$F. At $t = 0$ the charge on the capacitor is zero and the current is 2.00 A. (a) What is the maximum charge that will appear on the capacitor? (b) At what earliest time $t > 0$ is the rate at which energy is stored in the capacitor greatest, and (c) what is that greatest rate?

13. (a) The charge (as a function of time) is given by $q = Q\sin\omega t$, where $Q$ is the maximum charge on the capacitor and $\omega$ is the angular frequency of oscillation. A sine function was chosen so that $q = 0$ at time $t = 0$. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q\cos\omega t,$$

and at $t = 0$, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC}$,

$$Q = I\sqrt{LC} = (2.00\,\mathrm{A})\sqrt{(3.00\times10^{-3}\,\mathrm{H})(2.70\times10^{-6}\,\mathrm{F})} = 1.80\times10^{-4}\,\mathrm{C}.$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2 \sin^2 \omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2\omega \sin\omega t \cos\omega t}{C}$$

We use the trigonometric identity $\cos\omega t \sin\omega t = \frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C}\sin(2\omega t).$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2$ rad. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} = \frac{\pi}{4}\sqrt{(3.00\times10^{-3}\,\mathrm{H})(2.70\times10^{-6}\,\mathrm{F})} = 7.07\times10^{-5}\,\mathrm{s}.$$

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}.$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times10^{-3}\ \text{H})(2.70\times10^{-6}\ \text{F})} = 5.655\times10^{-4}\,\text{s}$, so

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{\pi\left(1.80\times10^{-4}\ \text{C}\right)^2}{\left(5.655\times10^{-4}\ \text{s}\right)\left(2.70\times10^{-6}\ \text{F}\right)} = 66.7\ \text{W}.$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

# Summary:

- One can build electrical oscillators by coupling coils and capacitors. Both components exchange electrical energy back and forth very much like mechanical oscillators do between kinetic and potential.
- Current and charge are out of phase, energy is constant.
- In real life oscillations die off due to dissipation, and one needs to drive the oscillator to keep it going. Next.