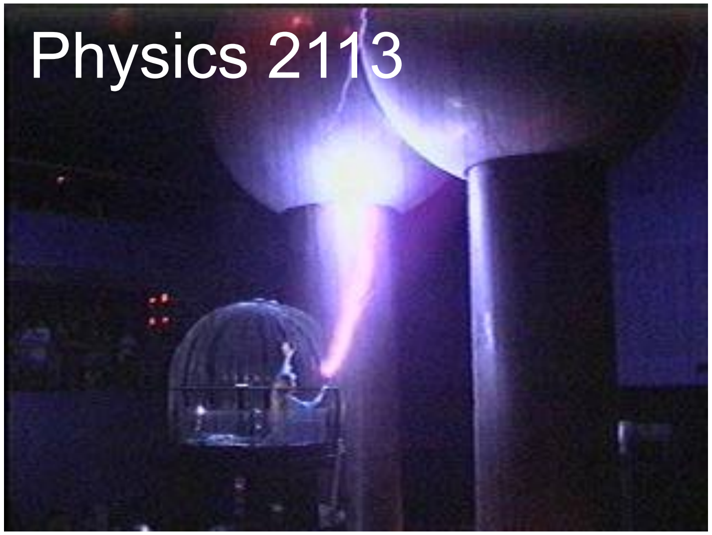
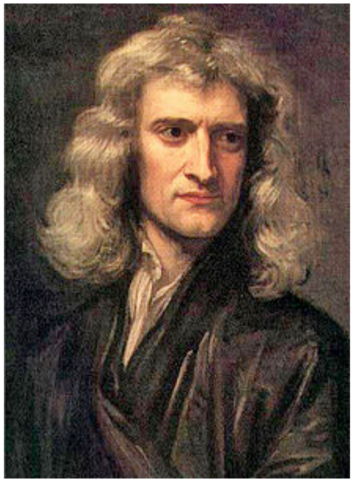




Isaac Newton
(1642–1727)

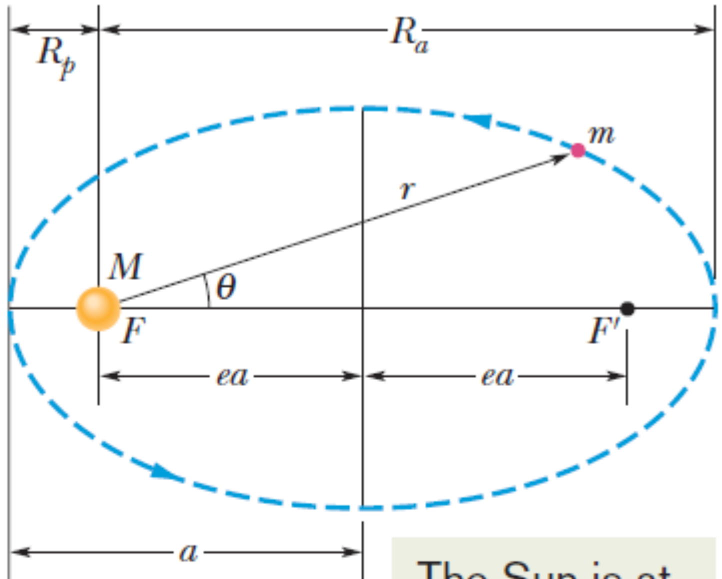
Physics 2113
Lecture 03: FRI 29 AUG

CH13: Gravitation III

Michael Faraday
(1791–1867)

13.7: Planets and Satellites: Kepler's 1st Law

1. THE LAW OF ORBITS: All planets move in elliptical orbits, with the Sun at one focus.



The Sun is at one of the two focal points.

Fig. 13-12 A planet of mass m moving in an elliptical orbit around the Sun. The Sun, of mass M, is at one focus F of the ellipse. The other focus is F', which is located in empty space. Each focus is a distance ea from the ellipse's center, with e being the eccentricity of the ellipse. The semimajor axis a of the ellipse, the perihelion (nearest the Sun) distance R_p, and the aphelion (farthest from the Sun) distance R_a are also shown.

Laws were based on data fits!

Tycho Brahe
1546–1601

Johannes Kepler
1571–1630

13.7: Planets and Satellites: Kepler's 2nd Law

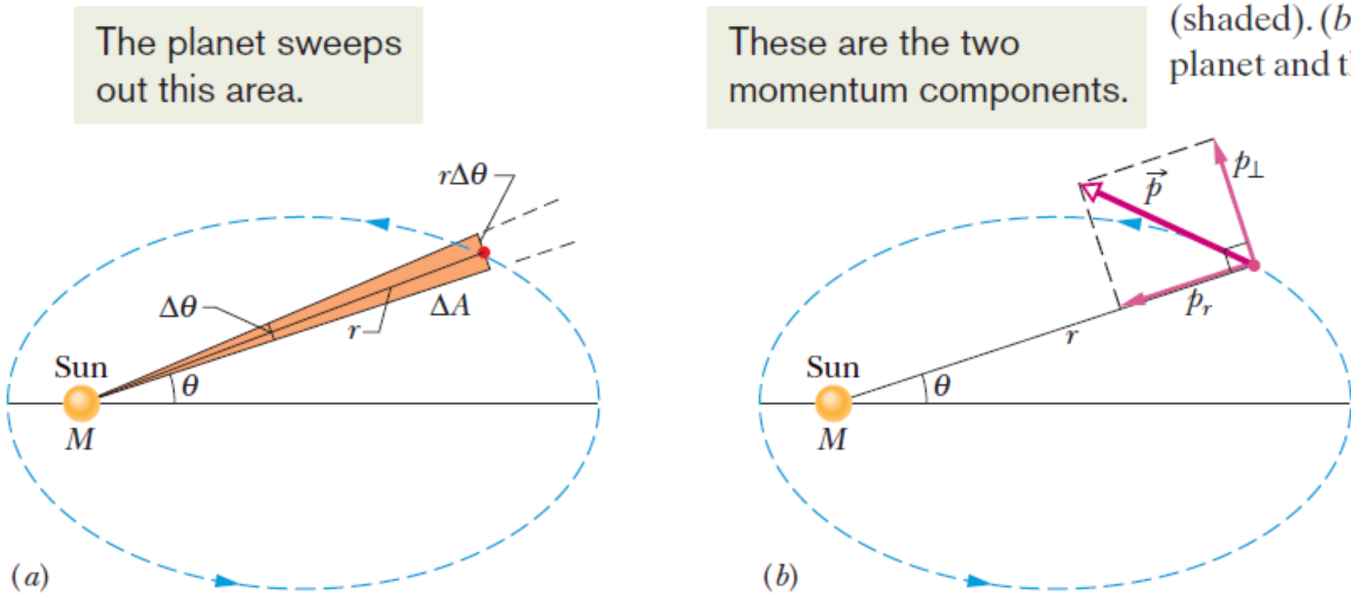


Fig. 13-13 (*a*) In time Δt, the line r connecting the planet to the Sun moves through an angle $\Delta\theta$, sweeping out an area ΔA (shaded). (*b*) The linear momentum $\vec{p}$ of the planet and the components of $\vec{p}$.

2. THE LAW OF AREAS:
A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate *dA/dt* at which it sweeps out area A is constant.

$$\Rightarrow A \propto t$$

$$\frac{dA}{dt} = \tfrac{1}{2} r^2 \frac{d\theta}{dt} = \tfrac{1}{2} r^2 \omega,$$

Angular momentum, L:

$$L = rp_\perp = (r)(mv_\perp) = (r)(m\omega r) = mr^2\omega,$$

$$\frac{dA}{dt} = \frac{L}{2m}.$$

13.7: Planets and Satellites: Kepler's 3rd Law

3. THE LAW OF PERIODS: The square of the period of any planet is proportional to the cube of the semi-major axis of its orbit.

Consider a *circular orbit* with radius *r* (the radius of a circle is equivalent to the semimajor axis of an ellipse). Applying Newton's second law to the orbiting planet yields

$$\boxed{\frac{GMm}{r^2}} = (m)(\omega^2 r).$$

$$T = \frac{2\pi}{\omega}$$

Using the relation of the angular velocity, ω, and the period, T, one gets:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad \text{(law of periods).}$$

Table 13-3

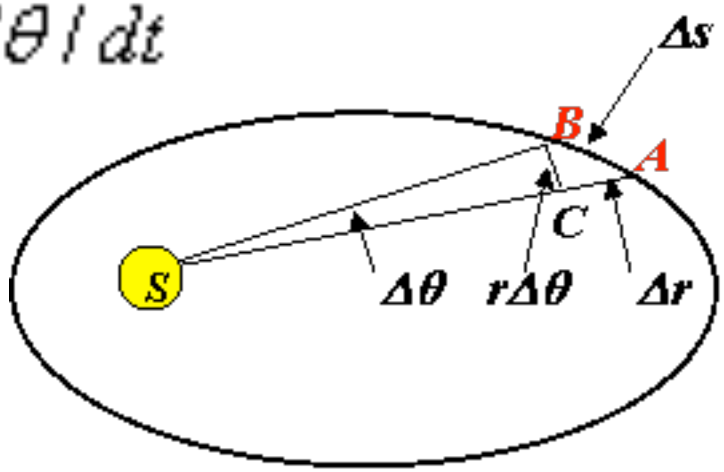
Kepler's Law of Periods for the Solar System

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

13.7: Newton Derived Kepler's Laws from Inverse Square Law!

http://galileo.phys.virginia.edu/classes/152.mf1i.spring02/KeplersLaws.htm

$$v_r = dr/dt, \quad v_\perp = r d\theta/dt = r\omega \qquad \omega = d\theta/dt$$



Kepler's Second Law First: Equal Areas Proportional to Equal Time!

$$\text{Area swept out in small } \Delta t \approx \tfrac{1}{2}r^2\Delta\theta = \tfrac{1}{2}r^2\frac{\Delta\theta}{\Delta t}\Delta t = \tfrac{1}{2}r^2\omega\Delta t$$

$L = mrv_\perp = mr^2\omega$ Angular Momentum

Rate of sweeping out of area,

$$dA/dt = c$$

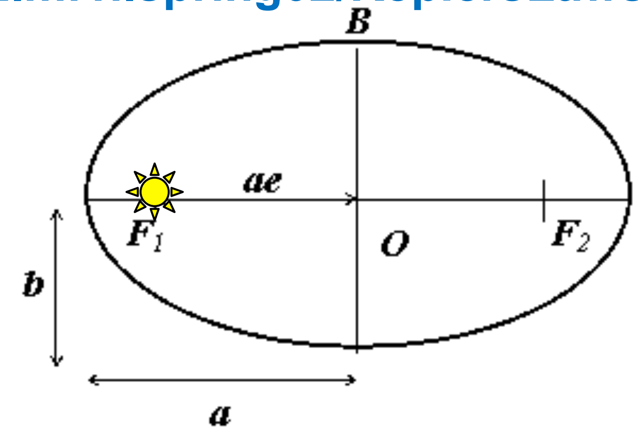
is proportional to the angular momentum L, and equal to $L/2m$ = Constant = C.

$$\Rightarrow A \propto t$$

13.7: Newton Derived Kepler's Laws from Inverse Square Law!

http://galileo.phys.virginia.edu/classes/152.mf1i.spring02/KeplersLaws.htm

Kepler's First Law: Ellipse with Sun at Focus



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{a\left(1-e^2\right)}{r}=1+e\cos\theta$$

$$\frac{d^2r}{dt^2}-r\omega^2=-\frac{GM}{r^2}$$

$$\frac{d^2r}{dt^2}=-\frac{GM}{r^2}+r\left(\frac{L}{mr^2}\right)^2$$
$$=-\frac{GM}{r^2}+\frac{L^2}{mr^3}$$

$$L=mr^2\omega=\frac{m}{u^2}\frac{d\theta}{dt}$$

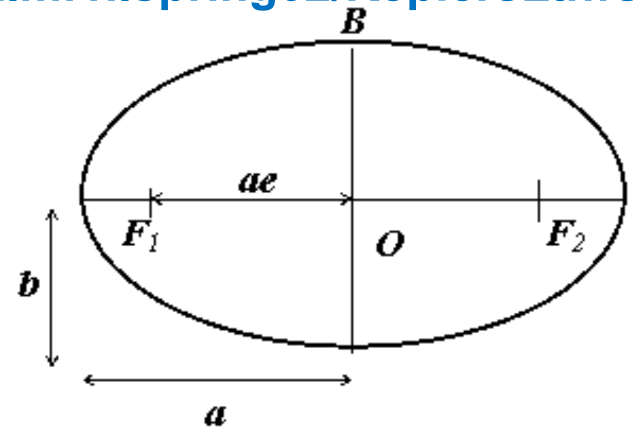
$$\frac{d^2u}{d\theta^2}+u=\frac{GMm^2}{L^2}$$

$$u=\frac{1}{r}=\frac{GMm^2}{L^2}+A\cos\theta$$

This is equivalent to the standard *(r, q)* equation of an ellipse of semi-major axis *a* and eccentricity *e*, with the origin — the Sun — at one focus. Note *1/L²* is from inverse Square Law.

13.7: Newton Derived Kepler's Laws from Inverse Square Law!

http://galileo.phys.virginia.edu/classes/152.mf1i.spring02/KeplersLaws.htm



Kepler's 3rd Law:
For Ellipse

$$T = \frac{\pi ab}{L/2m}$$

$$\frac{1}{r} = \frac{GMm^2}{L^2} + A\cos\theta$$

$$\frac{a(1-e^2)}{r} = 1 + e\cos\theta$$

$$L^2 / GMm^2 = a(1-e^2)$$

$$b^2 = a^2(1-e^2)$$

$$\begin{aligned} T^2 &= (2m\pi ab)^2 / L^2 \\ &= (2m\pi ab)^2 / GMm^2 a(1-e^2) \\ &= (2m\pi ab)^2 / GMm^2 a(b^2/a^2) \\ &= 4\pi^2 a^3 / GM. \end{aligned}$$

$$T^2 \propto a^3$$

Example, Halley's Comet

Comet Halley orbits the Sun with a period of 76 years and, in 1986, had a distance of closest approach to the Sun, its *perihelion distance* R_p, of 8.9×10^{10} m. Table 13-3 shows that this is between the orbits of Mercury and Venus.

(a) What is the comet's farthest distance from the Sun, which is called its *aphelion distance* R_a?

KEY IDEAS

From Fig. 13-12, we see that $R_a + R_p = 2a$, where a is the semimajor axis of the orbit. Thus, we can find R_a if we first find a. We can relate a to the given period via the law of periods (Eq. 13-34) if we simply substitute the semimajor axis a for r.

Calculations: Making that substitution and then solving for a, we have

$$a = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}. \tag{13-35}$$

If we substitute the mass M of the Sun, 1.99×10^{30} kg, and the period T of the comet, 76 years or 2.4×10^9 s, into Eq. 13-35, we find that $a = 2.7 \times 10^{12}$ m. Now we have

$$\begin{aligned} R_a &= 2a - R_p \\ &= (2)(2.7 \times 10^{12}\text{ m}) - 8.9 \times 10^{10}\text{ m} \\ &= 5.3 \times 10^{12}\text{ m.} \end{aligned} \qquad \text{(Answer)}$$

Table 13-3 shows that this is a little less than the semimajor axis of the orbit of Pluto. Thus, the comet does not get farther from the Sun than Pluto.

(b) What is the eccentricity e of the orbit of comet Halley?

KEY IDEA

We can relate e, a, and R_p via Fig. 13-12, in which we see that $ea = a - R_p$.

Calculation: We have

$$e = \frac{a - R_p}{a} = 1 - \frac{R_p}{a} \tag{13-36}$$

$$= 1 - \frac{8.9 \times 10^{10}\text{ m}}{2.7 \times 10^{12}\text{ m}} = 0.97. \qquad \text{(Answer)}$$

This tells us that, with an eccentricity approaching unity, this orbit must be a long thin ellipse.

ICPP: Estimate comet's speed at farthest distance?

$$v = v_\perp = \omega r$$

$$\omega = 2\pi / T$$

$$v = \frac{2\pi r}{T} \cong \frac{6 \times 9 \times 10^{10}\,\text{m}}{76\text{y} \times 3 \times 10^7\,\text{s/y}} \cong \frac{10^{12}\,\text{m}}{10^9\,\text{s}} \cong 1{,}000\,\text{m/s}$$

13.8: Satellites: Orbits and Energy

As a satellite orbits Earth in an elliptical path, the mechanical energy E of the satellite remains constant. Assume that the satellite's mass is so much smaller than Earth's mass.

The potential energy of the system is given by

$$U = -\frac{GMm}{r}$$

For a satellite in a circular orbit,

$$\frac{GMm}{r^2} = m\,\frac{v^2}{r},$$

Thus, one gets:

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r} \qquad \text{(circular orbit).}$$

For an elliptical orbit (semimajor axis a),

$$\boxed{E = -\frac{GMm}{2a}}$$

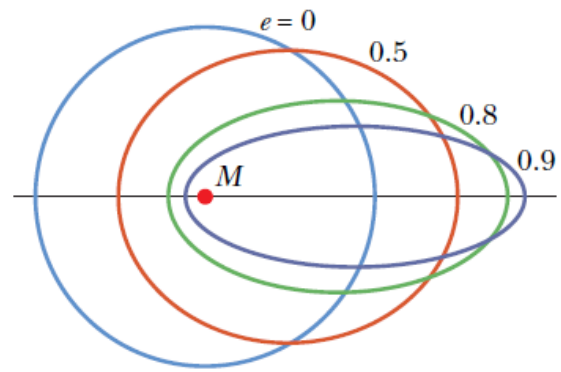
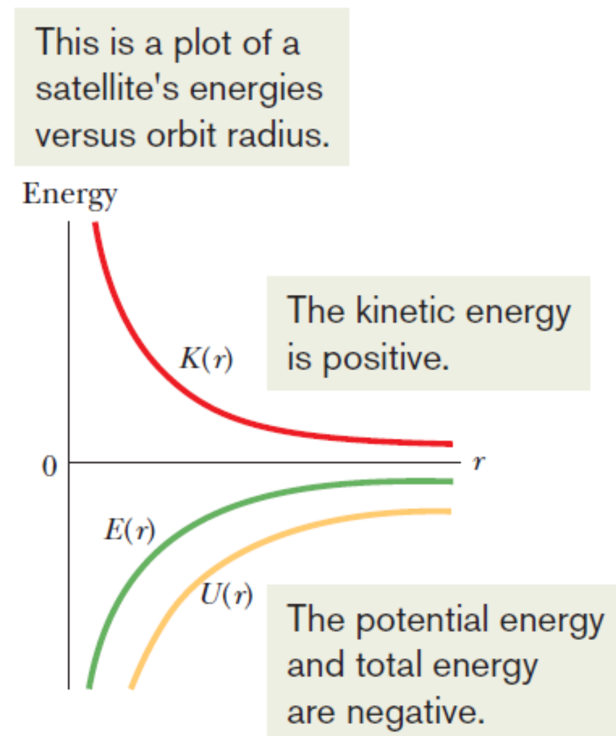


Fig. 13-15 Four orbits with different eccentricities e about an object of mass M. All four orbits have the same semimajor axis a and thus correspond to the same total mechanical energy E.



Example, Mechanical Energy of a Bowling Ball

A playful astronaut releases a bowling ball, of mass $m = 7.20$ kg, into circular orbit about Earth at an altitude h of 350 km.

(a) What is the mechanical energy E of the ball in its orbit?

KEY IDEA

We can get E from the orbital energy, given by Eq. 13-40 ($E = -GMm/2r$), if we first find the orbital radius r. (It is *not* simply the given altitude.)

Calculations: The orbital radius must be

$$r = R + h = 6370 \text{ km} + 350 \text{ km} = 6.72 \times 10^6 \text{ m},$$

in which R is the radius of Earth. Then, from Eq. 13-40, the mechanical energy is

$$\begin{aligned} E &= -\frac{GMm}{2r} \\ &= -\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(7.20\ \text{kg})}{(2)(6.72 \times 10^6\ \text{m})} \\ &= -2.14 \times 10^8\ \text{J} = -214\ \text{MJ}. \qquad \text{(Answer)} \end{aligned}$$

(b) What is the mechanical energy E_0 of the ball on the launchpad at Cape Canaveral (before it, the astronaut, and the spacecraft are launched)? From there to the orbit, what is the change ΔE in the ball's mechanical energy?

KEY IDEA

On the launchpad, the ball is *not* in orbit and thus Eq. 13-40 does *not* apply. Instead, we must find $E_0 = K_0 + U_0$, where K_0 is the ball's kinetic energy and U_0 is the gravitational potential energy of the ball–Earth system.

Calculations: To find U_0, we use Eq. 13-21 to write

$$\begin{aligned} U_0 &= -\frac{GMm}{R} \\ &= -\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(7.20\ \text{kg})}{6.37 \times 10^6\ \text{m}} \\ &= -4.51 \times 10^8\ \text{J} = -451\ \text{MJ}. \end{aligned}$$

The kinetic energy K_0 of the ball is due to the ball's motion with Earth's rotation. You can show that K_0 is less than 1 MJ, which is negligible relative to U_0. Thus, the mechanical energy of the ball on the launchpad is

$$E_0 = K_0 + U_0 \approx 0 - 451\ \text{MJ} = -451\ \text{MJ}. \qquad \text{(Answer)}$$

The *increase* in the mechanical energy of the ball from launchpad to orbit is

$$\begin{aligned} \Delta E &= E - E_0 = (-214\ \text{MJ}) - (-451\ \text{MJ}) \\ &= 237\ \text{MJ}. \qquad \text{(Answer)} \end{aligned}$$

This is worth a few dollars at your utility company. Obviously the high cost of placing objects into orbit is not due to their required mechanical energy.

13.9: Einstein and Gravitation

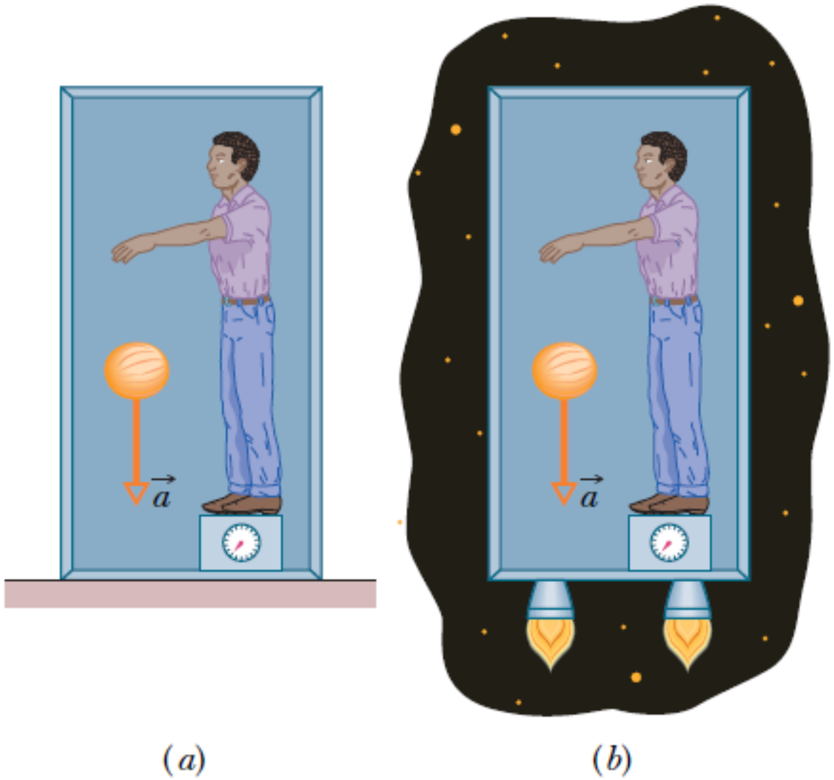


Fig. 13-17 (*a*) A physicist in a box resting on Earth sees a cantaloupe falling with acceleration $a = 9.8\ m/s^2$. (*b*) If he and the box accelerate in deep space at $9.8\ m/s^2$, the cantaloupe has the same acceleration relative to him. It is not possible, by doing experiments within the box, for the physicist to tell which situation he is in. For example, the platform scale on which he stands reads the same weight in both situations.

The fundamental postulate of Einstein's general theory of relativity about gravitation (the gravitating of objects toward each other) is called the **principle of equivalence,** which says that gravitation and acceleration are equivalent.

13.9: Einstein and Gravitation: Curvature of Space

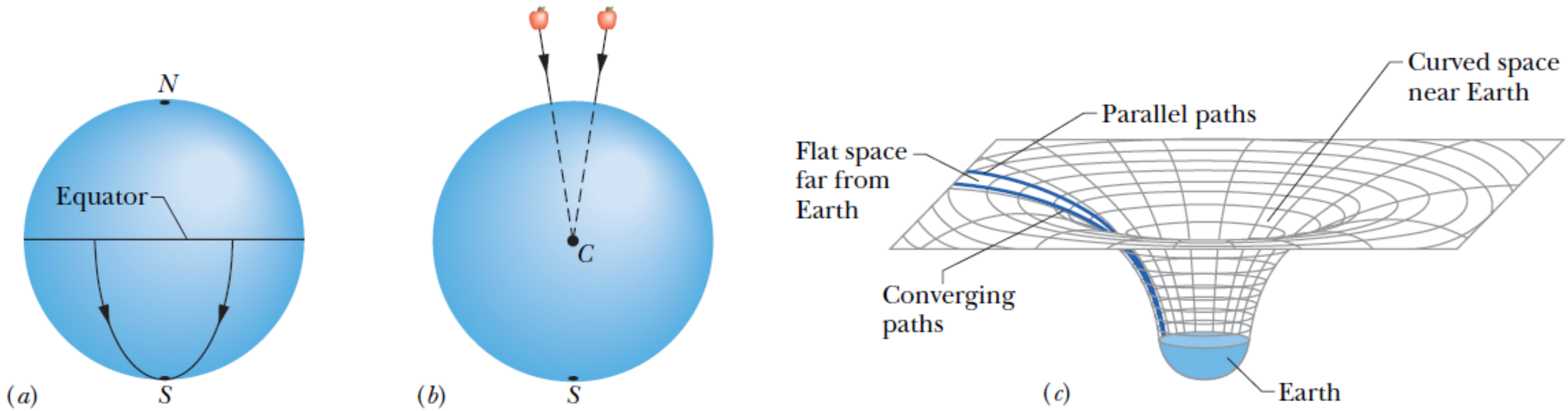
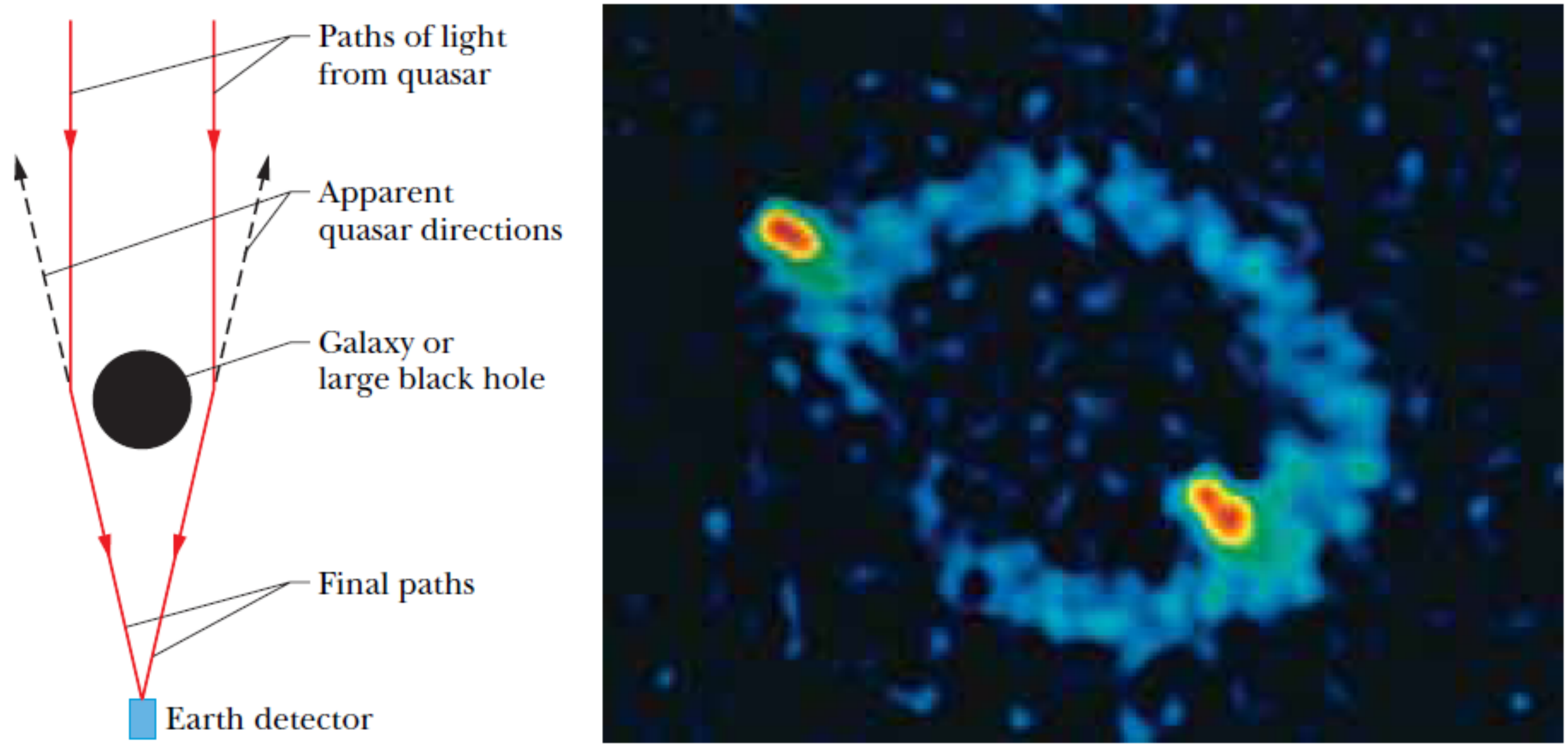


Fig. 13-18 (*a*) Two objects moving along lines of longitude toward the south pole converge because of the curvature of Earth's surface. (*b*) Two objects falling freely near Earth move along lines that converge toward the center of Earth because of the curvature of space near Earth. (*c*) Far from Earth (and other masses), space is flat and parallel paths remain parallel. Close to Earth, the parallel paths begin to converge because space is curved by Earth's mass.

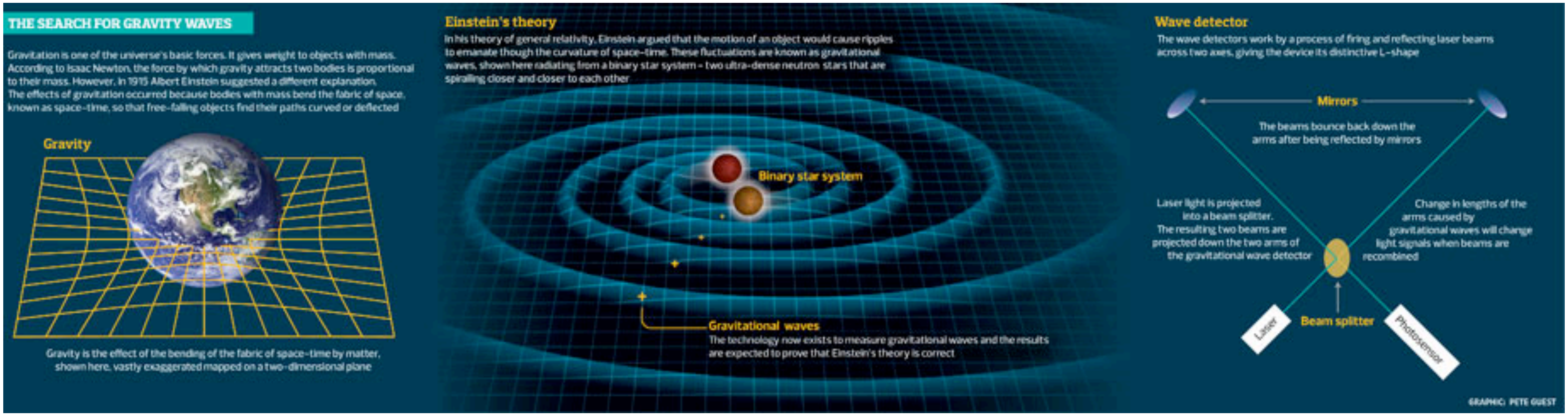
13.9: Einstein and Gravitation: Curvature of Space



(*a*) (*b*)

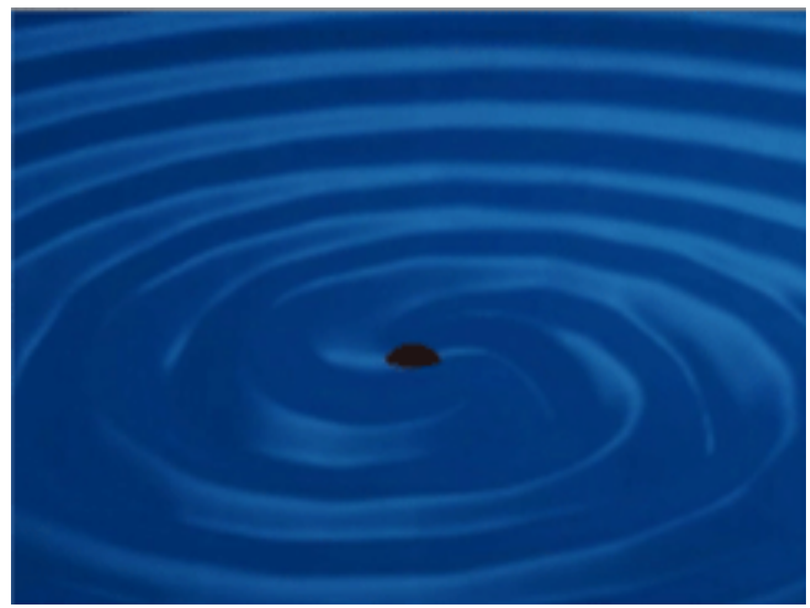
Fig. 13-19 (*a*) Light from a distant quasar follows curved paths around a galaxy or a large black hole because the mass of the galaxy or black hole has curved the adjacent space. If the light is detected, it appears to have originated along the backward extensions of the final paths (dashed lines). (*b*) The Einstein ring known as MG1131+0456 on the computer screen of a telescope. The source of the light (actually, radio waves, which are a form of invisible light) is far behind the large, unseen galaxy that produces the ring; a portion of the source appears as the two bright spots seen along the ring. (*Courtesy National Radio Astronomy Observatory*)

13.9: Einstein and Gravitation: Gravity Waves



LIVINGSTON LASER INTERFEROMETER GRAVITATIONAL-WAVE OBSERVATORY

Disturbances in the Gravitational Field Move Outward As Waves



Two Orbiting Black Holes

Summary:

- Kepler’s laws.
- They can be derived from Newton’s law.
- Einstein’s theory is the modern description of gravity. Gravity is not a force but a deformation of space-time.