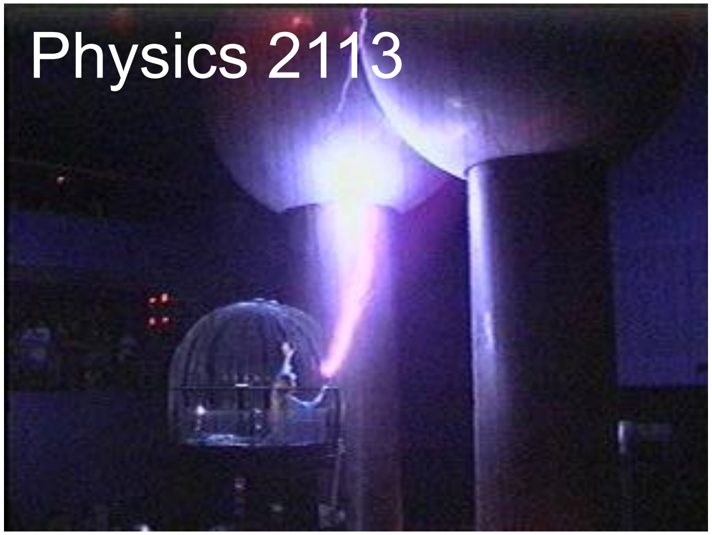
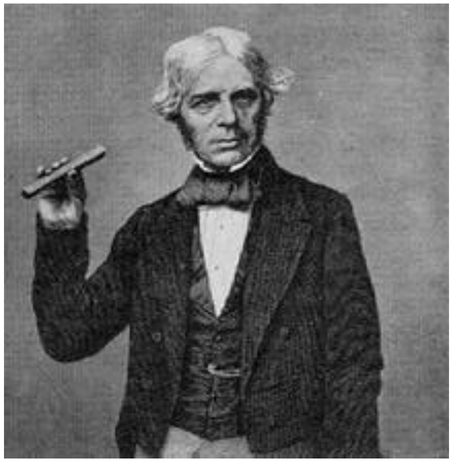
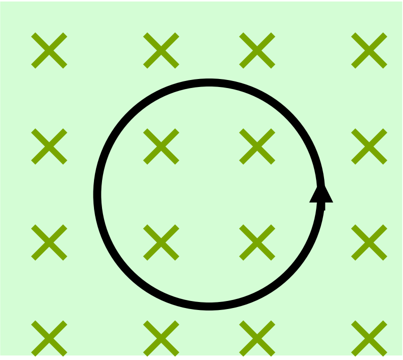


Michael Faraday 1791-1867

# Physics 2113
# Lecture 29: WED 5 NOV

## CH30: Induction and Inductance



We considered a region with a changing magnetic field and assume a charge moves through it.

There is an electric field in the region, and therefore the charge does work moving through it.

The work done in one revolution is $q\ E_0$ where $E_0$ is the emf that is generated ("induced emf") by the time-dependent magnetic field.

$$W = \oint \vec{F} \cdot d\vec{s} = q \oint \vec{E} \cdot d\vec{s} \qquad \text{or, } E_0 = \oint \vec{E} \cdot d\vec{s}$$

We can therefore rewrite Faraday's law of induction as,

$$\boxed{\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

We therefore by now have learnt that electric fields can be created in two different ways: a) by electric charges, b) by changing magnetic fields. Field lines behave similarly in both cases: they always end at charges or at infinity.

There is a difference in the kind of electric field we get in both cases. In the case of fields produced statically by charges, we introduced the notion of electric potential, $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

Which in turn meant that if one went around a closed loop,

$$\boxed{\oint \vec{E} \cdot d\vec{s} = 0}$$

Conservative

Doesn't this contradict?

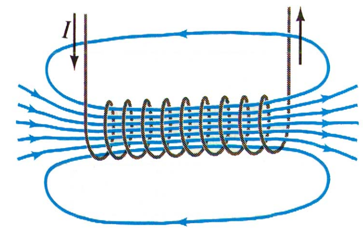
$$\boxed{\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

It does. It means that the concept of electric potential is only well defined for ***electrostatic*** fields (fields produced by charges), and not for electric fields produced by changing magnetic fields.

Is energy conserved? It is, but the discussion of energy conservation is more complex if there are time-dependent fields.

## Inductors

Are with respect to the magnetic field what capacitors are with respect to the electric field. They "pack a lot of field in a small region"



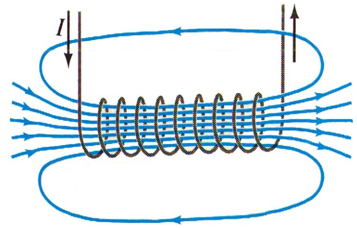
Capacitance -> how much potential for a given charge.
Inductance -> how much magnetic flux for a given current.

$$\text{For a solenoid,}\quad L = \frac{N\Phi}{i}$$

$$\text{Units}:\quad [L] = \frac{\text{Tesla}\cdot\text{m}^2}{\text{Ampere}} \equiv \text{H}\ \ (\text{Henry})$$

Joseph Henry
(1799-1878)

For a solenoid, $L = \dfrac{N\Phi}{i}$




If it has cross sectional area $A$,
and $n$ turns per unit length and length $l$,

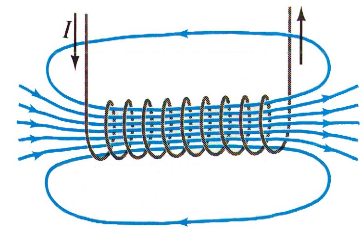
$$\Phi = BA, \qquad N = nl$$

Also, $B = \mu_0 i n$

Therefore: $$L = \frac{N\Phi}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 i n)A}{i} = \mu_0 n^2 A l$$

As the capacitance of a capacitor, the inductance of a solenoid depends only on the geometry of the coil.

# Self-inductance




Suppose you have a coil with a current that changes with time. The magnetic flux in it will changewith time too. Therefore it will induce an emf in the coil!
This effect is called self-induction.

1

Changing current → Changing flux

2

3

EMF

1 2

Combining, $L = \dfrac{N\Phi}{i}$ With Faraday's law, $emf = -N\dfrac{d\,\Phi}{dt}$

3

$$emf = -\frac{d(N\Phi)}{dt} = -\frac{d(Li)}{dt} = -L\frac{d\,i}{d\,t}$$

When we "take a walk" around a circuit to solve it, every time we find a solenoid, we add a term *-L di/dt*.

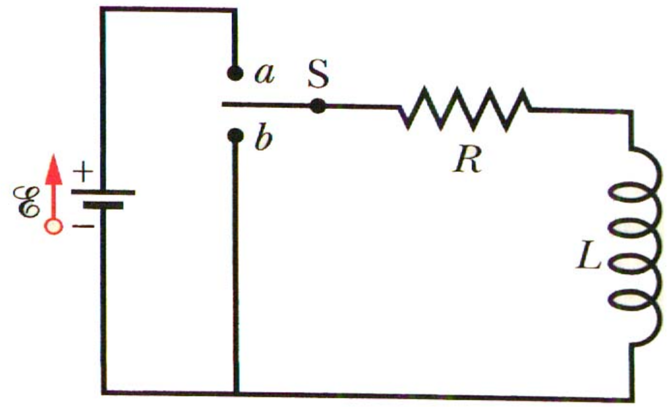
If the current is constant, "the coil is invisible" (piece of wire).
If we have sudden changes of current we can get large emf's with a coil.

# RL circuits

## Qualitative

When we studied capacitors, we saw that if one connected a capacitor to a resistor and to a battery, the charge took some time to build up, characterized by $\tau = RC$. It also took time to discharge the capacitor.

Similarly, in the circuit shown, it takes time for things to happen. Suppose you close the switch to the a position. If the inductor were not there, the current would rise rapidly to E/R.

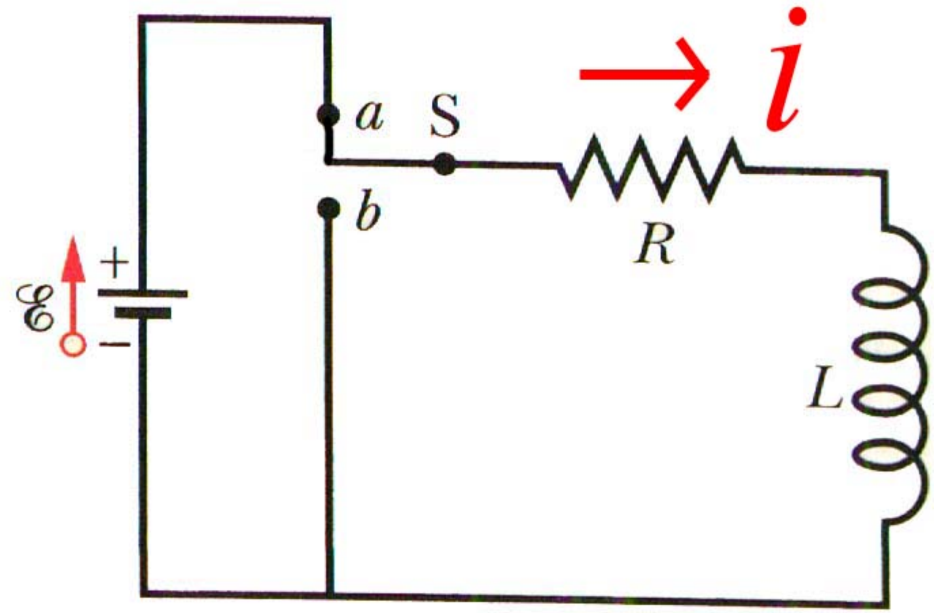



The presence of the inductor however, creates an emf. Due to Lenz' law, the emf opposes what is creating it, i.e. it opposes the emf of the battery. This "backlash" of the inductor forces the current to take time to reach its level of E/R.

Similarly, if we now flick the switch to b, the current does not immediately drop to zero, the collapsing magnetic field in L keeps the current going for a while.

## RL circuits

### Equations




$$E - iR - L\frac{di}{dt} = 0$$

Solving (similar to RC),

$$i = \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right), \quad \text{with} \quad \tau = \frac{\mathrm{L}}{\mathrm{R}}$$

T is the time constant of the circuit, has units of time and represents the amount of time it takes the current to reach 63% of its final value.

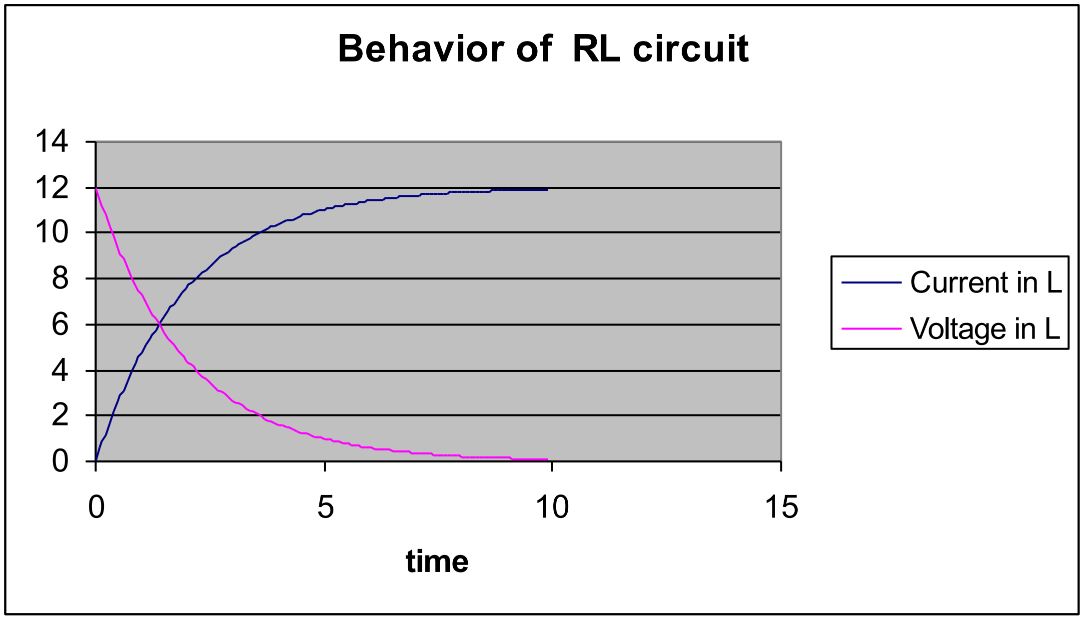
If we now connect the switch to b, the equation describing it is,

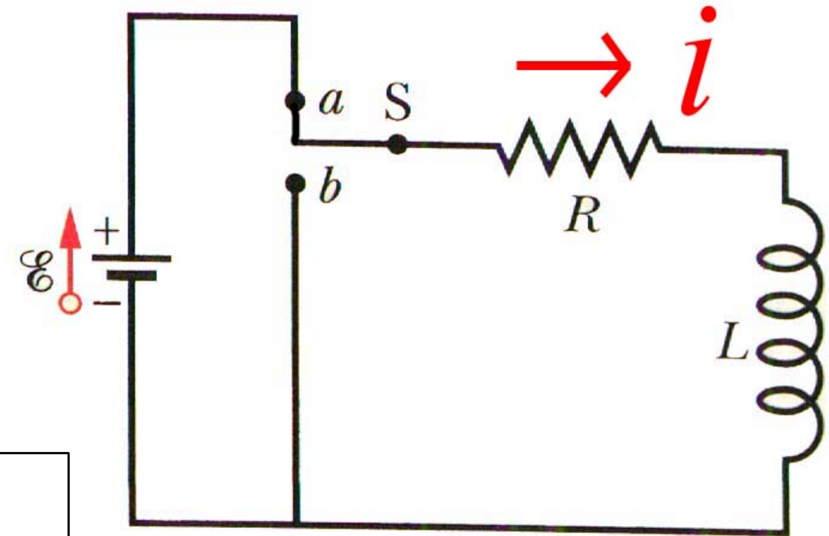
$$-iR - L\frac{di}{dt} = 0 \qquad \text{with solution,} \; i = \frac{E}{R}e^{-\frac{t}{\tau}}$$

# RL circuits

## Plots

Example, R=1Ω, τ=2s,L=2H



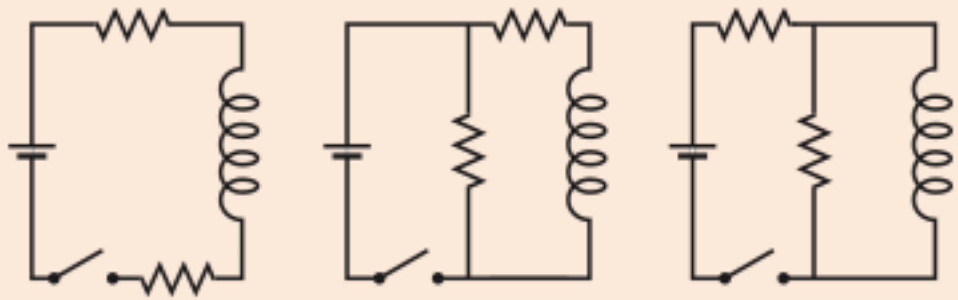

Initially, voltage in coil equals emf, no current flows through the circuit.

At late times, voltage in coil is zero, current through the circuit is like if the coil were not there.

At first connection, coils behave like an open circuit. After a long time, as a short-circuit.

## CHECKPOINT 6

The figure shows three circuits with identical batteries, inductors, and resistors. Rank the circuits according to the current through the battery (a) just after the switch is closed and (b) a long time later, greatest first. (If you have trouble here, work through the next sample problem and then try again.)

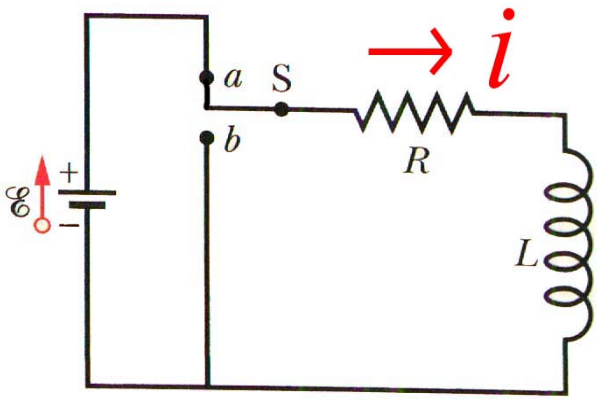


(1) (2) (3)

# Energy stored in a magnetic field

When one runs a current through a coil, it produces a magnetic field. We saw that if one disconnects the coil, the magnetic field "collapses", producing an induced emf. That is, one can store energy in a coil through its magnetic field.

Considering once more the circuit,




$$E - iR - L\frac{di}{dt} = 0$$

Multiply times i

$$\boxed{iE = i^2 R + Li\frac{di}{dt}}$$

We can interpret the last equation as follows: the left hand side represents the rate at which the battery doeswork. The first term on the right is the rate at which energy is dissipated in R. The other term has to be (by conservation of energy) the energy being stored in the magnetic field in the coil.

# Energy stored in a magnetic field

$$\frac{dU}{dt} = Li\frac{di}{dt} = \frac{L}{2}\frac{d\,i^2}{dt}$$

Therefore, $U = \frac{1}{2}Li^2$

$\left(\text{Compare with } U = \frac{1}{2}CV^2 \text{ for capacitor}\right)$

Consider a long solenoid. In a region near its middle, we can write for its energy per unit volume (energy density),

$$u = \frac{U}{Al} = \frac{L}{l}\frac{i^2}{2A} = \mu_0 n^2 A\frac{i^2}{2A} = \frac{1}{2}\mu_0 n^2 i^2$$

$\boxed{\text{Therefore, } u = \frac{B^2}{2\mu_0}}$

Now, $B = \mu_0 in$

$\left(\text{Compare with, } u = \frac{\varepsilon_0 E^2}{2} \text{ for electric field.}\right)$

## Sample Problem

### *RL* circuit, immediately after switching and after a long time

Figure 30-18*a* shows a circuit that contains three identical resistors with resistance $R = 9.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathscr{E} = 18$ V.

(a) What is the current $i$ through the battery just after the switch is closed?

**KEY IDEA**

Just after the switch is closed, the inductor acts to oppose a change in the current through it.

***Calculations:*** Because the current through each inductor is zero before the switch is closed, it will also be zero just afterward. Thus, immediately after the switch is closed, the inductors act as broken wires, as indicated in Fig. 30-18*b*. We then have a single-loop circuit for which the loop rule gives us

$$\mathscr{E} - iR = 0.$$

Substituting given data, we find that

$$i = \frac{\mathscr{E}}{R} = \frac{18\ \text{V}}{9.0\ \Omega} = 2.0\ \text{A}. \qquad \text{(Answer)}$$

(b) What is the current $i$ through the battery long after the switch has been closed?

**KEY IDEA**

Long after the switch has been closed, the currents in the circuit have reached their equilibrium values, and the inductors act as simple connecting wires, as indicated in Fig. 30-18*c*.

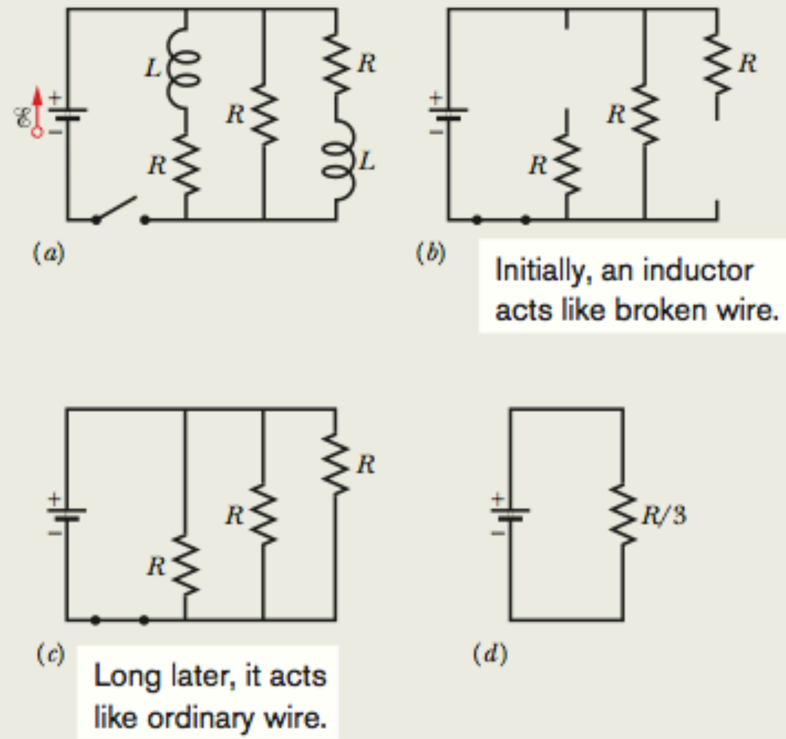



**Fig. 30-18** (*a*) A multiloop *RL* circuit with an open switch. (*b*) The equivalent circuit just after the switch has been closed. (*c*) The equivalent circuit a long time later. (*d*) The single-loop circuit that is equivalent to circuit (*c*).

***Calculations:*** We now have a circuit with three identical resistors in parallel; from Eq. 27-23, their equivalent resistance is $R_{eq} = R/3 = (9.0\ \Omega)/3 = 3.0\ \Omega$. The equivalent circuit shown in Fig. 30-18*d* then yields the loop equation $\mathscr{E} - iR_{eq} = 0$, or

$$i = \frac{\mathscr{E}}{R_{eq}} = \frac{18\ \text{V}}{3.0\ \Omega} = 6.0\ \text{A}. \qquad \text{(Answer)}$$

# Summary:

- Time dependent magnetic fields generate electric fields, with closed field lines and non-conservative.
- Inductors, like capacitors, are devices that store energy. They store it in the magnetic field they set up.
- As such, one can build circuits with inductors and resistors where the currents increase and decrease exponentially.