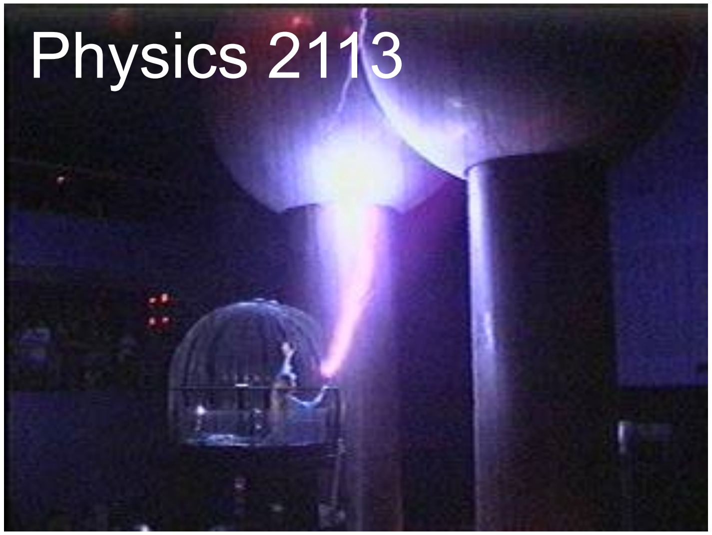
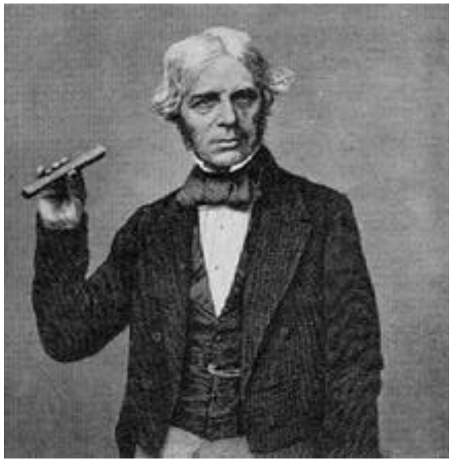




Michael Faraday 1791-1867

Physics 2113
Lecture 28: MON 3 NOV

CH30: Induction and Inductance

At the beginning of this course we argued that static charges give rise to electric fields.

We then went on to discuss how moving charges gave rise to magnetic fields.

A rather remarkable fact that we will discuss today is that moving magnetic fields, in turn produce electric fields!

This just further suggests that electric and magnetic effects are just but two manifestations of the same phenomena. This idea actually plays a key role in Einstein's theory of relativity.

To discuss how time dependent magnetic fields give rise to electric fields, we first introduce the concept of magnetic flux.

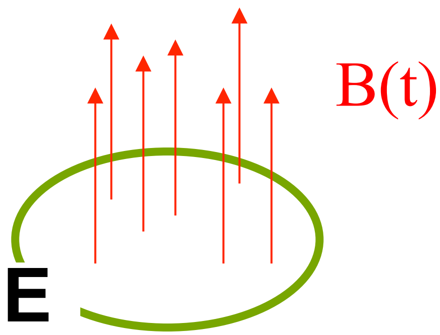
$$\Phi_B = \int \vec{B} \cdot d\vec{s} \qquad [\Phi_B] = T\,m^2 = \text{Weber}$$

Here is where we see why people defined this unit!

Faraday's law of induction:

Michael Faraday 1791-1867

Given a closed loop of wire, the area of which is pierced by a magnetic field in such a way that the magnetic flux is time-dependent, then an emf develops across the loop,



And the emf is proportional to the rate of change of flux.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

If instead of a loop one has a coil with N turns, one gets N times the emf.

The flux can change with time due to the fact that:

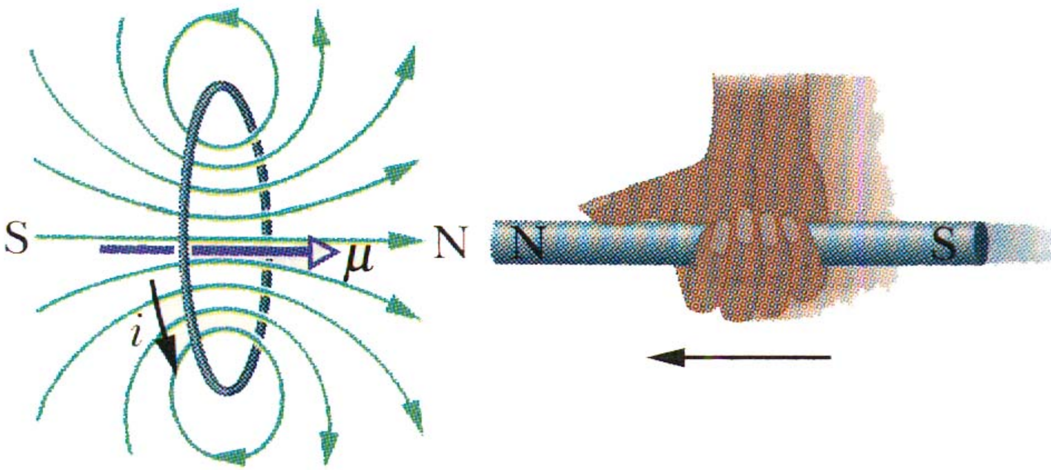
- The magnetic field changes.
- The area of the loop changes.
- The angle of B and the area c

Demo

Lenz' law:

Heinrich F.E. Lenz
(1804-1865)

The emf generated by a time dependent magnetic flux is such that the current generated on the wire produces a magnetic field that opposes the change in the magnetic flux.

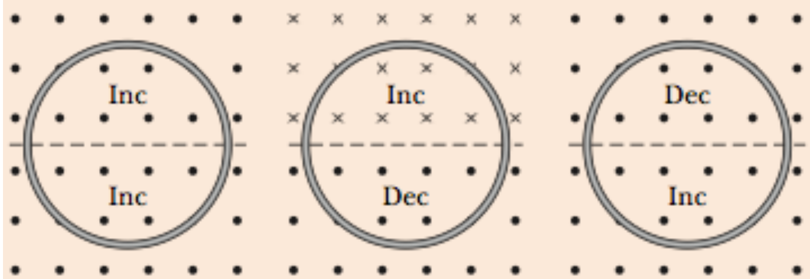


$$\mathbf{E} = -\frac{d\Phi_B}{dt}$$

Demo

CHECKPOINT 2

The figure shows three situations in which identical circular conducting loops are in uniform magnetic fields that are either increasing (Inc) or decreasing (Dec) in magnitude at identical rates. In each, the dashed line coincides with a diameter. Rank the situations according to the magnitude of the current induced in the loops, greatest first.



(*a*) (*b*) (*c*)

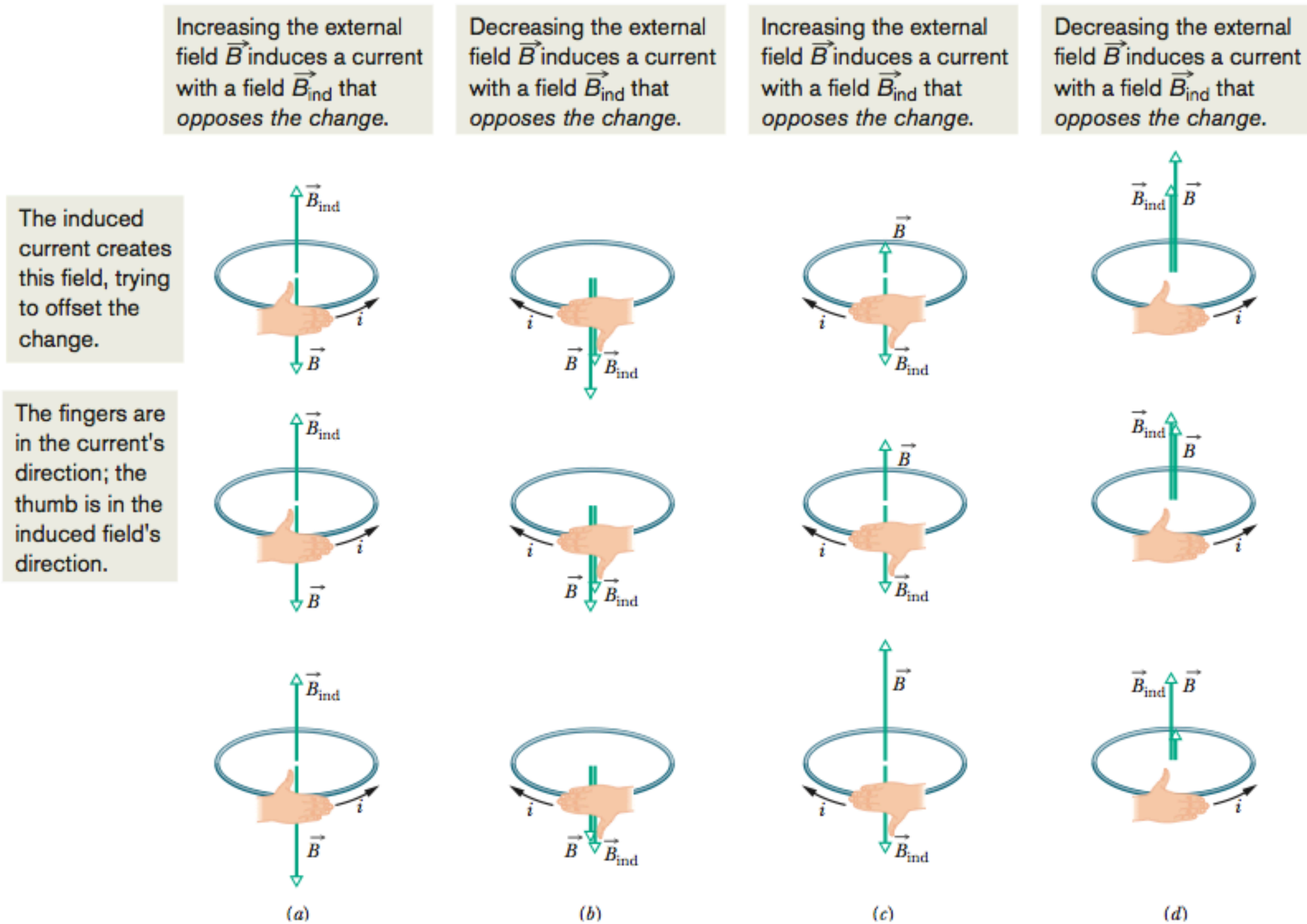


Fig. 30-5 The direction of the current i induced in a loop is such that the current's magnetic field $\vec{B}_{ind}$ opposes the *change* in the magnetic field $\vec{B}$ inducing i. The field $\vec{B}_{ind}$ is always directed opposite an increasing field $\vec{B}$ (a, c) and in the same direction as a decreasing field $\vec{B}$ (b, d). The curled–straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

Induced emf in coil due to a solenoid

The long solenoid S shown (in cross section) in Fig. 30-3 has 220 turns/cm and carries a current $i = 1.5$ A; its diameter D is 3.2 cm. At its center we place a 130-turn closely packed coil C of diameter $d = 2.1$ cm. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of the emf that is induced in coil C while the current in the solenoid is changing?

KEY IDEAS

1. Because it is located in the interior of the solenoid, coil C lies within the magnetic field produced by current i in the solenoid; thus, there is a magnetic flux Φ_B through coil C.
2. Because current i decreases, flux Φ_B also decreases.
3. As Φ_B decreases, emf $\mathscr{E}$ is induced in coil C.

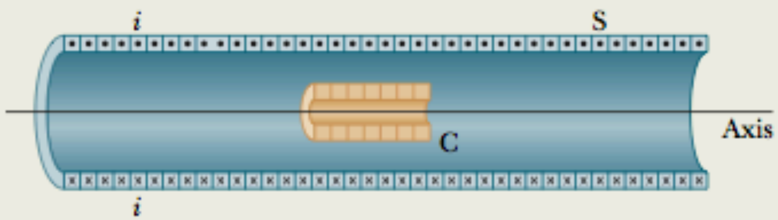


Fig. 30-3 A coil C is located inside a solenoid S, which carries current i.

4. The flux through each turn of coil C depends on the area A and orientation of that turn in the solenoid's magnetic field $\vec{B}$. Because $\vec{B}$ is uniform and directed perpendicular to area A, the flux is given by Eq. 30-2 ($\Phi_B = BA$).
5. The magnitude B of the magnetic field in the interior of a solenoid depends on the solenoid's current i and its number n of turns per unit length, according to Eq. 29-23 ($B = \mu_0 in$).

Calculations: Because coil C consists of more than one turn, we apply Faraday's law in the form of Eq. 30-5 ($\mathscr{E} = -N\, d\Phi_B/dt$), where the number of turns N is 130 and $d\Phi_B/dt$ is the rate at which the flux changes.

Because the current in the solenoid decreases at a steady rate, flux Φ_B also decreases at a steady rate, and so we can write $d\Phi_B/dt$ as $\Delta\Phi_B/\Delta t$. Then, to evaluate $\Delta\Phi_B$, we need the final and initial flux values. The final flux $\Phi_{B,f}$ is zero because the final current in the solenoid is zero. To find the initial flux $\Phi_{B,i}$, we note that area A is $\frac{1}{4}\pi d^2$ ($= 3.464 \times 10^{-4}$ m^2) and the number n is 220 turns/cm, or 22 000 turns/m. Substituting Eq. 29-23 into Eq. 30-2 then leads to

$$\begin{aligned}\Phi_{B,i} &= BA = (\mu_0 in)A \\ &= (4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(1.5\,\mathrm{A})(22\,000\ \mathrm{turns/m}) \\ &\quad \times (3.464 \times 10^{-4}\,\mathrm{m^2}) \\ &= 1.44 \times 10^{-5}\,\mathrm{Wb}.\end{aligned}$$

Now we can write

$$\begin{aligned}\frac{d\Phi_B}{dt} &= \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} \\ &= \frac{(0 - 1.44 \times 10^{-5}\,\mathrm{Wb})}{25 \times 10^{-3}\,\mathrm{s}} \\ &= -5.76 \times 10^{-4}\,\mathrm{Wb/s} = -5.76 \times 10^{-4}\,\mathrm{V}.\end{aligned}$$

We are interested only in magnitudes; so we ignore the minus signs here and in Eq. 30-5, writing

$$\begin{aligned}\mathscr{E} &= N\frac{d\Phi_B}{dt} = (130\ \mathrm{turns})(5.76 \times 10^{-4}\,\mathrm{V}) \\ &= 7.5 \times 10^{-2}\,\mathrm{V} = 75\,\mathrm{mV}. \qquad \text{(Answer)}\end{aligned}$$

5E. The magnetic flux through the loop shown in Fig. 31-41 increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s? (b) What is the direction of the current through R?

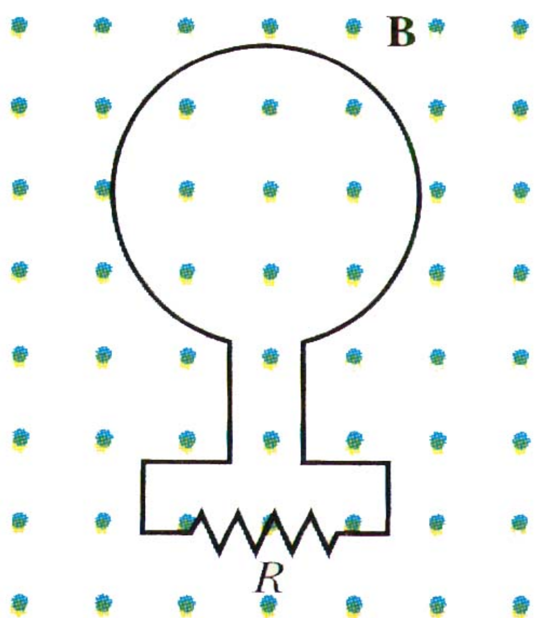


FIGURE 31-41 Exercise 5 and Problem 19.

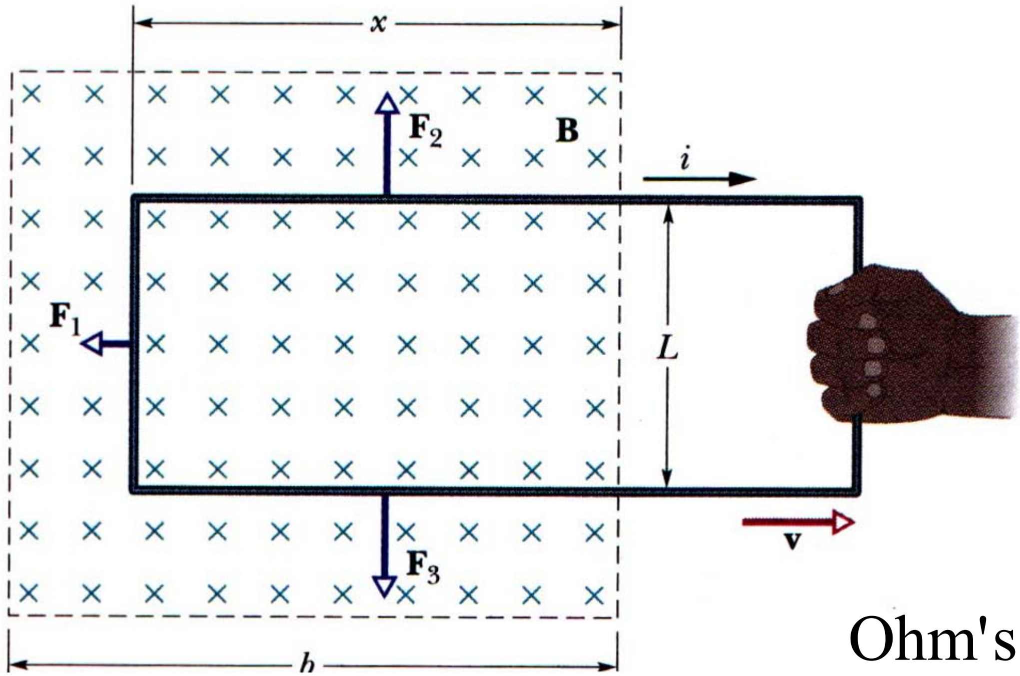
$$emf = \left|\frac{d\Phi}{dt}\right| = 12t + 7,$$

at t = 2s, $emf = 31mV$

Direction:

Φ grows with time, therefore the current will be such that it will try to induce a magnetic field into blackboard, i.e. it will flow clockwise.

Energy balance in a moving loop:



Power supplied by human: Fv

$$\Phi = BA = BLx$$

$$emf = \frac{d\Phi}{dt} = \frac{d(BLx)}{dt}$$

$$= BL\frac{dx}{dt} = BLv$$

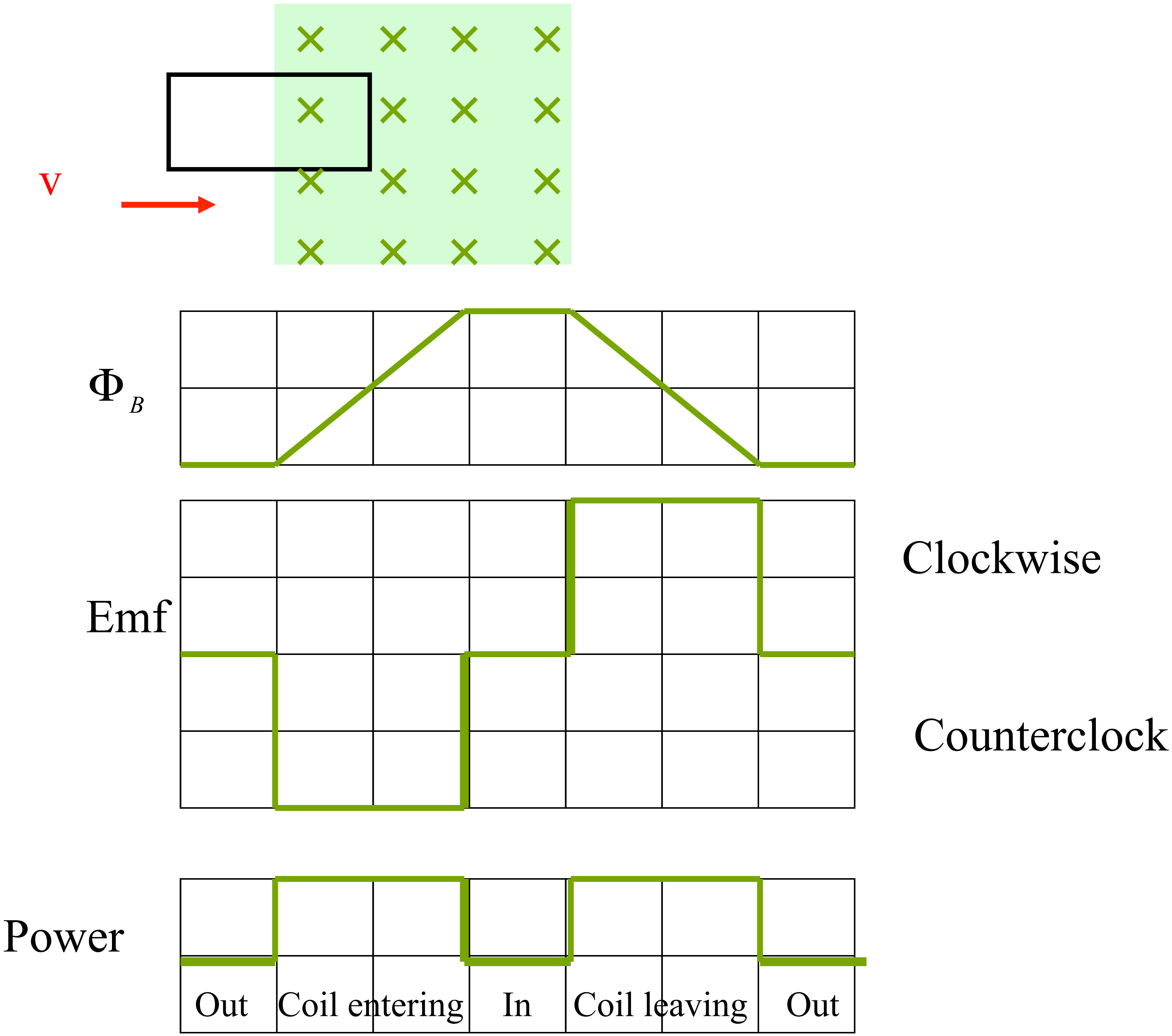
Ohm's law: $i = \dfrac{emf}{R} = \dfrac{BLv}{R}$

$$F_1 = iLB = \frac{BLv}{R}LB = \frac{B^2L^2v}{R}$$

Power supplied: $\dfrac{B^2L^2v^2}{R}$

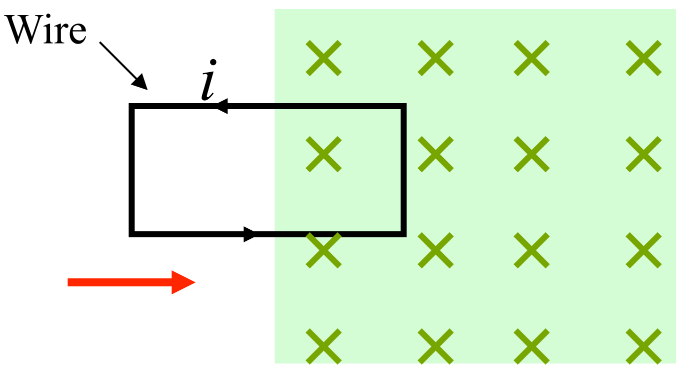
Power dissipated in wire's resistance: $i^2R = \left(\dfrac{BLv}{R}\right)^2 R = \dfrac{B^2L^2v^2}{R}$

And energy is conserved

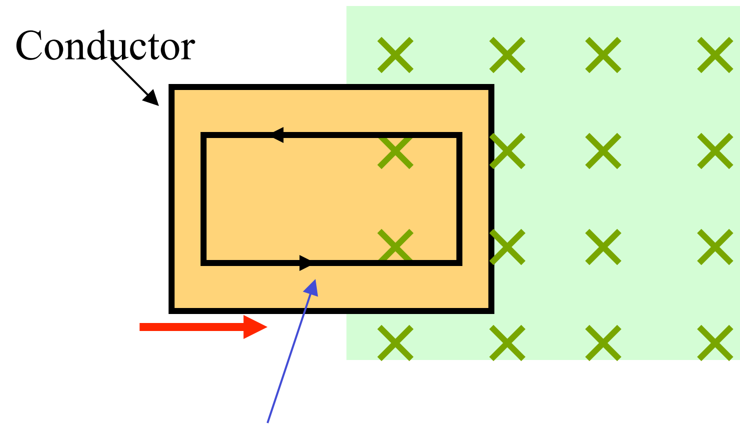


V
Φ_B
Emf
Clockwise
Counterclock
Power
Out
Coil entering
In
Coil leaving
Out

Eddy currents:



Last problem



Eddy current loop

As the eddy current circulates, it dissipates power through the resistance of the conductor. The piece of metal therefore slows down. Shuttling a piece of conductor through a magnetic field is a way to slow it down. This is the basis of the "Foucault brake".

Notice that the conductor can be a non-magnetic material, since the effect is due to the induced current, which will be there as long as there is a conductor.

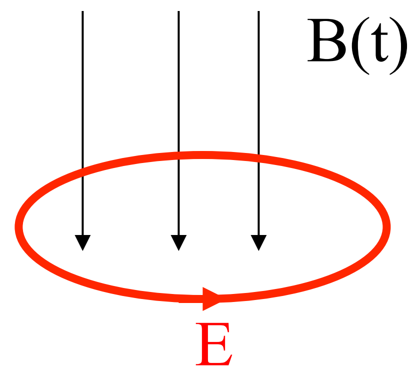
So we by now discussed several examples that focused on the fact that every time there is a changing flux, there will be an emf as soon as one inserts a conductor in the region of changing flux.

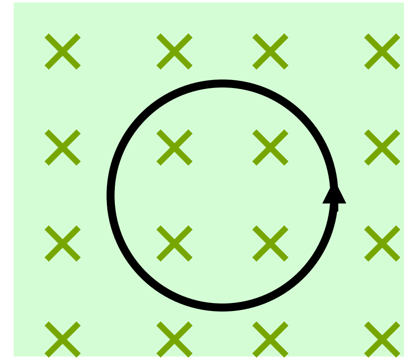
What this means is that the space is endowed with the property that if you put a conductor in that space, a current flows through it. Where did we hear this before?

When we had an electric field. If you have an electric field in a region of space and you put a conductor in it, current will flow through the conductor.

Therefore we can view a time dependent magnetic field as creating an electric field.

Since there are no charges present, the electric field lines are closed.





Let us consider a region with a changing magnetic field and assume a charge moves through it.

As we just argued, there is an electric field in the region, and therefore the charge does work moving through it.

The work done in one revolution is $q\,E_0$ where E_0 is the emf that is generated ("induced emf") by the time-dependent magnetic field.

$$W = \oint \vec{F}\cdot d\vec{s} = q\oint \vec{E}\cdot d\vec{s} \qquad \text{or, } E_0 = \oint \vec{E}\cdot d\vec{s}$$

We can therefore rewrite Faraday's law of induction as,

$$\boxed{\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

Summary:

- A changing magnetic flux produces an emf.
- The direction of the emf is such that it produces a magnetic field opposing the change that produced it.
- Fluxes can change due to changes in the field, of the area, or of the orientation.
- The induced emf generates currents that dissipate energy if there is a metal present.