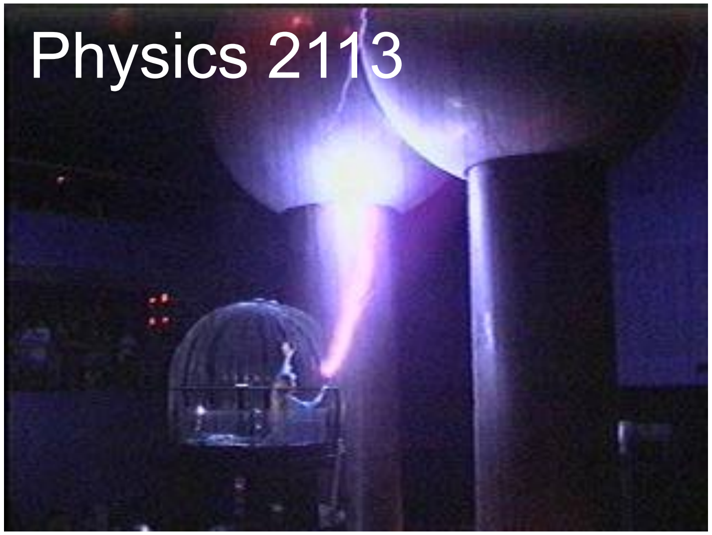


Physics 2113
Lecture 24: FRI 24 OCT

CH28: Magnetism

Potential difference set up across a moving conductor

Figure 28-9*a* shows a solid metal cube, of edge length $d = 1.5$ cm, moving in the positive y direction at a constant velocity $\vec{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\vec{B}$ of magnitude 0.050 T in the positive z direction.

(a) Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

KEY IDEA

Because the cube is moving through a magnetic field $\vec{B}$, a magnetic force $\vec{F}_B$ acts on its charged particles, including its conduction electrons.

Reasoning: When the cube first begins to move through the magnetic field, its electrons do also. Because each electron has charge q and is moving through a magnetic field with velocity $\vec{v}$, the magnetic force $\vec{F}_B$ acting on the electron is given by Eq. 28-2. Because q is negative, the direction of $\vec{F}_B$ is opposite the cross product $\vec{v} \times \vec{B}$, which is in the positive direction of the x axis (Fig. 28-9*b*). Thus, $\vec{F}_B$ acts in the negative direction of the x axis, toward the left face of the cube (Fig. 28-9*c*).

Most of the electrons are fixed in place in the atoms of the cube. However, because the cube is a metal, it contains conduction electrons that are free to move. Some of those conduction electrons are deflected by $\vec{F}_B$ to the left cube face, making that face negatively charged and leaving the right face positively charged (Fig. 28-9*d*). This charge separation produces an electric field $\vec{E}$ directed from the positively charged right face to the negatively charged left face (Fig. 28-9*e*). Thus, the left face is at a lower electric potential, and the right face is at a higher electric potential.

(b) What is the potential difference between the faces of higher and lower electric potential?

KEY IDEAS

1. The electric field $\vec{E}$ created by the charge separation produces an electric force $\vec{F}_E = q\vec{E}$ on each electron

KEY IDEAS

1. The electric field $\vec{E}$ created by the charge separation produces an electric force $\vec{F}_E = q\vec{E}$ on each electron (Fig. 28-9*f*). Because q is negative, this force is directed opposite the field $\vec{E}$—that is, rightward. Thus on each electron, $\vec{F}_E$ acts toward the right and $\vec{F}_B$ acts toward the left.

2. When the cube had just begun to move through the magnetic field and the charge separation had just begun, the magnitude of $\vec{E}$ began to increase from zero. Thus, the magnitude of $\vec{F}_E$ also began to increase from zero and was initially smaller than the magnitude $\vec{F}_B$. During this early stage, the net force on any electron was dominated by $\vec{F}_B$, which continuously moved additional electrons to the left cube face, increasing the charge separation (Fig. 28-9*g*).

3. However, as the charge separation increased, eventually magnitude F_E became equal to magnitude F_B (Fig. 28-9*h*). The net force on any electron was then zero, and no additional electrons were moved to the left cube face. Thus, the magnitude of $\vec{F}_E$ could not increase further, and the electrons were then in equilibrium.

Calculations: We seek the potential difference V between the left and right cube faces after equilibrium was reached (which occurred quickly). We can obtain V with Eq. 28-9 ($V = Ed$) provided we first find the magnitude E of the electric field at equilibrium. We can do so with the equation for the balance of forces ($F_E = F_B$).

For F_E, we substitute $|q|E$, and then for F_B, we substitute $|q|vB \sin \phi$ from Eq. 28-3. From Fig. 28-9*a*, we see that the angle ϕ between velocity vector $\vec{v}$ and magnetic field vector $\vec{B}$ is 90°; thus $\sin \phi = 1$ and $F_E = F_B$ yields

$$|q|E = |q|vB \sin 90° = |q|vB.$$

This gives us $E = vB$; so $V = Ed$ becomes

$$V = vBd. \tag{28-13}$$

Substituting known values gives us

$$V = (4.0 \text{ m/s})(0.050 \text{ T})(0.015 \text{ m})$$
$$= 0.0030 \text{ V} = 3.0 \text{ mV}. \quad \text{(Answer)}$$

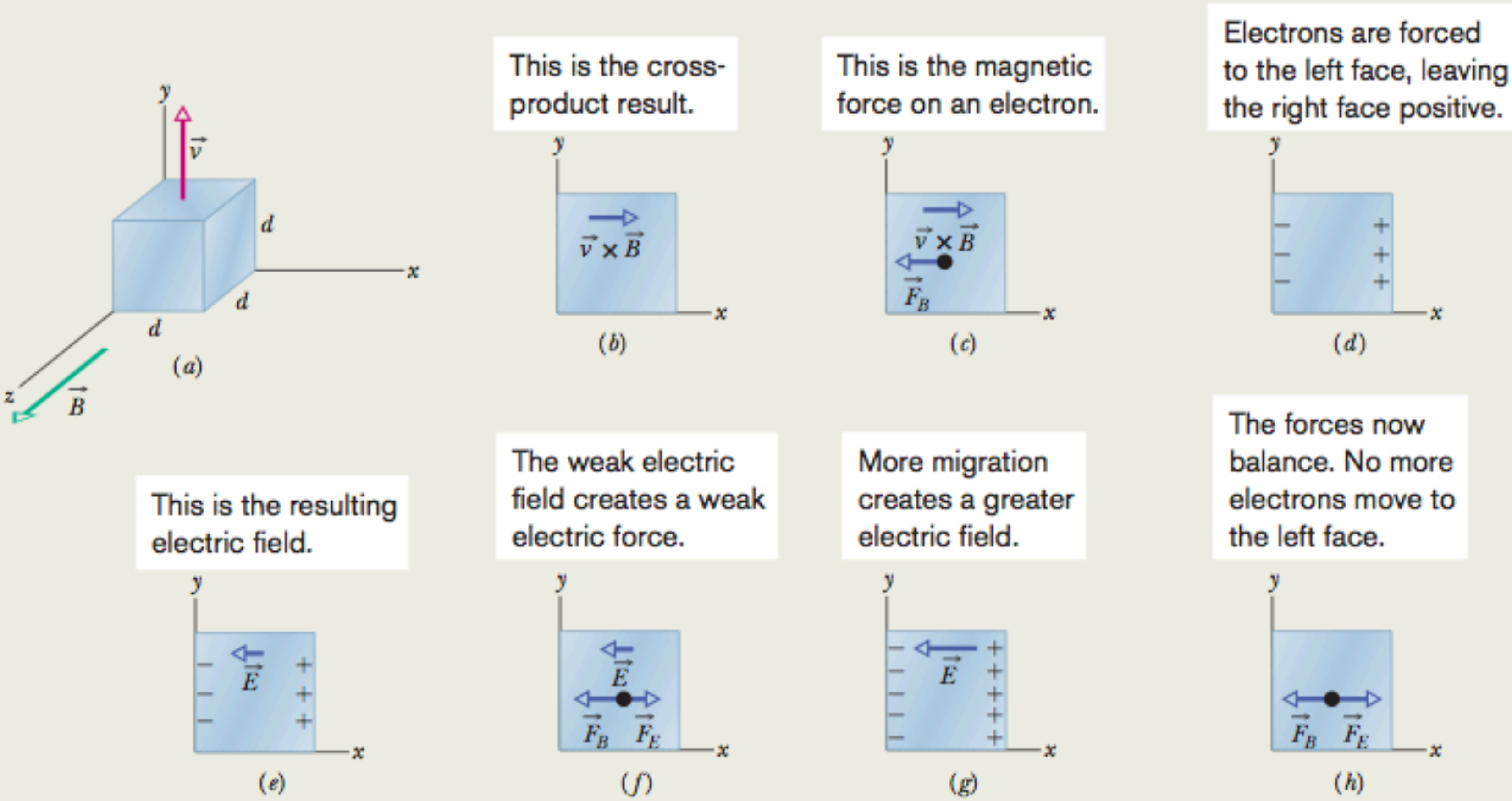


Fig. 28-9 (*a*) A solid metal cube moves at constant velocity through a uniform magnetic field. (*b*) – (*d*) In these front views, the magnetic force acting on an electron forces the electron to the left face, making that face negative and leaving the opposite face positive. (*e*) – (*f*) The resulting weak electric field creates a weak electric force on the next electron, but it too is forced to the left face. Now (*g*) the electric field is stronger and (*h*) the electric force matches the magnetic force.

Helical motion of a charged particle in a magnetic field

An electron with a kinetic energy of 22.5 eV moves into a region of uniform magnetic field $\vec{B}$ of magnitude 4.55×10^{-4} T. The angle between the directions of $\vec{B}$ and the electron's velocity $\vec{v}$ is 65.5°. What is the pitch of the helical path taken by the electron?

KEY IDEAS

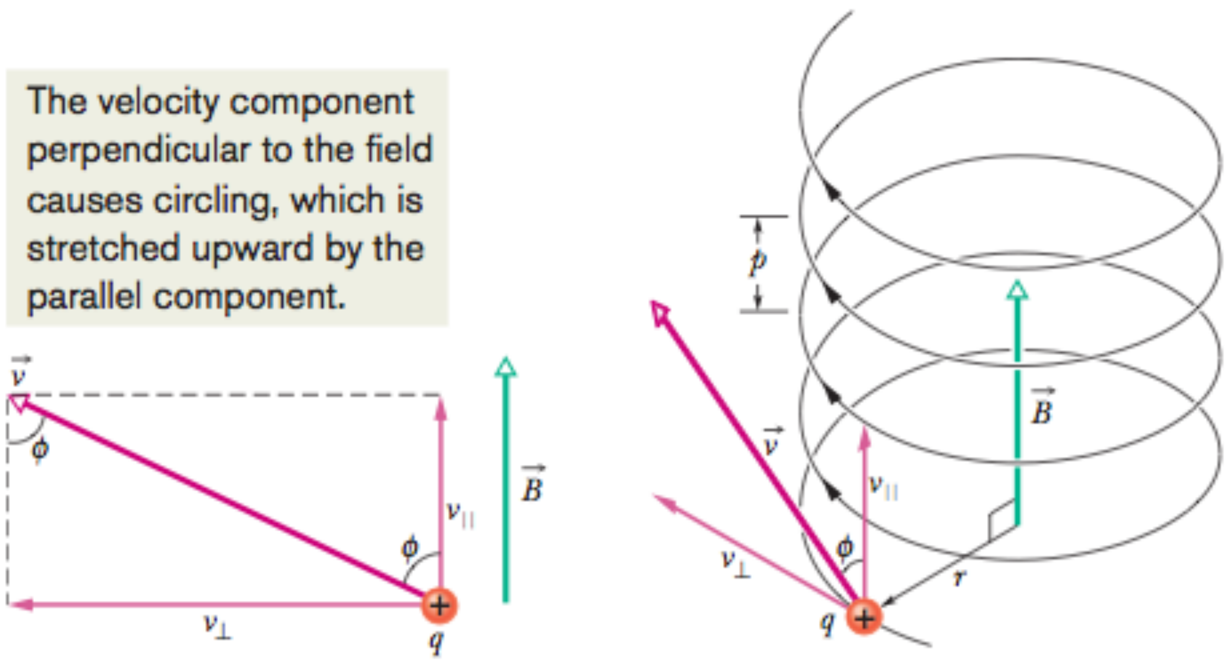
(1) The pitch p is the distance the electron travels parallel to the magnetic field $\vec{B}$ during one period T of circulation. (2) The period T is given by Eq. 28-17 regardless of the angle between the directions of $\vec{v}$ and $\vec{B}$ (provided the angle is not zero, for which there is no circulation of the electron).

Calculations: Using Eqs. 28-20 and 28-17, we find

$$p = v_{\parallel}T = (v \cos \phi)\,\frac{2\pi m}{|q|B}. \tag{28-21}$$

Calculating the electron's speed v from its kinetic energy, find that $v = 2.81 \times 10^6$ m/s. Substituting this and known data in Eq. 28-21 gives us

$$\begin{aligned} p &= (2.81 \times 10^6 \text{ m/s})(\cos 65.5°) \\ &\quad \times \frac{2\pi(9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(4.55 \times 10^{-4} \text{ T})} \\ &= 9.16 \text{ cm.} \end{aligned} \qquad \text{(Answer)}$$



(*a*) (*b*)

Accelerating a charged particle in a cyclotron

Suppose a cyclotron is operated at an oscillator frequency of 12 MHz and has a dee radius $R = 53$ cm.

(a) What is the magnitude of the magnetic field needed for deuterons to be accelerated in the cyclotron? The deuteron mass is $m = 3.34 \times 10^{-27}$ kg (twice the proton mass).

KEY IDEA

For a given oscillator frequency f_{osc}, the magnetic field magnitude B required to accelerate any particle in a cyclotron depends on the ratio $m/|q|$ of mass to charge for the particle, according to Eq. 28-24 ($|q|B = 2\pi m f_{osc}$).

Calculation: For deuterons and the oscillator frequency $f_{osc} =$ 12 MHz, we find

$$B = \frac{2\pi m f_{osc}}{|q|} = \frac{(2\pi)(3.34 \times 10^{-27}\,\text{kg})(12 \times 10^{6}\,\text{s}^{-1})}{1.60 \times 10^{-19}\,\text{C}}$$

$$= 1.57\,\text{T} \approx 1.6\,\text{T}. \qquad \text{(Answer)}$$

Note that, to accelerate protons, B would have to be reduced by a factor of 2, provided the oscillator frequency remained fixed at 12 MHz.

(b) What is the resulting kinetic energy of the deuterons?

KEY IDEAS

(1) The kinetic energy ($\frac{1}{2}mv^2$) of a deuteron exiting the cyclotron is equal to the kinetic energy it had just before exiting, when it was traveling in a circular path with a radius approximately equal to the radius R of the cyclotron dees. (2) We can find the speed v of the deuteron in that circular path with Eq. 28-16 ($r = mv/|q|B$).

Calculations: Solving that equation for v, substituting R for r, and then substituting known data, we find

$$v = \frac{R|q|B}{m} = \frac{(0.53\,\text{m})(1.60 \times 10^{-19}\,\text{C})(1.57\,\text{T})}{3.34 \times 10^{-27}\,\text{kg}}$$

$$= 3.99 \times 10^{7}\,\text{m/s}.$$

This speed corresponds to a kinetic energy of

$$K = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}(3.34 \times 10^{-27}\,\text{kg})(3.99 \times 10^{7}\,\text{m/s})^2$$

$$= 2.7 \times 10^{-12}\,\text{J}, \qquad \text{(Answer)}$$

or about 17 MeV.

Rotating a magnetic dipole in a magnetic field

Figure 28-21 shows a circular coil with 250 turns, an area A of $2.52 \times 10^{-4}\ \mathrm{m}^2$, and a current of 100 μA. The coil is at rest in a uniform magnetic field of magnitude $B = 0.85$ T, with its magnetic dipole moment $\vec{\mu}$ initially aligned with $\vec{B}$.

(a) In Fig. 28-21, what is the direction of the current in the coil?

Right-hand rule: Imagine cupping the coil with your right hand so that your right thumb is outstretched in the direction of $\vec{\mu}$. The direction in which your fingers curl around the coil is the direction of the current in the coil. Thus, in the wires on the near side of the coil—those we see in Fig. 28-21—the current is from top to bottom.

(b) How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its initial orientation, so that $\vec{\mu}$ is perpendicular to $\vec{B}$ and the coil is again at rest?

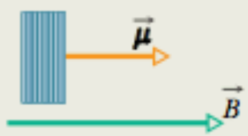


Fig. 28-21 A side view of a circular coil carrying a current and oriented so that its magnetic dipole moment is aligned with magnetic field $\vec{B}$.

KEY IDEA

The work W_a done by the applied torque would be equal to the change in the coil's orientation energy due to its change in orientation.

Calculations: From Eq. 28-39 ($W_a = U_f - U_i$), we find

$$\begin{aligned} W_a &= U(90^\circ) - U(0^\circ) \\ &= -\mu B \cos 90^\circ - (-\mu B \cos 0^\circ) = 0 + \mu B \\ &= \mu B. \end{aligned}$$

Substituting for μ from Eq. 28-35 ($\mu = NiA$), we find that

$$\begin{aligned} W_a &= (NiA)B \\ &= (250)(100 \times 10^{-6}\ \mathrm{A})(2.52 \times 10^{-4}\ \mathrm{m}^2)(0.85\ \mathrm{T}) \\ &= 5.355 \times 10^{-6}\ \mathrm{J} \approx 5.4\ \mu\mathrm{J}. \qquad \text{(Answer)} \end{aligned}$$

•3 An electron that has velocity

$$\vec{v} = (2.0 \times 10^6\ \mathrm{m/s})\hat{\mathrm{i}} + (3.0 \times 10^6\ \mathrm{m/s})\hat{\mathrm{j}}$$

moves through the uniform magnetic field $\vec{B} = (0.030\ \mathrm{T})\hat{\mathrm{i}} - (0.15\ \mathrm{T})\hat{\mathrm{j}}$. (a) Find the force on the electron due to the magnetic field. (b) Repeat your calculation for a proton having the same velocity.

$$\begin{aligned}\vec{F}_B &= q\vec{v}\times\vec{B} = q\left(v_x\hat{\mathrm{i}}+v_y\hat{\mathrm{j}}\right)\times\left(B_x\hat{\mathrm{i}}+B_y\vec{j}\right) = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}} \\ &= \left(-1.6\times10^{-19}\,\mathrm{C}\right)\left[\left(2.0\times10^{6}\ \mathrm{m/s}\right)\left(-0.15\ \mathrm{T}\right)-\left(3.0\times10^{6}\ \mathrm{m/s}\right)\left(0.030\ \mathrm{T}\right)\right] \\ &= \left(6.2\times10^{-14}\ \mathrm{N}\right)\hat{\mathrm{k}}.\end{aligned}$$

•7 An electron has an initial velocity of $(12.0\hat{j} + 15.0\hat{k})$ km/s and a constant acceleration of $(2.00 \times 10^{12}\ \mathrm{m/s^2})\hat{i}$ in a region in which uniform electric and magnetic fields are present. If $\vec{B} = (400\ \mu\mathrm{T})\hat{i}$, find the electric field $\vec{E}$.

7. We apply $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = m_e\vec{a}$ to solve for $\vec{E}$:

$$\begin{aligned}
\vec{E} &= \frac{m_e\vec{a}}{q} + \vec{B} \times \vec{v} \\
&= \frac{\left(9.11 \times 10^{-31}\,\mathrm{kg}\right)\left(2.00 \times 10^{12}\ \mathrm{m/s^2}\right)\hat{i}}{-1.60 \times 10^{-19}\,\mathrm{C}} + (400\mu\mathrm{T})\hat{i} \times \ (12.0\,\mathrm{km/s})\hat{j} + (15.0\,\mathrm{km/s})\hat{k} \\
&= \left(-11.4\hat{i} - 6.00\hat{j} + 4.80\hat{k}\right)\mathrm{V/m}.
\end{aligned}$$

•18 In Fig. 28-35, a particle moves along a circle in a region of uniform magnetic field of magnitude $B = 4.00$ mT. The particle is either a proton or an electron (you must decide which). It experiences a magnetic force of magnitude 3.20×10^{-15} N. What are (a) the particle's speed, (b) the radius of the circle, and (c) the period of the motion?

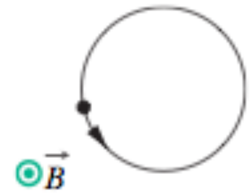


Fig. 28-35 Problem 18.

18. With the $\vec{B}$ pointing "out of the page," we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle's path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent toward the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 28-3 (with angle ϕ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e|\vec{B}|} = 4.99 \times 10^{6} \text{ m/s}.$$

(b) Using either Eq. 28-14 or Eq. 28-16, we find $r = 0.00710$ m.

(c) Using Eq. 28-17 (in either its first or last form) readily yields $T = 8.93 \times 10^{-9}$ s.

••37 Estimate the total path length traveled by a deuteron in a cyclotron of radius 53 cm and operating frequency 12 MHz during the (entire) acceleration process. Assume that the accelerating potential between the dees is 80 kV.

37. We approximate the total distance by the number of revolutions times the circumference of the orbit corresponding to the average energy. This should be a good approximation since the deuteron receives the same energy each revolution and its period does not depend on its energy. The deuteron accelerates twice in each cycle, and each time it receives an energy of $qV = 80 \times 10^3$ eV. Since its final energy is 16.6 MeV, the number of revolutions it makes is

$$n = \frac{16.6 \times 10^6 \text{ eV}}{2(80 \times 10^3 \text{ eV})} = 104 .$$

Its average energy during the accelerating process is 8.3 MeV. The radius of the orbit is given by $r = mv/qB$, where v is the deuteron's speed. Since this is given by $v = \sqrt{2K/m}$, the radius is

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km} .$$

For the average energy

$$r = \frac{\sqrt{2(8.3 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})(3.34 \times 10^{-27} \text{ kg})}}{(1.60 \times 10^{-19} \text{ C})(1.57 \text{ T})} = 0.375 \text{m} .$$

The total distance traveled is about

$$n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2 \text{ m}.$$

••46 In Fig. 28-43, a metal wire of mass $m = 24.1$ mg can slide with negligible friction on two horizontal parallel rails separated by distance $d = 2.56$ cm. The track lies in a vertical uniform magnetic field of magnitude 56.3 mT. At time $t = 0$, device G is connected to the rails, producing a constant current $i = 9.13$ mA in the wire and rails (even as the wire moves). At $t = 61.1$ ms, what are the wire's (a) speed and (b) direction of motion (left or right)?

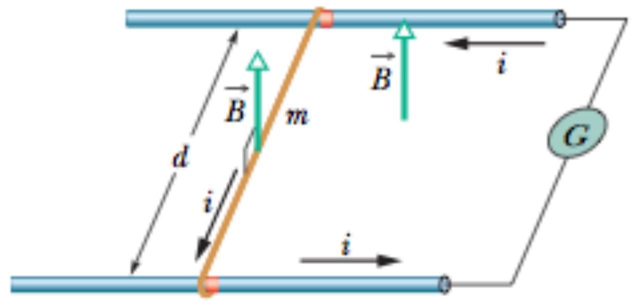


Fig. 28-43 Problem 46.

46. (a) The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus

$$v = at = \frac{F_B t}{m} = \frac{idBt}{m} = \frac{(9.13\times10^{-3}\,\text{A})(2.56\times10^{-2}\,\text{m})(5.63\times10^{-2}\,\text{T})(0.0611\,\text{s})}{2.41\times10^{-5}\,\text{kg}}$$

$$= 3.34\times10^{-2}\,\text{m/s}.$$

(b) The direction is to the left (away from the generator).

Naval Surface Warfare Center test firing in January 2008[1]

••52 In Fig. 28-46, a rectangular loop carrying current lies in the plane of a uniform magnetic field of magnitude 0.040 T. The loop consists of a single turn of flexible conducting wire that is wrapped around a flexible mount such that the dimensions of the rectangle can be changed. (The total length of the wire is not changed.) As edge length x is varied from approximately zero to its maximum value of approximately 4.0 cm, the magnitude τ of the torque on the loop changes. The maximum value of τ is 4.80×10^{-8} N·m. What is the current in the loop?

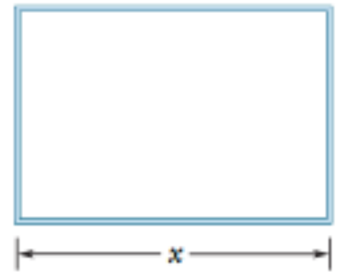


Fig. 28-46 Problem 52.

52. The insight central to this problem is that for a given length of wire (formed into a rectangle of various possible aspect ratios), the maximum possible area is enclosed when the ratio of height to width is 1 (that is, when it is a square). The maximum possible value for the width, the problem says, is x = 4 cm (this is when the height is very close to zero, so the total length of wire is effectively 8 cm). Thus, when it takes the shape of a square the value of x must be ¼ of 8 cm; that is, x = 2 cm when it encloses maximum area (which leads to a maximum torque by Eq. 28-35 and Eq. 28-37) of $A = (0.020\text{ m})^2 = 0.00040\text{ m}^2$. Since $N = 1$ and the torque in this case is given as 4.8×10^{-4} N·m, then the aforementioned equations lead immediately to $i = 0.0030$ A.

••61 SSM The coil in Fig. 28-49 carries current $i = 2.00$ A in the direction indicated, is parallel to an xz plane, has 3.00 turns and an area of 4.00×10^{-3} m^2, and lies in a uniform magnetic field $\vec{B} = (2.00\hat{i} - 3.00\hat{j} - 4.00\hat{k})$ mT. What are (a) the orientation energy of the coil in the magnetic field and (b) the torque (in unit-vector notation) on the coil due to the magnetic field?

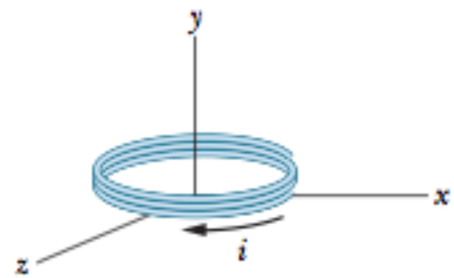


Fig. 28-49 Problem 61.

EXPRESS The magnetic potential energy of the dipole is given by $U = -\vec{\mu}\cdot\vec{B}$, where $\vec{\mu}$ is the magnetic dipole moment of the coil and $\vec{B}$ is the magnetic field. The magnitude of $\vec{\mu}$ is $\mu = NiA$, where i is the current in the coil, N is the number of turns, A is the area of the coil. On the other hand, the torque on the coil is given by the vector product $\vec{\tau} = \vec{\mu}\times\vec{B}$.

ANALYZE (a) By using the right-hand rule, we see that $\vec{\mu}$ is in the $-y$ direction. Thus, we have

$$\vec{\mu} = (NiA)(-\hat{j}) = -(3)(2.00\text{ A})(4.00\times10^{-3}\text{ m}^2)\hat{j} = -(0.0240\text{ A}\cdot\text{m}^2)\hat{j}.$$

The corresponding magnetic energy is

$$U = -\vec{\mu}\cdot\vec{B} = -\mu_y B_y = -(-0.0240\text{ A}\cdot\text{m}^2)(-3.00\times10^{-3}\text{ T}) = -7.20\times10^{-5}\text{ J}.$$

(b) Using the fact that $\hat{j}\cdot\hat{i} = 0$, $\hat{j}\times\hat{j} = 0$, and $\hat{j}\times\hat{k} = \hat{i}$, the torque on the coil is

$$\begin{aligned}\vec{\tau} = \vec{\mu}\times\vec{B} &= \mu_y B_z\hat{i} - \mu_y B_x\hat{k}\\ &= (-0.0240\text{ A}\cdot\text{m}^2)(-4.00\times10^{-3}\text{ T})\hat{i} - (-0.0240\text{ A}\cdot\text{m}^2)(2.00\times10^{-3}\text{ T})\hat{k}\\ &= (9.60\times10^{-5}\text{ N}\cdot\text{m})\hat{i} + (4.80\times10^{-5}\text{ N}\cdot\text{m})\hat{k}.\end{aligned}$$

Summary:

The Magnetic Field *B*

- Defined in terms of the force $\boldsymbol{F_B}$ acting on a test particle with charge q moving through the field with velocity $\boldsymbol{v}$

$$\vec{F}_B = q\vec{v} \times \vec{B}.$$

A Charge Particle Circulating in a Magnetic Field

- Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r}$$

- from which we find the radius r of the orbit circle to be

$$r = \frac{mv}{|q|B}.$$

Magnetic Force on a Current Carrying wire

- A straight wire carrying a current i in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}.$$

- The force acting on a current element i dL in a magnetic field is

$$d\vec{F}_B = i\, d\vec{L} \times \vec{B}.$$

Torque on a Current Carrying Coil

- A coil (of area A and N turns, carrying current i) in a uniform magnetic field $\boldsymbol{B}$ will experience a torque $\boldsymbol{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

Summary:

The Hall Effect

- When a conducting strip carrying a current i is placed in a uniform magnetic field $\boldsymbol{B}$, some charge carriers (with charge e) build up on one side of the conductor, creating a potential difference V across the strip. The polarities of the sides indicate the sign of the charge carriers.

Orientation Energy of a Magnetic Dipole

- The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

- If an external agent rotates a magnetic dipole from an initial orientation θ_i to some other orientation θ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i.$$