Physics 2113


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Lecture 19: MON 13 OCT

## CH27: Circuits



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Last class we considered this circuit:


Here the battery operated as a "pump" that moved positive charges from lower to higher electric potential. As we established, the battery "did work" on the circuit.

Any device that operates in this way is generically called an "electromotive force" (EMF) device. These come in various kinds and demand of a source of energy to operate. In the case of a battery it is chemical energy, in the case of a generator it is mechanical energy, in the case of a solar cell it is energy from light, etc.

The difference in potential energy that the device is able to establish is called the EMF
 and denoted by $E$.

$$
V_{a}+\mathrm{E}-i R=V_{a} \quad \mathrm{E}=i R
$$




What we just did is something we will repeat throughout this course: figuring out potential differences in a circuit. In more complicated circuits this can become tricky.

A goof-proof procedure consists in "taking a walk" along the circuit. Say you start from $a$ and return to $a$.

As one "walks" through the circuit (in any direction) one needs to follow two rules:

## When walking through an EMF, add $+E$ if you flow with the current or $-E$

 otherwise. How to remember: current "gains" potential in a battery.When walking through a resistor, add $-i R$, if flowing with the current or $+i R$ otherwise. How to remember: resistors are passive, current flows "potential down".

## Example:

Walking clockwise from $a:+\mathrm{E}-i R=0$.
Walking counter-clockwise from $a$ : $-\mathrm{E}+i R=0$.


Ideally, an EMF device will "shuttle around" as many charges as needed to establish the specified EMF. In the real world, there are practical limitations to EMF devices.
These limitations manifest themselves in the fact that if one connects resistors of lower and lower value of R, eventually the EMF source fails to establish the potential difference $E$, and settles for a lower value.

One can represent a "real EMF device" as an ideal one attached to a resistor, called "internal resistance" of the EMF device:

$\mathrm{E}_{\text {true }}=\mathrm{E}-i r_{\text {int }} \quad$ The true EMF is a function of current.

## Resistors in series:



Taking a walk:

$$
+\mathrm{E}-i R_{1}-i R_{2}=0 \Rightarrow \mathrm{E}=i\left(R_{1}+R_{2}\right)=i R_{\mathrm{tot}}
$$

$$
R_{\mathrm{tot}}=R_{1}+R_{2}
$$

Behave like capacitors
in parallel!
If you have $n$ resistors in series : $R_{\text {tot }}=\sum_{i=1}^{n} R_{i}$

Multi-loop circuits:


## Strategy:

Assign a different current to each branch of the circuit.

Write equations for cable junctions.
Take walks along each loop in the circuit. Each walk will produce an equation.
At the end of the day you should have enough equations to solve for the unknowns!

## Let's do it!



Cable junctions:

$$
i_{1}+i_{3}=i_{2}
$$

Walk along the left loop:

$-i_{1} R_{1}+i_{3} R_{3}+\mathrm{E}_{1}=0$

Walk along the right loop:


$$
-i_{3} R_{3}-i_{2} R_{2}-E_{2}=0
$$

Given the values of the EMF's and the resistors, we are left with three equations with three unknowns, i.e., the three currents.

Is the procedure unique? No. Typically one can choose more than one set of loops. For instance, we could have taken the left loop and the "exterior loop". The result is unique.


The equations that result from solving circuits in this way are known as Kirchoff's laws.


Gustav Robert
Kirchhoff (1824-1887)

Some loops are definitely more convenient than others!
Example: monster resistor maze.
Typically: batteries are easier to deal with. We know V from the outset!
$+V-V-V-V-i R=0$
$-8 V-i 4 \Omega=0$
$|i|=2 A$
12. Res-monster maze. In Fig. 28-21, all the resistors have a resistance of $4.0 \Omega$ and all the (ideal) batteries have an emf of 4.0 V. What is the current through resistor $R$ ? (If you can find the proper loop through this mare, you call answer the question with a few seconds of mental calculation.)


## Single-loop circuit with two real batteries

The emfs and resistances in the circuit of Fig. 27-8a have the following values:

$$
\begin{gathered}
\mathscr{E}_{1}=4.4 \mathrm{~V}, \quad \mathscr{E}_{2}=2.1 \mathrm{~V} \\
r_{1}=2.3 \Omega, \quad r_{2}=1.8 \Omega, \quad R=5.5 \Omega
\end{gathered}
$$

(a) What is the current $i$ in the circuit?

KEY IDEA
We can get an expression involving the current $i$ in this singleloop circuit by applying the loop rule.

Calculations: Although knowing the direction of $i$ is not necessary, we can easily determine it from the emfs of the two batteries. Because $\mathscr{E}_{1}$ is greater than $\mathscr{E}_{2}$, battery 1 controls the direction of $i$, so the direction is clockwise. (These decisions about where to start and which way you go are arbitrary but, once made, you must be consistent with decisions about the plus and minus signs.) Let us then apply the loop rule by going counterclockwiseagainst the current-and starting at point $a$. We find

$$
-\mathscr{E}_{1}+i r_{1}+i R+i r_{2}+\mathscr{E}_{2}=0 .
$$

Check that this equation also results if we apply the loop rule clockwise or start at some point other than $a$. Also, take the time to compare this equation term by term with Fig. 27-8b, which shows the potential changes graphically (with the potential at point $a$ arbitrarily taken to be zero).

Solving the above loop equation for the current $i$, we obtain

$$
\begin{aligned}
i & =\frac{\mathscr{E}_{1}-\mathscr{C}_{2}}{R+r_{1}+r_{2}}=\frac{4.4 \mathrm{~V}-2.1 \mathrm{~V}}{5.5 \Omega+2.3 \Omega+1.8 \Omega} \\
& =0.2396 \mathrm{~A} \approx 240 \mathrm{~mA} . \quad \text { (Answer) }
\end{aligned}
$$

(b) What is the potential difference between the terminals of battery 1 in Fig. 27-8a?

## KEY IDEA

We need to sum the potential differences between points $a$ and $b$.

Calculations: Let us start at point $b$ (effectively the negative terminal of battery 1) and travel clockwise through battery 1 to point $a$ (effectively the positive terminal), keeping track of potential changes. We find that

$$
V_{b}-i r_{1}+\mathscr{E}_{1}=V_{a},
$$


(a)


Fig. 27-8 (a) A single-loop circuit containing two real batteries and a resistor. The batteries oppose each other; that is, they tend to send current in opposite directions through the resistor. (b) A graph of the potentials, counterclockwise from point $a$, with the potential at $a$ arbitrarily taken to be zero. (To better link the circuit with the graph, mentally cut the circuit at $a$ and then unfold the left side of the circuit toward the left and the right side of the circuit toward the right.)
which gives us

$$
\begin{aligned}
V_{a}-V_{b} & =-i r_{1}+\mathscr{E}_{1} \\
& =-(0.2396 \mathrm{~A})(2.3 \Omega)+4.4 \mathrm{~V} \\
& =+3.84 \mathrm{~V} \approx 3.8 \mathrm{~V}
\end{aligned}
$$

(Answer)
which is less than the emf of the battery. You can verify this result by starting at point $b$ in Fig. 27-8a and traversing the circuit counterclockwise to point $a$. We learn two points here. (1) The potential difference between two points in a circuit is independent of the path we choose to go from one to the other. (2) When the current in the battery is in the "proper" direction, the terminal-to-terminal potential difference is low.

Resistors in parallel:


Node a: $i_{1}=i_{2}+i_{3}$
Left loop: $\mathrm{E}-i_{2} R_{1}=0 \Rightarrow i_{2}=\frac{\mathrm{E}}{R_{1}}$ Outer loop: $\mathrm{E}-i_{3} R_{2}=0 \Rightarrow i_{3}=\frac{\mathrm{E}}{R_{2}}$


Same as capacitors in series.

For $n$ resistors : $\quad R_{\mathrm{tot}}=\frac{1}{\left(\sum_{i=1}^{n} \frac{1}{R_{i}}\right)}$

## Summary:

- To solve a circuit, start at any point in it and "take a walk" around it.
- Add potential every time you traverse an EMF from - to + .
- Deduct potential every time you move along a resistor "with the flow".
- Take as many walks along independent loops as needed to solve the circuit.

