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Physics 2113

Lecture 18: FRI 10 OCT

CH26: Current and resistance



Sample of problems

Electric current:

••2 An isolated conducting sphere has a 10 cm radius. One wire carries a current of 1.000 002 0 A into it. Another wire carries a current of 1.000 000 0 A out of it. How long would it take for the sphere to increase in potential by 1000 V?

$$\Delta V = \frac{\Delta q}{4\pi\epsilon_0 r},$$

$$\begin{aligned}\Delta t &= \frac{\Delta q}{i_{\text{in}} - i_{\text{out}}} = \frac{4\pi\epsilon_0 r \Delta V}{i_{\text{in}} - i_{\text{out}}} = \frac{(0.10 \text{ m})(1000 \text{ V})}{(8.99 \times 10^9 \text{ F/m})(1.0000020 \text{ A} - 1.0000000 \text{ A})} \\ &= 5.6 \times 10^{-3} \text{ s.}\end{aligned}$$


Current density:

•11 What is the current in a wire of radius $R = 3.40$ mm if the magnitude of the current density is given by (a) $J_a = J_0 r/R$ and (b) $J_b = J_0(1 - r/R)$, in which r is the radial distance and $J_0 = 5.50 \times 10^4$ A/m²? (c) Which function maximizes the current density near the wire's surface?

$$i = \int_{\text{cylinder}} J_a dA = \frac{J_0}{R} \int_0^R r \cdot 2\pi r dr = \frac{2}{3} \pi R^2 J_0 = \frac{2}{3} \pi (3.40 \times 10^{-3} \text{ m})^2 (5.50 \times 10^4 \text{ A/m}^2) \\ = 1.33 \text{ A.}$$

$$i = \int_{\text{cylinder}} J_b dA = \int_0^R J_0 \left(1 - \frac{r}{R}\right) 2\pi r dr = \frac{1}{3} \pi R^2 J_0 = \frac{1}{3} \pi (3.40 \times 10^{-3} \text{ m})^2 (5.50 \times 10^4 \text{ A/m}^2) \\ = 0.666 \text{ A.}$$

Resistance and resistivity:

•14  A human being can be electrocuted if a current as small as 50 mA passes near the heart. An electrician working with sweaty hands makes good contact with the two conductors he is holding, one in each hand. If his resistance is 2000 Ω , what might the fatal voltage be?

$$V = (50 \times 10^{-3} \text{ A})(2000 \Omega) = 100 \text{ V}.$$

•18 A wire 4.00 m long and 6.00 mm in diameter has a resistance of 15.0 m Ω . A potential difference of 23.0 V is applied between the ends. (a) What is the current in the wire? (b) What is the magnitude of the current density? (c) Calculate the resistivity of the wire material. (d) Using Table 26-1, identify the material.

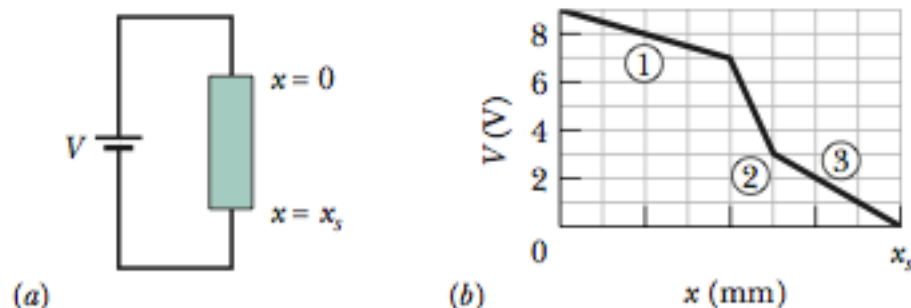
$$i = V/R = 23.0 \text{ V}/15.0 \times 10^{-3} \Omega = 1.53 \times 10^3 \text{ A}.$$

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4(1.53 \times 10^3 \text{ A})}{\pi(6.00 \times 10^{-3} \text{ m})^2} = 5.41 \times 10^7 \text{ A/m}^2.$$

$$\rho = \frac{RA}{L} = \frac{(15.0 \times 10^{-3} \Omega)\pi(6.00 \times 10^{-3} \text{ m})^2}{4(4.00 \text{ m})} = 10.6 \times 10^{-8} \Omega \cdot \text{m}.$$

Platinum	10.6×10^{-8}	3.9×10^{-3}
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••26 In Fig. 26-25a, a 9.00 V battery is connected to a resistive



strip that consists of three sections with the same cross-sectional areas but different conductivities. Figure 26-25b gives the electric potential $V(x)$ versus position x along the strip. The horizontal scale is set by $x_s = 8.00$ mm. Section 3 has conductivity 3.00×10^7 $(\Omega \cdot \text{m})^{-1}$. What is the conductivity of section (a) 1 and (b) 2?

$$J_1 = \frac{i}{A} = \sigma_1 E_1 = \sigma_1 (0.50 \times 10^3 \text{ V/m})$$

$$J_2 = \frac{i}{A} = \sigma_2 E_2 = \sigma_2 (4.0 \times 10^3 \text{ V/m})$$

$$J_3 = \frac{i}{A} = \sigma_3 E_3 = \sigma_3 (1.0 \times 10^3 \text{ V/m}) .$$

••31 An electrical cable consists of 125 strands of fine wire, each having $2.65 \mu\Omega$ resistance. The same potential difference is applied between the ends of all the strands and results in a total current of 0.750 A. (a) What is the current in each strand? (b) What is the applied potential difference? (c) What is the resistance of the cable?

31. (a) The current in each strand is $i = 0.750 \text{ A}/125 = 6.00 \times 10^{-3} \text{ A}$.

(b) The potential difference is $V = iR = (6.00 \times 10^{-3} \text{ A})(2.65 \times 10^{-6} \Omega) = 1.59 \times 10^{-8} \text{ V}$.

(c) The resistance is $R_{\text{total}} = 2.65 \times 10^{-6} \Omega/125 = 2.12 \times 10^{-8} \Omega$.

•42 In Fig. 26-32, a battery of potential difference $V = 12\text{ V}$ is connected to a resistive strip of resistance $R = 6.0\ \Omega$. When an electron moves through the strip from one end to the other, (a) in which direction in the figure does the electron move, (b) how much work is done on the electron by the electric field in the strip, and (c) how much energy is transferred to the thermal energy of the strip by the electron?

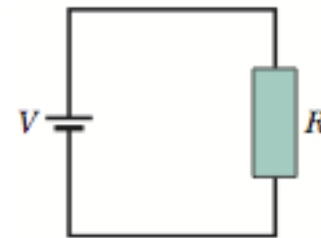


Fig. 26-32
Problem 42.

42. (a) Referring to Fig. 26-33, the electric field would point down (toward the bottom of the page) in the strip, which means the current density vector would point down, too (by Eq. 26-11). This implies (since electrons are negatively charged) that the conduction electrons would be “drifting” upward in the strip.

(b) Equation 24-6 immediately gives 12 eV, or (using $e = 1.60 \times 10^{-19}\text{ C}$) $1.9 \times 10^{-18}\text{ J}$ for the work done by the field (which equals, in magnitude, the potential energy change of the electron).


(c) Since the electrons don’t (on average) gain kinetic energy as a result of this work done, it is generally dissipated as heat. The answer is as in part (b): 12 eV or $1.9 \times 10^{-18}\text{ J}$.

47 A heating element is made by maintaining a potential difference of 75.0 V across the length of a Nichrome wire that has a $2.60 \times 10^{-6} \text{ m}^2$ cross section. Nichrome has a resistivity of $5.00 \times 10^{-7} \Omega \cdot \text{m}$. (a) If the element dissipates 5000 W, what is its length? (b) If 100 V is used to obtain the same dissipation rate, what should the length be?

47. (a) From $P = V^2/R = AV^2 / \rho L$, we solve for the length:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60 \times 10^{-6} \text{ m}^2)(75.0 \text{ V})^2}{(5.00 \times 10^{-7} \Omega \cdot \text{m})(5000 \text{ W})} = 5.85 \text{ m}.$$

(b) Since $L \propto V^2$ the new length should be $L' = L \left(\frac{V'}{V} \right)^2 = (5.85 \text{ m}) \left(\frac{100 \text{ V}}{75.0 \text{ V}} \right)^2 = 10.4 \text{ m}.$

74  The current density in a wire is uniform and has magnitude $2.0 \times 10^6 \text{ A/m}^2$, the wire's length is 5.0 m, and the density of conduction electrons is $8.49 \times 10^{28} \text{ m}^{-3}$. How long does an electron take (on the average) to travel the length of the wire?

$$v_d = \frac{|\vec{J}|}{ne} = \frac{2.0 \times 10^6 \text{ A/m}^2}{(8.49 \times 10^{28} / \text{m}^3)(1.6 \times 10^{-19} \text{ C})} = 1.47 \times 10^{-4} \text{ m/s}.$$

At this (average) rate, the time required to travel $L = 5.0 \text{ m}$ is

$$t = \frac{L}{v_d} = \frac{5.0 \text{ m}}{1.47 \times 10^{-4} \text{ m/s}} = 3.4 \times 10^4 \text{ s}.$$

65 A potential difference V is applied to a wire of cross-sectional area A , length L , and resistivity ρ . You want to change the applied potential difference and stretch the wire so that the energy dissipation rate is multiplied by 30.0 and the current is multiplied by 4.00. Assuming the wire's density does not change, what are (a) the ratio of the new length to L and (b) the ratio of the new cross-sectional area to A ?

$$\left(\frac{L}{A}\right)_{\text{new}} = \left(\frac{P}{i^2 \rho}\right)_{\text{new}} = \frac{30}{4^2} \left(\frac{P}{i^2 \rho}\right)_{\text{old}} = \frac{30}{16} \left(\frac{L}{A}\right)_{\text{old}} .$$

Consequently, $(L/A)_{\text{new}} = 1.875(L/A)_{\text{old}}$, and

$$L_{\text{new}} = \sqrt{1.875} L_{\text{old}} = 1.37 L_{\text{old}} \Rightarrow \frac{L_{\text{new}}}{L_{\text{old}}} = 1.37 .$$