Physics 2113

# Physics 2113
# Lecture 10: WED17 SEP

## CH23: Gauss' Law

Isaac Newton
(1642–1727)

Michael Faraday
(1791–1867)

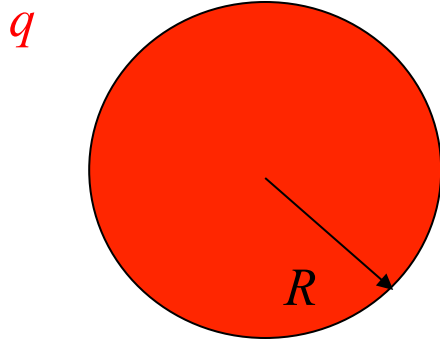## Gauss' law:

Given an arbitrary closed surface, the electric flux through it is proportional to the charge enclosed by the surface.

$q$

Flux=0!

$q$
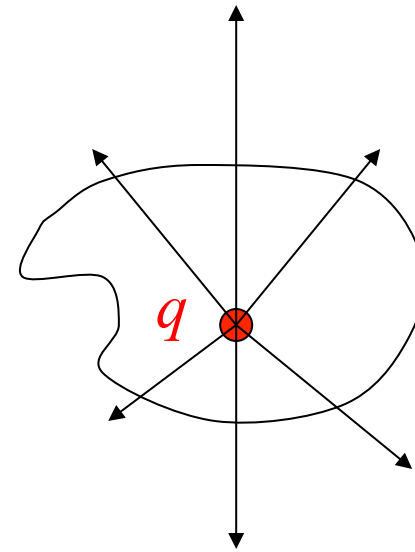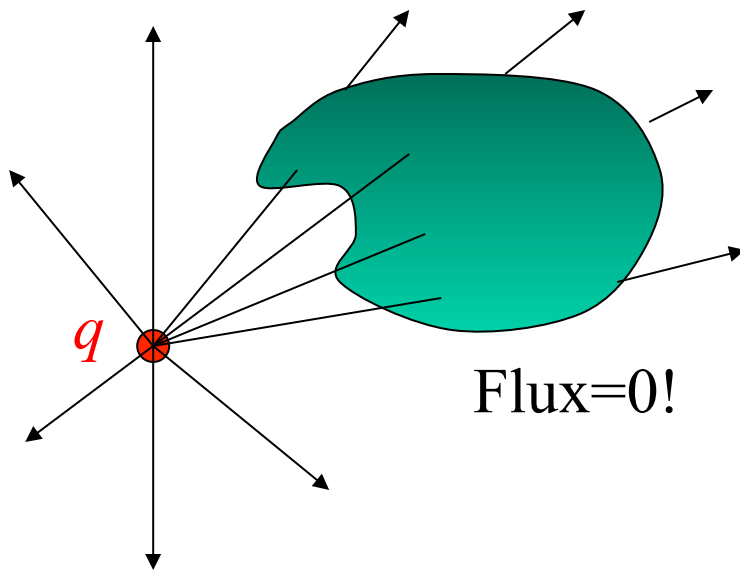
$$\Phi \equiv \oint_{Surface} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

**Example:** Charged spherical ball, compute field as a function of r (including inside).

Charge per unit volume : $\rho = \dfrac{q}{\dfrac{4\pi R^3}{3}}$

$q$

$R$

Inside the sphere: pick spherical Gaussian surface of radius r. The amount of charge enclosed by the surface is $\dfrac{4\pi r^3}{3}\rho = \dfrac{r^3}{R^3}q$

Applying Gauss' law: $\Phi = 4\pi r^2 E = \dfrac{q}{\varepsilon_0}\dfrac{r^3}{R^3} \Rightarrow E = \dfrac{qr}{4\pi\varepsilon_0 R^3}$

Field grows linearly.

Outside the sphere, pick a spherical Gaussian surface, the enclosed charge is q, therefore again the field is the same as for a point charge q.

$E$

$\dfrac{qr}{4\pi\varepsilon_0 R^3}$

$\dfrac{q}{4\pi\varepsilon_0 r^2}$

R          r
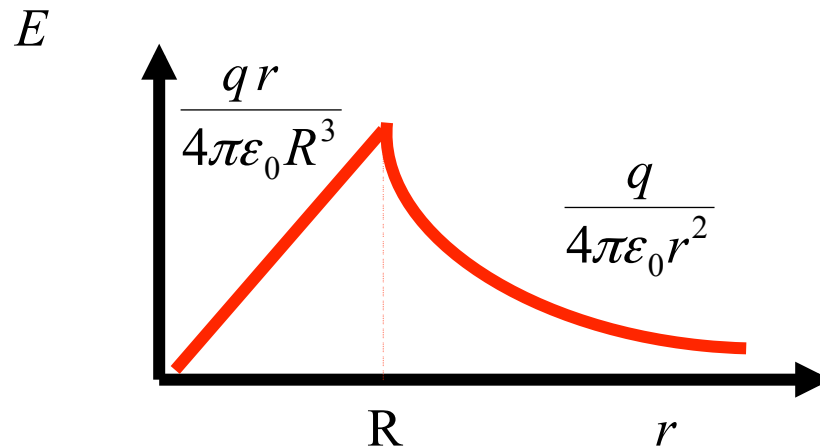
# Faraday's cage

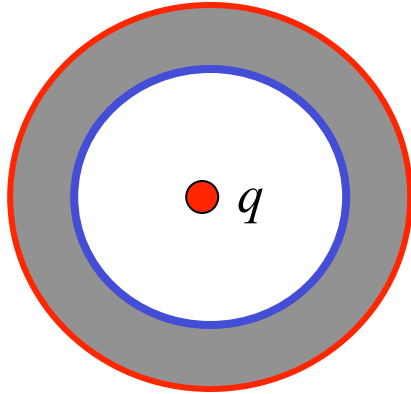Since electric fields must vanish inside conductors (even if they are hollow), metal enclosures can effectively be used to shield against electric fields.

Example:

Stations in car radios fade when driving through suspension bridges. The cabling in the bridge is the "metal enclosure".

# Example: a charged conducting spherical sheet with a charge inside

The presence of the central charge attracts electrons to the inner surface of the metal sheet. How much charge is there on the inner surface of the sheet?
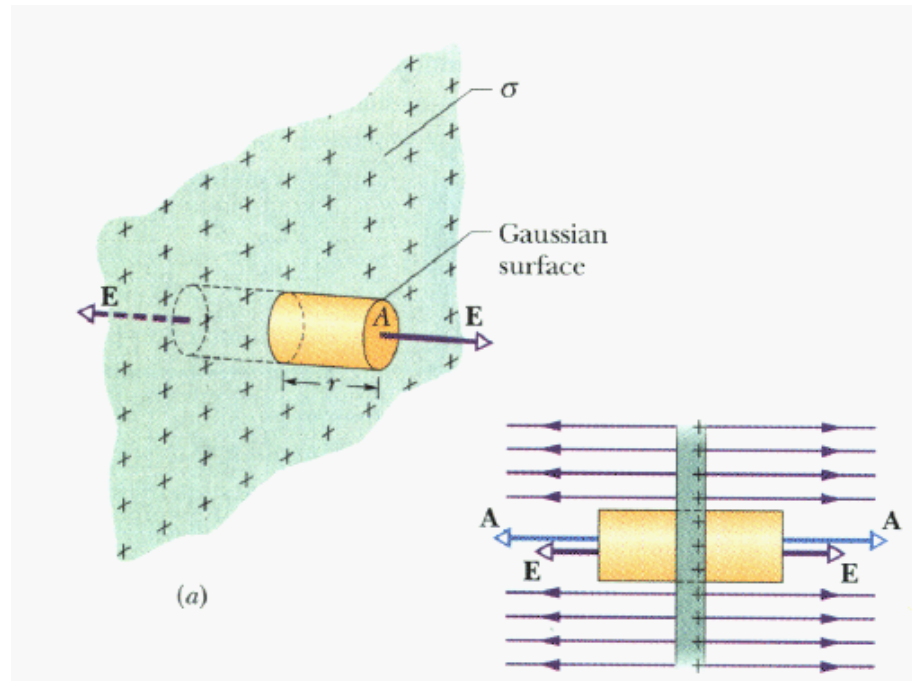
Construct a spherical Gaussian surface inside the sheet. Since it is a conductor, the field there vanishes. Therefore the flux vanishes. By Gauss' law, the enclosed charge should be zero. Therefore the amount of charge on the inner surface is -q.

Charge of sheet : $q_s$

Charge in the outside surface: construct a spherical Gaussian surface outside the sheet. The enclosed charge is q +qs. The external field will be equal to that of a point charge of value q+qs.

Now, the external field is entirely due to the charge on the outside of the sheet (since the field due to the inner surface cancelled with that of the point charge). Therefore the amount of charge deposited on the outside is q+qs.

## Planar symmetry:



Construct a "pill box" (cylindrical) Gaussian surface.

The surface encloses an amount of charge equal to the area of the cover of the cylinder times the surface charge density, $A\sigma$.

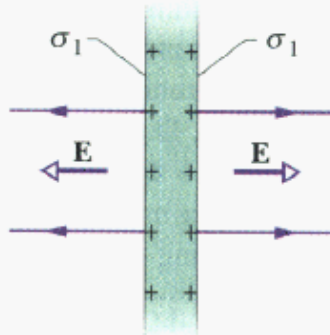The flux of the electric field only has contributions from the covers. The flux is the same in both covers, $EA$. Therefore the total flux is $2EA$.

Applying Gauss' law, we have, $\dfrac{A\sigma}{\varepsilon_0} = 2AE$

Solving for the electric field, we get $\boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

The result coincides with the one we found (with great effort) by taking the infinite radius limit of a disk!

# Planar symmetry: the case of conductors



In conductors, charges have to arrange themselves to cancel the field in the interior. Therefore one has, even for a thin plate, "double" the charge density than for an insulator.

We now repeat the "pill box" construction. Now, however, we consider one cover of the cylinder in the interior of the conductor. The flux there is therefore zero, since there is no field. The total flux is now $AE$. The charge enclosed is still $\sigma A$.



(a)           (b)

Applying Gauss' law, we have, $\dfrac{A\sigma}{\varepsilon_0} = AE$

Solving for the electric field, we get $\boxed{E = \dfrac{\sigma}{\varepsilon_0}}$
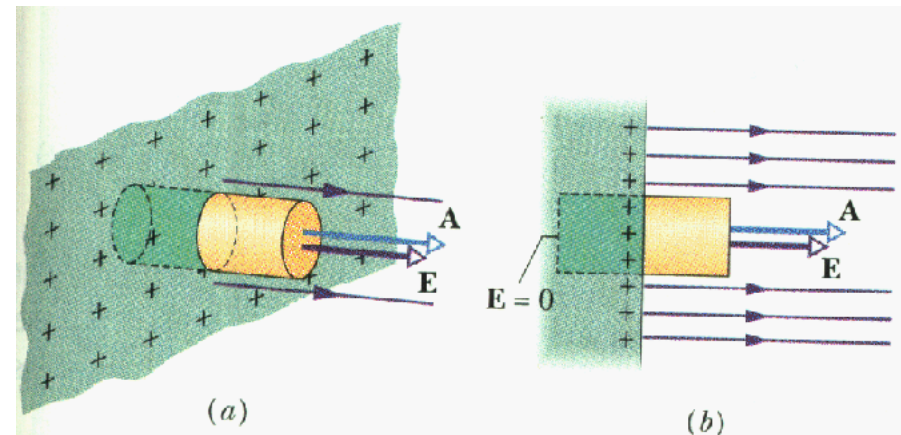
So there is a factor of 2 difference between the fields of insulator and conducting infinite planes.

# Two conducting Plates

**Figure (a)** shows a cross section of a thin, infinite conducting plate with excess positive charge. **Figure (b)** shows an identical plate with excess negative charge having the same magnitude of surface charge density $\sigma 1$.

Suppose we arrange for the plates of **Figs. a** and **b** to be close to each other and parallel **(c)**. Since the plates are conductors, when we bring them into this arrangement, the excess charge on one plate attracts the excess charge on the other plate, and all the excess charge moves onto the inner faces of the plates as in Fig.c. With twice as much charge now on each inner face, the electric field at any point between the plates has the magnitude



(a)                    (b)

(c)

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$
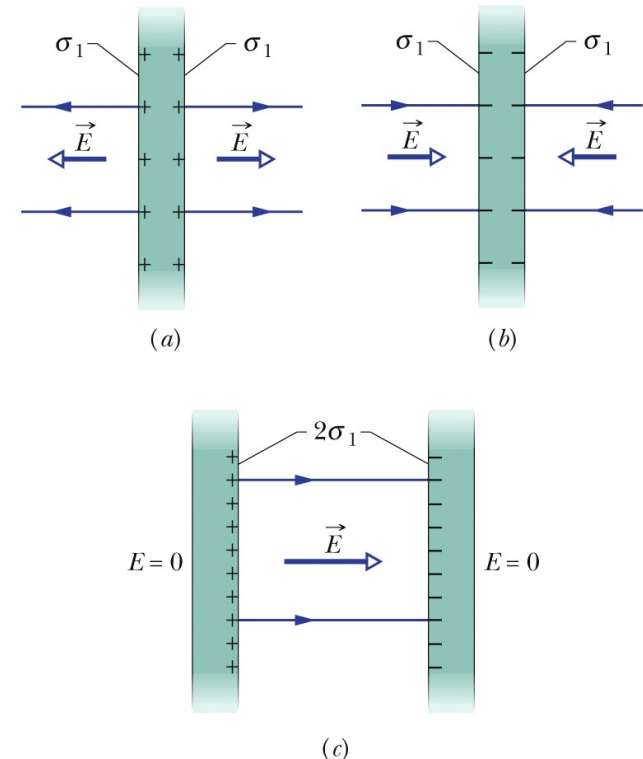
# Insulating plates

Figure 23-17a shows portions of two large, parallel, non-conducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8\ \mu C/m^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\ \mu C/m^2$ for the negatively charged sheet.

Find the electric field $\vec{E}$ (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.

### KEY IDEA

With the charges fixed in place (they are on nonconductors), we can find the electric field of the sheets in Fig. 23-17a by (1) finding the field of each sheet as if that sheet were isolated and (2) algebraically adding the fields of the isolated sheets via the superposition principle. (We can add the fields algebraically because they are parallel to each other.)

**Calculations:** At any point, the electric field $\vec{E}_{(+)}$ due to the positive sheet is directed *away* from the sheet and, from Eq. 23-13, has the magnitude

$$E_{(+)} = \frac{\sigma_{(+)}}{2\varepsilon_0} = \frac{6.8 \times 10^{-6}\ C/m^2}{(2)(8.85 \times 10^{-12}\ C^2/N \cdot m^2)}$$

$$= 3.84 \times 10^5\ N/C.$$

Similarly, at any point, the electric field $\vec{E}_{(-)}$ due to the negative sheet is directed *toward* that sheet and has the magnitude

$$E_{(-)} = \frac{\sigma_{(-)}}{2\varepsilon_0} = \frac{4.3 \times 10^{-6}\ C/m^2}{(2)(8.85 \times 10^{-12}\ C^2/N \cdot m^2)}$$

$$= 2.43 \times 10^5\ N/C.$$

Figure 23-17b shows the fields set up by the sheets to the left of the sheets (L), between them (B), and to their right (R).

The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

$$E_L = E_{(+)} - E_{(-)}$$

$$= 3.84 \times 10^5\ N/C - 2.43 \times 10^5\ N/C$$

$$= 1.4 \times 10^5\ N/C. \hspace{2cm} \text{(Answer)}$$

Because $E_{(+)}$ is larger than $E_{(-)}$, the net electric field $\vec{E}_L$ in this region is directed to the left, as Fig. 23-17c shows. To the right of the sheets, the electric field has the same magnitude but is directed to the right, as Fig. 23-17c shows.

Between the sheets, the two fields add and we have

$$E_B = E_{(+)} + E_{(-)}$$

$$= 3.84 \times 10^5\ N/C + 2.43 \times 10^5\ N/C$$

$$= 6.3 \times 10^5\ N/C. \hspace{2cm} \text{(Answer)}$$

The electric field $\vec{E}_B$ is directed to the right.



**Fig. 23-17** (a) Two large, parallel sheets, uniformly charged on one side. (b) The individual electric fields resulting from the two charged sheets. (c) The net field due to both charged sheets, found by superposition
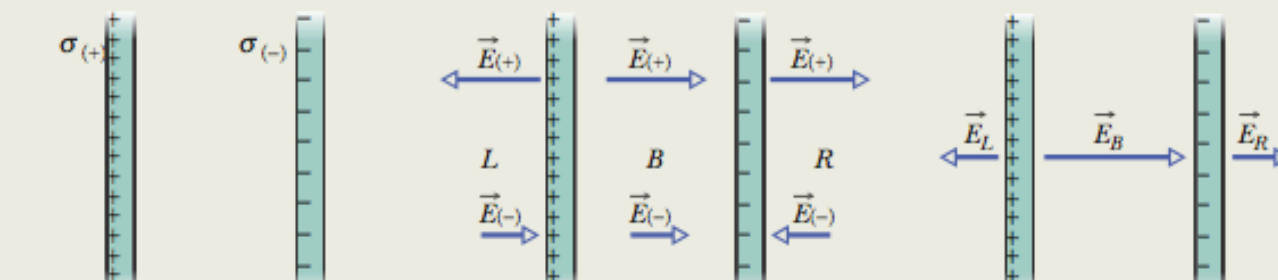
## Cylindrical symmetry:
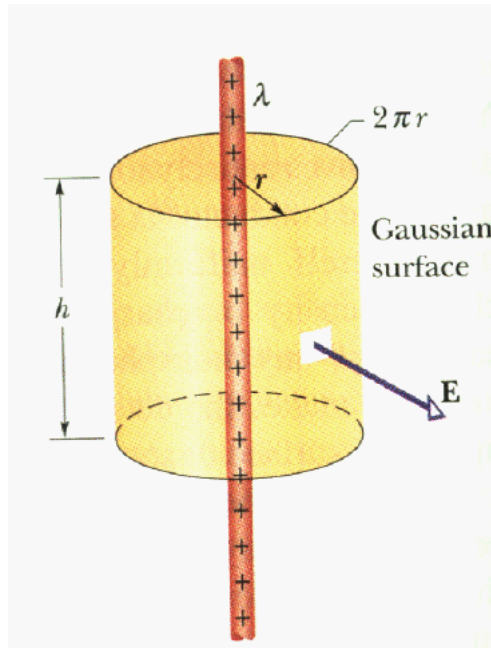


Consider an infinite wire of charge per unit length $\lambda$.

We construct a cylindrical Gaussian surface of finite height $h$.

The amount of charge enclosed is $h\lambda$.

The field points radially outwards, and is constant along the surface of the cylinder. The covers give no contribution to the flux. The total flux is the area of the side of the cylinder times the field: $\Phi = 2\pi r h E$

Applying Gauss' law, we get : $2\pi r h E = \lambda h$, therefore we get for the field $E = \dfrac{\lambda}{2\pi r}$

Again we reproduce easily a result we had arrived to with effort using Coulomb's law.

**Everything together:**

- Gauss' law is $\varepsilon_0 \Phi = q_{enc}$

- the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

**Applications of Gauss' Law**
- surface of a charged conductor

$$E = \frac{\sigma}{\varepsilon_0}$$

- Within the surface $E=0$.

- line of charge $\quad E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

- Infinite non-conducting sheet

$$E = \frac{\sigma}{2\varepsilon_0}$$

- Outside a spherical shell of charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- Inside a uniform spherical shell

$$E = 0$$

- Inside a uniform sphere of charge

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r.$$

# Summary

- In situations of high symmetry (planar, spherical, cylindrical), Gauss' law allows to compute quantitatively the electric field in a straightforward manner.

- It allows us to understand also quantitatively behavior of charges in conductors (very hard to study otherwise).