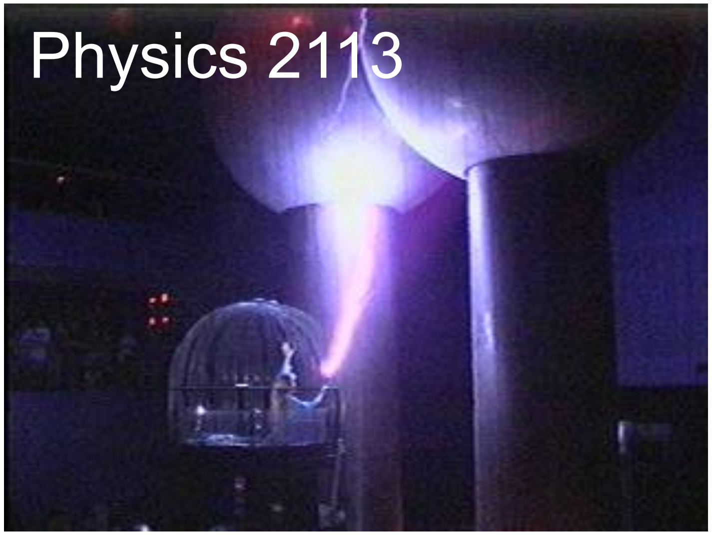
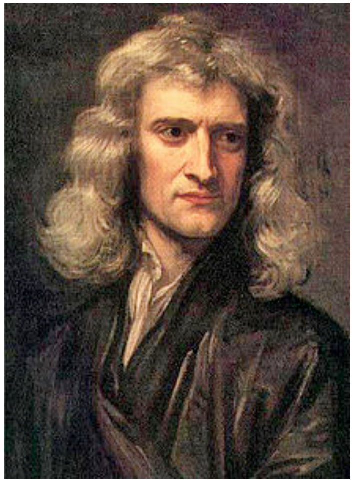




Isaac Newton
(1642–1727)

Physics 2113
Lecture 01: MON 25 AUG

CH13: Gravitation I

Michael Faraday
(1791–1867)

Who am I?

Prof. Jorge Pullin

1988: Ph.D. Argentina
1989-2001 Researcher at Syracuse NY, Salt Lake City, UT
1993–2001: Faculty at PennState
2001–Present: Director, Hearne Institute for Theoretical Physics, LSU

Office hours: Nicholson Hall 241C, MWF 2:30PM–3:30PM (or by appointment made by email)
Email: pullin@lsu.edu

My Own Research: General relativity, the big bang, black holes.
Other interests:

Course Details

• Main **Class Website for All Sections**:
www.phys.lsu.edu/classes/phys2113/
Syllabus, Schedule, Grading Policy, Exam Solutions, …

• **Lectures** will be posted in this section's website.

• **Text:** Fundamentals of Physics, Halliday, Resnick, and Walker, 9th edition. We will cover chapters 13, 21-33.

Exam I: 6:00–7:00PM TUE 16 SEP 2014
Exam II: 6:00–7:00PM TUE 14 OCT 2014
Exam III: 6:00–7:00PM TUE 11 NOV 2014
FINAL: 5:30-7:30PM MON 08 DEC 2014

• **Lab: PHYS 2109 Meets <u>This Week!</u> Show up or be dropped!**
• **Tutoring: Free Tutors in Middleton & 102 Nicholson Hall.**

Course Details: Homework

Web-based system: Web Assign

To register:

- Go to **http://www.webassign.net/student.html**
- On the left frame, "student login"
    - *Username*: pawsusername@lsu
    - *Institution*: lsu
    - *Password*: lsuidnumber
    - Section 3 Class Key: lsu 9088 4324
- Choose "credit card registration"

- One Assignment Per Week Due 11:59PM Fridays.

- First HW Is Posted Due THIS Friday 11:59PM.

Course Details: Grading

Midterms - 100 points each	Total: 300 points
Final Exam - 200 points	Total: 200 points
Homework - 100 points	Total: 100 points
No quizzes in this section.	
TOTAL:	600 points

Your numerical grade will be the total number of points you obtain, divided by 6.0.

Given your numerical grade, your letter grade will be at least:

A: 90–100 B: 75–89 C: 60–74 D: 50–59 F: <50

What Are We Going to Learn?

Ch. 13: Gravitation, Newton's Law of Gravity, superposition, gravitational potential energy, Gauss's Law, Kepler's Laws, escape speed, orbital mechanics (i.e., Rocket Science)

Ch. 21: Electric charge, conductors and insulators, Coulomb's Law, quantization and conservation principles for charge

Ch. 22: Electric fields, field maps, fields due to various charge geometries, point charges and dipoles in an electric field

Ch. 23: Electric flux, Gauss's Law for electric fields, Coulomb's Law from Gauss's Law, isolated charged conductors, considerations of symmetry

Ch. 24: Electric potential energy and work, electric potential, equipotentials, potentials due to discrete and continuous charge distributions, isolated conductors, determining the electric field from the potential

Ch. 25: Capacitors and capacitance, series and parallel arrangements, stored energy, dielectric materials, Gauss's Law with dielectric

Ch. 26: Electric current, current density, non-perfect conductors, resistivity and resistance, Ohm's Law, power and energy in electric circuits, semiconductor materials, superconductors

Ch. 27: DC circuits, energy and work, electromotive force, single and multi-loop circuits, parallel and series combinations of resistances, Kirchoff's Laws, RC circuits, time constant

Ch. 28: Magnetic fields, forces on moving charges, crossed fields, Hall effect, cyclotrons, force and torque on current carrying wires and loops, magnetic dipoles and dipole moment

Ch. 29: Sources of magnetic field, Biot-Savart Law, calculating the magnetic field for various current geometries, Ampere's Law, consideration of symmetry, forces between parallel currents, solenoids and toroids, a coil as a dipole

Ch. 30: Electromagnetic induction, Faraday's Law, Lenz's Law, induced electric fields, induction and inductors, RL circuits and time constants, energy stored in magnetic fields, energy density in magnetic fields, mutual inductance

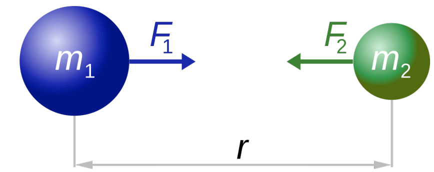
Ch. 31:Electromagnetic oscillators, series RLC circuit, transformers, forced oscillators, resonant circuits, damped oscillators

Ch. 32: Gauss's Law for magnetism, displacement currents, induced magnetic fields, Maxwell's equations, magnets and magnetic materials.

Ch. 33: Electromagnetic waves, electromagnetic spectrum, travelling EM waves, Poynting Vector, energy transport, radiation pressure, polarization, reflection and refraction.

13.2 Newton's Law of Gravitation:

$$F = G\frac{m_1 m_2}{r^2} \qquad \text{(Newton's law of gravitation).}$$



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Here m_1 and m_2 are the masses of the particles, r is the distance between them, and G is the gravitational constant.

$$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$
$$= 6.67 \times 10^{-11}\ \mathrm{m^3/kg\ s^2}.$$

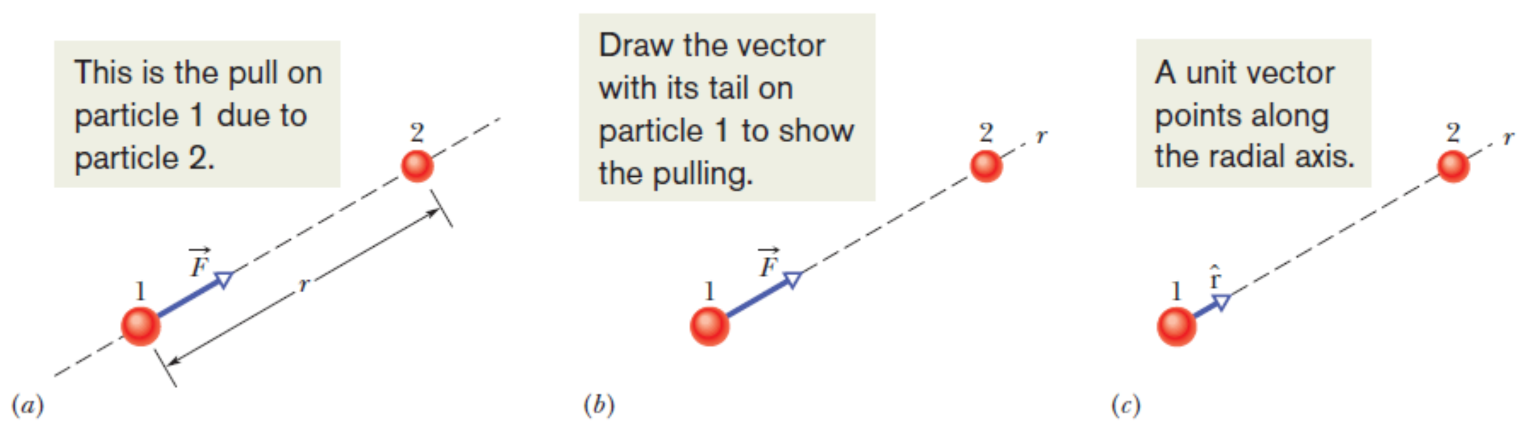


Fig. 13-2 (a) The gravitational force on particle 1 due to particle 2 is an attractive force because particle 1 is attracted to particle 2. (b) Force is directed along a radial coordinate axis r extending from particle 1 through particle 2. (c) is in the direction of a unit vector along the r axis.

13.2 Gravitation and the Principle of Superposition

For *n* interacting particles, we can write the principle of superposition for the gravitational forces on particle 1 as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}.$$

Here $\boldsymbol{F}_{1,\text{net}}$ is the net force on particle 1 due to the other particles and, for example, $\boldsymbol{F}_{13}$ is the force on particle 1 from particle 3, etc. Therefore,

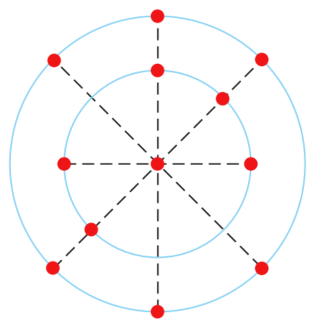
$$\vec{F}_{1,\text{net}} = \sum_{i=2}^{n} \vec{F}_{1i}.$$

The gravitational force on a particle from a real (extended) object can be expressed as:

$$\vec{F}_1 = \int d\vec{F},$$

Here the integral is taken over the entire extended object

How would you calculate total force on central mass if all masses equal?



13.2 Newton's Law of Gravitation: The Shell Game

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

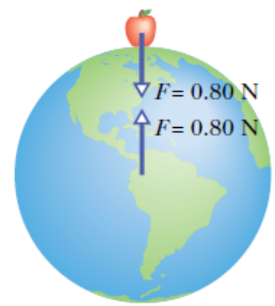
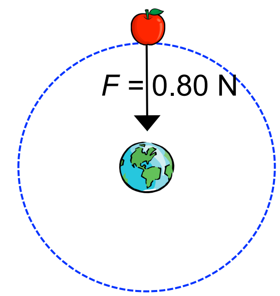
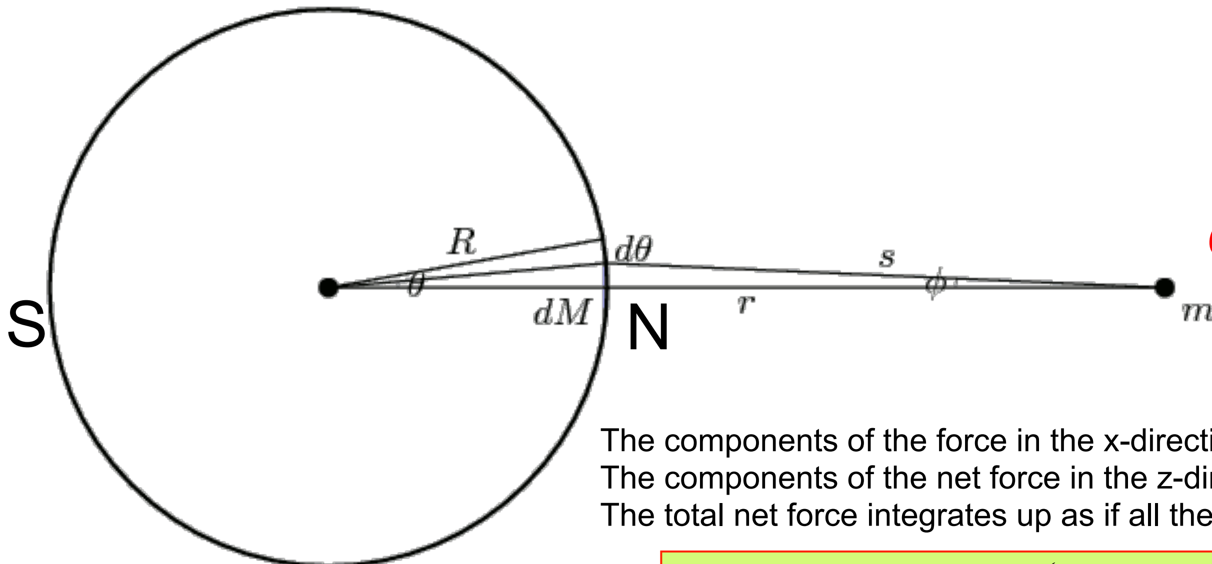
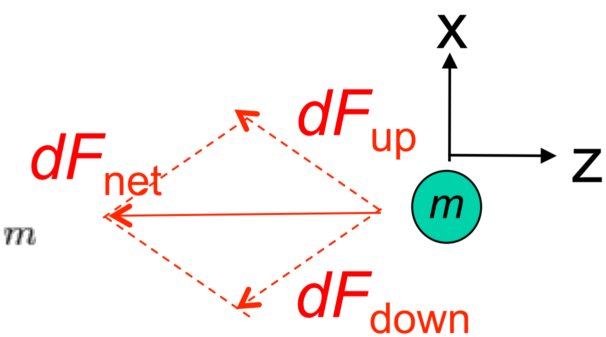


Fig. 13-3 The apple pulls up on Earth just as hard as Earth pulls down on the apple.



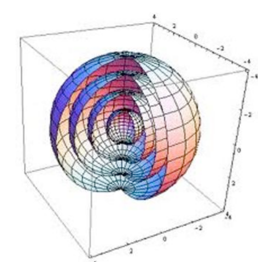
The force on the apple is the same if all the Earth's mass is squashed to the center!





The components of the force in the x-direction *cancel out* by symmetry.
The components of the net force in the z-direction *add up* by symmetry.
The total net force integrates up as if all the mass of the shell were at the center.

$$dF_{\text{net}}^{N} + dF_{\text{net}}^{S} = G \cdot dM \cdot m\left(\frac{1}{(r-R)^2} + \frac{1}{(r+R)^2}\right) \underset{r \gg R}{\cong} G \cdot dM \cdot m\frac{2}{r^2}$$



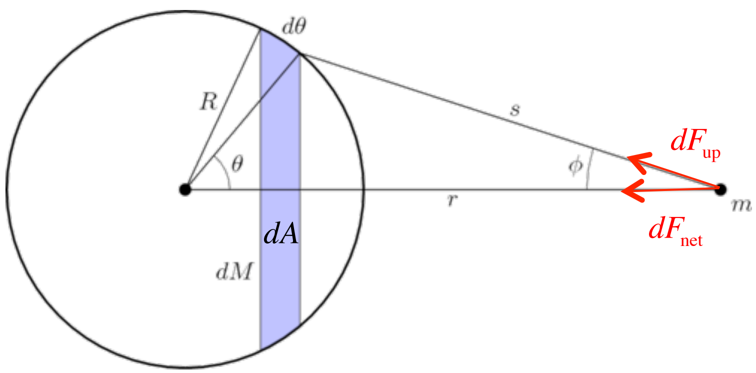
Applying the shell law to concentric shells proves can treat Earth (uniform sphere) as if all mass is at center.

13.2 Newton's Law of Gravitation: The Shell Game — Sketch of Proof

http://en.wikipedia.org/wiki/Shell_theorem

Calc III & trigmarole!

$$dF_{\text{up}} = \frac{G \cdot m \cdot dM}{s^2} \qquad \text{(Law of Gravitation)}$$



$$dF_{\text{net}} = dF_{\text{up}} \cos\phi = \frac{G \cdot m \cdot dM}{s^2} \cos\phi \qquad \text{(trig)}$$

$$\sigma = \frac{M}{4\pi R^2} \quad \text{(surface mass density of shell)}$$

$$dA = 2\pi R^2 \sin\theta d\theta \qquad dM = \sigma dA = \tfrac{1}{2} M \sin\theta d\theta$$

Law of Cosines

$$\boxed{\cos\phi = \frac{r^2 + s^2 - R^2}{2rs}}$$

$$F_{\text{net}} = \int_{\text{shell}} dF_{\text{net}} = \frac{GMm}{2} \int_{\text{shell}} \frac{\boxed{\cos\phi}\boxed{\sin\theta d\theta}}{s^2}$$

$$\cos\theta = \frac{r^2 + R^2 - s^2}{2rR}.$$

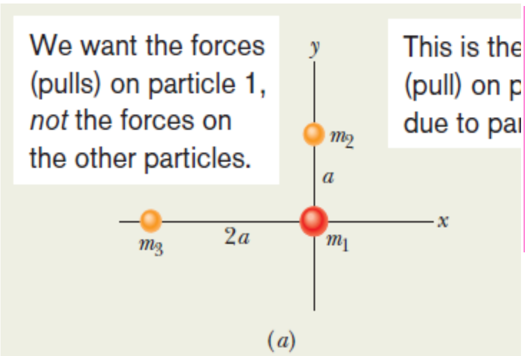
$$\boxed{\sin\theta \, d\theta = \frac{s}{rR} ds.}$$

$$F_{\text{net}} = \frac{GMm}{4r^2 R} \int_{r-R}^{r+R} \left(1 + \frac{r^2 - R^2}{s^2}\right) ds = \frac{GMm}{r^2}$$

QED

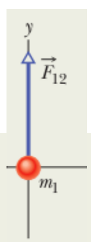
Sample Problem: Net Gravitational Force:

Figure 13-4a shows an arrangement of three particles, particle 1 of mass m_1= 6.0 kg and particles 2 and 3 of mass m_2=m_3=4.0 kg, and distance a =2.0 cm. What is the net gravitational force 1,net on particle 1 due to the other particles?

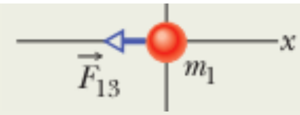


Which way does the force vector on m_1 point?

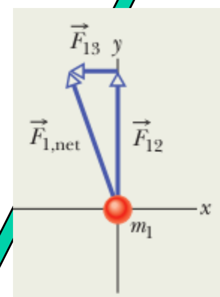
Calculations:



$$F_{12} = \frac{Gm_1m_2}{a^2}$$
$$= \frac{(6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2})(6.0\ \mathrm{kg})(4.0\ \mathrm{kg})}{(0.020\ \mathrm{m})^2}$$
$$= 4.00 \times 10^{-6}\ \mathrm{N}.$$



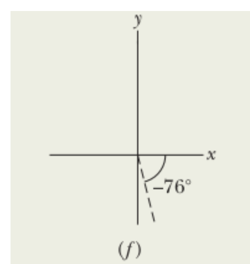
$$F_{13} = \frac{Gm_1m_3}{(2a)^2}$$
$$= \frac{(6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2})(6.0\ \mathrm{kg})(4.0\ \mathrm{kg})}{(0.040\ \mathrm{m})^2}$$
$$= 1.00 \times 10^{-6}\ \mathrm{N}.$$



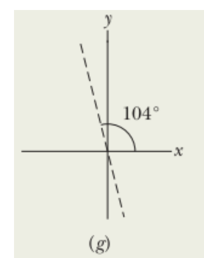
$$F_{1,\mathrm{net}} = \sqrt{(F_{12})^2 + (-F_{13})^2}$$
$$= \sqrt{(4.00 \times 10^{-6}\ \mathrm{N})^2 + (-1.00 \times 10^{-6}\ \mathrm{N})^2}$$
$$= 4.1 \times 10^{-6}\ \mathrm{N}. \qquad \text{(Answer)}$$

Relative to the positive direction of the x axis, the direction of $F_{1,\mathrm{net}}$ is:

$$\theta = \tan^{-1}\frac{F_{12}}{-F_{13}} = \tan^{-1}\frac{4.00 \times 10^{-6}\,\mathrm{N}}{-1.00 \times 10^{-6}\,\mathrm{N}} = -76°.$$



A calculator's inverse tangent can give this for the angle.



But this is the correct angle.

Note: In order to get the correct angle you MUST draw the vectors!

13.4: Gravitation Near Earth's Surface

If the particle is released, it will fall toward the center of Earth, as a result of the gravitational force , with an acceleration we shall call the gravitational acceleration a_g. Newton's second law tells us that magnitudes F and a_g are related by

$$F = ma_g.$$

If the Earth is a uniform sphere of mass M, the magnitude of the gravitational force from Earth on a particle of mass m, located outside Earth a distance r from Earth's center, is

$$F = G\frac{Mm}{r^2}.$$

Therefore,

$$a_g = \frac{GM}{r^2}.$$

Table 13-1

Variation of a_g with Altitude

Altitude (km)	a_g (m/s²)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

Earth is not uniform.

Earth is rotating.

$$F = ma_g - m\omega^2 R$$

Example, Difference in Accelerations

(a) An astronaut whose height h is 1.70 m floats "feet down" in an orbiting space shuttle at distance $r = 6.77 \times 10^6$ m away from the center of Earth. What is the difference between the gravitational acceleration at her feet and at her head?

KEY IDEAS

We can approximate Earth as a uniform sphere of mass M_E. Then, from Eq. 13-11, the gravitational acceleration at any distance r from the center of Earth is

$$a_g = \frac{GM_E}{r^2}. \qquad (13\text{-}15)$$

We might simply apply this equation twice, first with $r = 6.77 \times 10^6$ m for the location of the feet and then with $r = 6.77 \times 10^6 \text{ m} + 1.70 \text{ m}$ for the location of the head. However, a calculator may give us the same value for a_g twice, and thus a difference of zero, because h is so much smaller than r. Here's a more promising approach: Because we have a differential change dr in r between the astronaut's feet and head, we should differentiate Eq. 13-15 with respect to r.

Calculations: The differentiation gives us

$$da_g = -2\frac{GM_E}{r^3}\,dr, \qquad (13\text{-}16)$$

where da_g is the differential change in the gravitational acceleration due to the differential change dr in r. For the astronaut, $dr = h$ and $r = 6.77 \times 10^6$ m. Substituting data into Eq. 13-16, we find

$$da_g = -2\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.77 \times 10^6\ \text{m})^3}(1.70\ \text{m})$$
$$= -4.37 \times 10^{-6}\ \text{m/s}^2, \qquad \text{(Answer)}$$

where the M_E value is taken from Appendix C. This result means that the gravitational acceleration of the astronaut's feet toward Earth is slightly greater than the gravitational acceleration of her head toward Earth. This difference in acceleration (often called a *tidal effect*) tends to stretch her body, but the difference is so small that she would never even sense the stretching, much less suffer pain from it.

(b) If the astronaut is now "feet down" at the same orbital radius $r = 6.77 \times 10^6$ m about a black hole of mass $M_h = 1.99 \times 10^{31}$ kg (10 times our Sun's mass), what is the difference between the gravitational acceleration at her feet and at her head? The black hole has a mathematical surface (*event horizon*) of radius $R_h = 2.95 \times 10^4$ m. Nothing, not even light, can escape from that surface or anywhere inside it. Note that the astronaut is well outside the surface (at $r = 229R_h$).

Calculations: We again have a differential change dr in r between the astronaut's feet and head, so we can again use Eq. 13-16. However, now we substitute $M_h = 1.99 \times 10^{31}$ kg for M_E. We find

$$da_g = -2\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99 \times 10^{31}\ \text{kg})}{(6.77 \times 10^6\ \text{m})^3}(1.70\ \text{m})$$
$$= -14.5\ \text{m/s}^2. \qquad \text{(Answer)}$$

This means that the gravitational acceleration of the astronaut's feet toward the black hole is noticeably larger than that of her head. The resulting tendency to stretch her body would be bearable but quite painful. If she drifted closer to the black hole, the stretching tendency would increase drastically.

Gravitational Force vs. Gravitational Field

Gravitational Force:
(Units: Newtons = N)

$$F = -\frac{GmM}{r^2}$$

Gravitational Field:
(Units: m/s^2 = 100 Gal)

$$g = -\frac{GM}{r^2}$$

Given the Field, Find the Force:

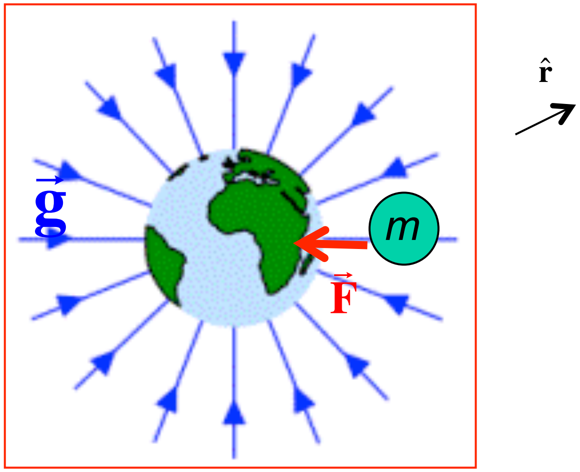
$$F = mg$$

Find the Force: (Vector Form)

$$\vec{\mathbf{g}} = -GM\frac{\hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{F}} = -m\vec{\mathbf{g}} = -GmM\frac{\hat{\mathbf{r}}}{r^2}$$

Gravitational Field Lines



Note: Field Exists in Empty Space Whether Test Mass *m* is There or Not!

Field strength~ how tightly packed lines are.

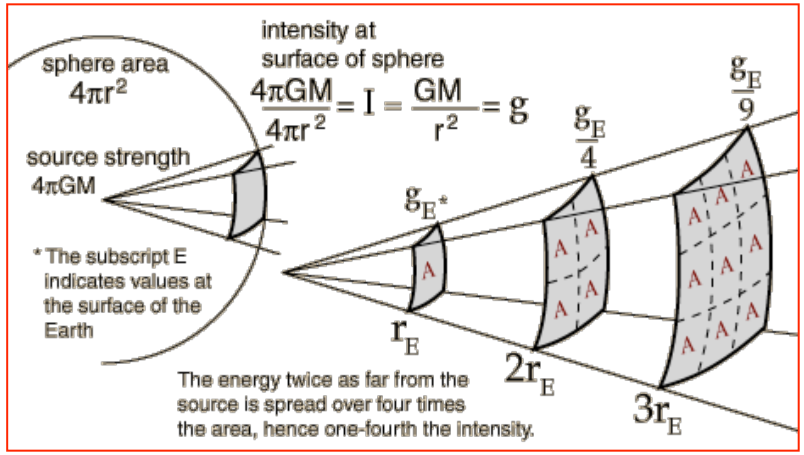
Gravitational Field:
Flux and Inverse Square Law

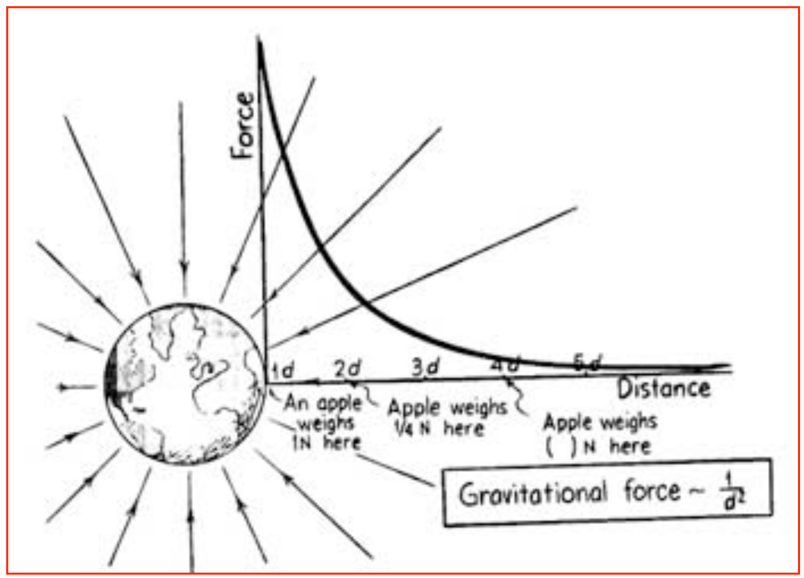
Total Number of Field Lines (= Flux) Is A Constant!

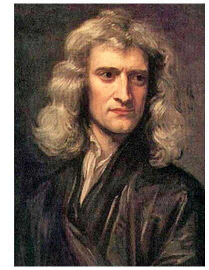
They Spread Out Over Surface of Expanding (Imaginary) Sphere!

Surface Area of Sphere Increases Like r^2

Hence Gravitational Field Strength (#Lines / Area) Decreases Like $1/r^2$

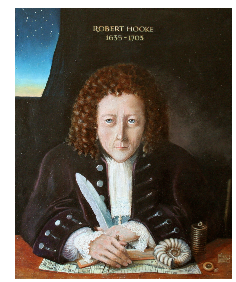






Isaac Newton

vs



Robert Hooke

Summary:

- Newton's law, force proportional to masses, inversely proportional to distance square, attractive, along line connecting particles.
- Forces can be superposed, summing them as vectors.
- Shells behave like point masses and as a consequence, spheres also do.