Formula Sheet for LSU Physics 2113, Final Exam, Fall '14

- **Constants, definitions:**

$g = 9.8\,\frac{\text{m}}{\text{s}^2}$ $\quad R_{Earth} = 6.37\times10^6\,\text{m}$ $\quad M_{Earth} = 5.98\times10^{24}\,\text{kg}$

$G = 6.67\times10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$ $\quad R_{Moon} = 1.74\times10^6\,\text{m}$ $\quad$ Earth-Sun distance $= 1.50\times10^{11}$ m

$M_{Sun} = 1.99\times10^{30}\,\text{kg}$ $\quad M_{Moon} = 7.36\times10^{22}\,\text{kg}$ $\quad$ Earth-Moon distance $= 3.82\times10^8$ m

$\epsilon_o = 8.85\times10^{-12}\,\frac{\text{C}^2}{\text{Nm}^2}$ $\quad k = \frac{1}{4\pi\epsilon_o} = 8.99\times10^9\,\frac{\text{Nm}^2}{\text{C}^2}$ $\quad e = 1.60\times10^{-19}\,\text{C}$

$c = 3.00\times10^8\,\text{m/s}$ $\quad m_p = 1.67\times10{-27}\,\text{kg}$ $\quad 1\text{ eV} = \text{e}(1\text{V}) = 1.60\times10^{-19}\,\text{J}$

dipole moment: $\vec{p} = q\vec{d}$ $\quad m_e = 9.11\times10^{-31}\,\text{kg}$ $\quad$ charge densities: $\lambda = \frac{Q}{L},\ \ \sigma = \frac{Q}{A},\ \ \rho = \frac{Q}{V}$

Area of a circle: $A = \pi r^2$ $\quad$ Area of a sphere: $A = 4\pi r^2$ $\quad$ Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Area of a cylinder: $A = 2\pi r\ell$ $\quad$ Volume of a cylinder: $V = \pi r^2\ell$

- **Units:**

Joule = J = N·m

- **Kinematics (constant acceleration):**

$v = v_o + at \qquad x - x_o = \frac{1}{2}(v_o + v)t \qquad x - x_o = v_o t + \frac{1}{2}at^2 \qquad v^2 = v_o^2 + 2a(x - x_o)$

- **Circular motion:**

$F_c = ma_c = \frac{mv^2}{r}, \quad T = \frac{2\pi r}{v}, \quad v = \omega r$

- **General (work, def. of potential energy, kinetic energy):**

$K = \frac{1}{2}mv^2 \qquad \vec{F}_{\text{net}} = m\vec{a} \qquad E_{\text{mech}} = K + U$

$W = -\Delta U$ (by field) $\quad W_{ext} = \Delta U = -W$ (if objects are initially and finally at rest)

- **Gravity:**

Newton's law: $|\vec{F}| = G\frac{m_1 m_2}{r^2}$

Gravitational acceleration (planet of mass M): $a_g = \frac{GM}{r^2}$

Gravitational Field: $\vec{g} = -G\frac{M}{r^2}\hat{r} = -\frac{dV_g}{dr}$

Gravitational potential: $V_g = -\frac{GM}{r}$

Law of periods: $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$

Potential Energy: $U = -G\frac{m_1 m_2}{r_{12}}$

Potential Energy of a System (more than 2 masses): $U = -\left(G\frac{m_1 m_2}{r_{12}} + G\frac{m_1 m_3}{r_{13}} + G\frac{m_2 m_3}{r_{23}} + \ldots\right)$

Gauss' law for gravity: $\oint_S \vec{g}\cdot d\vec{S} = -4\pi G M_{ins}$

- **Electrostatics:**

Coulomb's law: $|\vec{F}| = k\frac{|\,q_1\,||\,q_2\,|}{r^2}$ $\qquad$ Force on a charge in an electric field: $\vec{F} = q\vec{E}$

Electric field of a point charge: $|\vec{E}| = k\frac{|\,q\,|}{r^2}$

Electric field of a dipole on axis, far away from dipole: $\vec{E} = \frac{2k\vec{p}}{z^3}$

Electric field of an infinite line charge: $|\vec{E}| = \frac{2k\lambda}{r}$

Torque on a dipole in an electric field: $\vec{\tau} = \vec{p}\times\vec{E}$

Potential energy of a dipole in $\vec{E}$ field: $U = -\vec{p}\cdot\vec{E}$

- **Electric flux:** $\Phi = \int \vec{E} \cdot d\vec{A}$
- **Gauss' law:** $\epsilon_o \oint \vec{E} \cdot d\vec{A} = q_{enc}$
- **Electric field of an infinite non-conducting plane with a charge density** σ: $E = \dfrac{\sigma}{2\epsilon_o}$
- **Electric field of infinite conducting plane or close to the surface of a conductor:** $E = \dfrac{\sigma}{\epsilon_o}$
- **Electric potential, potential energy, and work:**

  $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$ **In a uniform field:** $\Delta V = -\vec{E} \cdot \Delta\vec{s} = -Ed\cos\theta$

  $\vec{E} = -\vec{\nabla}V, \quad E_x = -\dfrac{\partial V}{\partial x}, \quad E_y = -\dfrac{\partial V}{\partial y}, \quad E_z = -\dfrac{\partial V}{\partial z}$

  Potential of a point charge q: $V = k\dfrac{q}{r}$ **Potential of** n **point charges:** $V = \sum_{i=1}^{n} V_i = k\sum_{i=1}^{n} \dfrac{q_i}{r_i}$

  Electric potential energy: $\Delta U = q\Delta V \quad \Delta U = -W_{\text{field}}$

  Potential energy of two point charges: $U_{12} = W_{\text{ext}} = q_2 V_1 = q_1 V_2 = k\dfrac{q_1 q_2}{r_{12}}$
- **Capacitance: definition:** $q = CV$

  Capacitor with a dielectric: $C = \kappa C_{air}$ **Parallel plate:** $C = \varepsilon_\circ \dfrac{A}{d}$

  Potential Energy in Cap: $U = \dfrac{q^2}{2C} = \dfrac{1}{2}qV = \dfrac{1}{2}CV^2$ **Energy density of electric field:** $u = \dfrac{1}{2}\kappa\varepsilon_o|\vec{E}|^2$

  Capacitors in parallel: $C_{eq} = \sum C_i$ **Capacitors in series:** $\dfrac{1}{C_{eq}} = \sum \dfrac{1}{C_i}$
- **Current:** $i = \dfrac{dq}{dt} = \int \vec{J} \cdot d\vec{A}$, **Const. curr. density:** $J = \dfrac{i}{A}$, **Charge carrier's drift speed:** $\vec{v}_d = \dfrac{\vec{J}}{ne}$
- **Definition of resistance:** $R = \dfrac{V}{i}$ **Definition of resistivity:** $\rho = \dfrac{|\vec{E}|}{|\vec{J}|}$
- **Resistance in a conducting wire:** $R = \rho\dfrac{L}{A}$ **Temperature dependence:** $\rho - \rho_\circ = \rho_\circ\alpha(T - T_\circ)$
- **Power in an electrical device:** $P = iV$ **Power dissipated in a resistor:** $P = i^2 R = \dfrac{V^2}{R}$
- **Definition of** *emf* **:** $\mathcal{E} = \dfrac{dW}{dq}$
- **Resistors in series:** $R_{eq} = \sum R_i$ **Resistors in parallel:** $\dfrac{1}{R_{eq}} = \sum \dfrac{1}{R_i}$
- **Loop rule in DC circuits:** the sum of changes in potential across any closed loop of a circuit must be zero.
- **Junction rule in DC circuits:** the sum of currents entering any junction must be equal to the sum of currents leaving that junction.
- **RC circuit: Charging:** $q(t) = C\mathcal{E}(1 - e^{-\frac{t}{\tau_C}})$, Time constant $\tau_C = RC$, **Discharging:** $q(t) = q_o e^{-\frac{t}{\tau_C}}$
- **Magnetic Fields:**

  Magnetic force on a charge q: $\vec{F} = q\vec{v} \times \vec{B}$ **Lorentz force:** $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

  Hall voltage: $V = v_d B d = \dfrac{i}{nle}B$ d = width $\perp$ to field and i, l = thickness $\parallel$ to field and $\perp$ to i

Circular motion in a magnetic field: $qv_\perp B = \dfrac{mv_\perp^2}{r}$ with period: $T = \dfrac{2\pi m}{qB}$

Magnetic force on a length of wire: $\vec{F} = i\vec{L} \times \vec{B}$

Magnetic Dipole: $\vec{\mu} = Ni\vec{A}$ **Torque:** $\vec{\tau} = \vec{\mu} \times \vec{B}$ **Potential energy:** $U = -\vec{\mu} \cdot \vec{B}$

- **Generating Magnetic Fields:** $(\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{T \cdot m}}{\mathrm{A}})$

  Biot-Savart Law: $d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{i d\vec{s} \times \vec{r}}{r^3}$

  Magnetic field of a long straight wire: $B = \dfrac{\mu_0}{4\pi} \dfrac{2i}{r}$ **Magnetic field of a circular arc:** $B = \dfrac{\mu_0}{4\pi} \dfrac{i}{r} \phi$

  Force between parallel current carrying wires: $F_{ab} = \dfrac{\mu_0 i_a i_b}{2\pi d} L$

  Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$ **Magnetic field of a solenoid:** $B = \mu_0 i n$

  Magnetic field of a toroid: $B = \dfrac{\mu_0 i N}{2\pi r}$, **Magnetic field of a dipole on axis, far away:** $\vec{B} = \dfrac{\mu_0}{2\pi} \dfrac{\vec{\mu}}{z^3}$

- **Induction:**

  Magnetic Flux: $\Phi = \int \vec{B} \cdot d\vec{A}$

  Faraday's law: $\mathcal{E} = -\dfrac{d\Phi}{dt}$ $(= -N\dfrac{d\Phi}{dt}$ for a coil with N turns) **Motional emf:** $\mathcal{E} = BLv$

  Induced Electric Field: $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi}{dt}$

  Definition of Self-Inductance: $L = \dfrac{N\Phi}{i}$ **Inductance of a solenoid:** $L = \mu_0 n^2 A l$

  EMF (Voltage) across an inductor: $\mathcal{E} = -L\dfrac{di}{dt}$

  RL Circuit: Rise of current: $i = \dfrac{\mathcal{E}}{R}(1 - e^{-\frac{tR}{L}})$, Time constant: $\tau_L = \dfrac{L}{R}$, **Decay** of current: $i = i_0 e^{-\frac{tR}{L}}$

  Magnetic Energy: $U_B = \dfrac{1}{2} L i^2$ **Magnetic energy density:** $u_B = \dfrac{B^2}{2\mu_0}$

- **LC circuits:**

  Electric energy in a capacitor: $U_E = \dfrac{q^2}{2C} = \dfrac{CV^2}{2}$ **Magnetic energy in an inductor:** $U_B = \dfrac{Li^2}{2}$

  LC circuit oscillations: $q = Q\cos(\omega t + \phi)$ $\left(i = \dfrac{dq}{dt},\quad q = Cv\right)$ $\omega = \dfrac{1}{\sqrt{LC}}$ $T = \dfrac{2\pi}{\omega}$ $f = \dfrac{1}{T}$

- **Series RLC circuit:** $q(t) = Qe^{-Rt/(2L)} \cos(\omega' t + \phi)$ where $\omega' = \sqrt{\omega^2 - \left(\dfrac{R}{2L}\right)^2}$

- **Transformers:**

  Transformation of voltage: $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}$ **Turns ratio:** $\dfrac{N_p}{N_s}$ **Energy conservation:** $I_p V_p = I_s V_s$

- **Maxwell's Equations:**

  $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\epsilon_0}$ $\oint \vec{B} \cdot d\vec{A} = 0$ $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$ $\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \dfrac{d\Phi_E}{dt} + \mu_0 i_{enc}$

Displacement current: $i_d = \epsilon_0 \dfrac{d\Phi_E}{dt}$

Magnetization: $\vec{M} = \dfrac{\vec{\mu}}{\text{volume}}$

- **Electromagnetic Waves:**

  Wave traveling in +x direction: $E = E_m \sin(kx - \omega t)$ $\quad B = B_m \sin(kx - \omega t)$

  where $\vec{E} \perp \vec{B}$, the direction of travel is $\vec{E} \times \vec{B}$, $E_m/B_m = c$, $f\lambda = c$, $\lambda = 2\pi/k$

  Velocity of light in vacuum $= c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}}$

  Energy flow: $\vec{S} = \dfrac{1}{\mu_0} \vec{E} \times \vec{B}$ $\quad I = \dfrac{1}{2c\mu_0} E_m^2$ $\quad E_{rms} = \dfrac{E_m}{\sqrt{2}}$ $\quad I = \dfrac{P}{Area}$

  Radiation force and pressure: total absorption: $F_r = \dfrac{IA}{c}$, $p_r = \dfrac{I}{c}$ total reflection: $F_r = \dfrac{2IA}{c}$, $p_r = \dfrac{2I}{c}$

- **Polarizing Sheets:**

  Unpolarized → polarized: $I = \frac{1}{2} I_0$

  Polarized → polarized: $I = I_0 \cos^2 \theta$

- **Reflection/refraction:**

  Law of reflection: $\theta_i = \theta_r$

  Law of refraction: $n_2 \sin \theta_2 = n_1 \sin \theta_1$

  Total internal reflection (critical angle): $\theta_c = \sin^{-1} \dfrac{n_2}{n_1}$

  Polarization by reflection (Brewster's angle): $\theta_B = \tan^{-1} \dfrac{n_2}{n_1}$