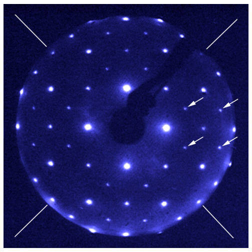
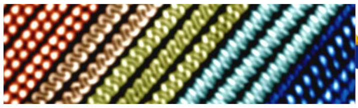
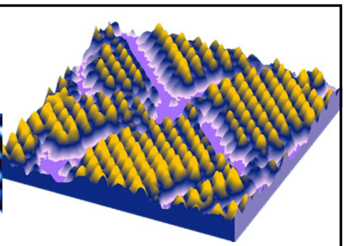


Physics 2203, Fall 2012
Modern Physics





- Monday, Sept. 10th, 2012:
- --Duality and then Ch. 5
  - --Experimental Rydberg Series.
  - --Bohr Model of H atom
  - --Shortcomings of Bohr Model
  - --Hydrogen-Like Ions –He^{+1} spectrum
  - --Moseley diagram.
  - --Electron Loss Spectroscopy
  - --Franck-Hertz Experiment
- Wednesday:
  - --meet in 106 and then go to 335 for computer exercise

Example for Term paper: Plasmonics

1. Find a general review article: Scientific American, Physics Today, web.
2. Write the introduction to the paper about the basic physics. In this case what is a plasmon? How can you control the plasmon energy by the size of the nanoparticle? What materials are good for plasmonics and why?
3. Pick one special area to describe in detail. For example, with plasmonics it could be quantum computing, or medical physics? If you are interested in the medical applications see the web site of Naomi Halas from Rice.
4. The paper should have the format of a Physical Review Letter—4 pages, two columns, 4 figures and full references.

Particle-Wave Duality

EM waves can act as waves and as particles: The Compton Experiment showed this.

The wave nature is characterized by

$y(x,t) = A\sin(kx - \omega t)$

$\lambda \rightarrow$ wavelength

$k = \frac{2\pi}{\lambda} \rightarrow \textit{wave}$ number

$\mathrm{c} \rightarrow$ the speed of light

$\mathrm{T} \rightarrow$ period

$\omega = \frac{2\pi}{T} \rightarrow \textit{angular}$ frequency

The particle nature is characterized by

$p \rightarrow$ momentum

$E \rightarrow$ Energy

$pc = E$

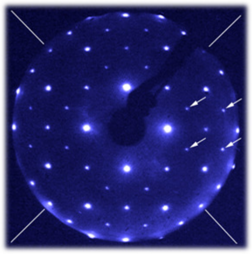
$E = hf = \hbar\omega =$

$P = \frac{E}{c} = \frac{hf}{c} = \frac{h}{cT} = \frac{h}{\lambda}$

Particles have wave nature: De Broglie

$$E = hf = \hbar\omega$$

$$p = \frac{h}{\lambda} = \hbar k$$



Electron Diffraction

Heisenberg Uncertainty Principle

You can't simultaneously measure the position and momentum.

You can't simultaneously measure the energy and the time.

$$\boxed{\begin{aligned}\Delta x \Delta k &\approx \frac{1}{2} \\ \Delta x \Delta p &\approx \frac{\hbar}{2}\end{aligned}}$$

$$\boxed{\begin{aligned}\Delta t \Delta \omega &\approx \frac{1}{2} \\ \Delta t \Delta E &\approx \frac{\hbar}{2}\end{aligned}}$$

Werner Heisenberg
(1901-1976) German

1932 Nobel Prize
"for the creation of Quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen"

Particle-Wave Duality

1911 Solvay Conference, held in Brussels from 29 October to 4 November. There were twenty-two physicists from Holland, France, England, Germany, Austria, and Denmark being convened to discuss 'current questions concerning the molecular and kinetic theories'. It was the first international meeting devoted to a specific agenda in contemporary physics: the quantum.

Planck and Einstein were among the eight asked to prepare reports on a particular topic. To be written in French, German, or English they were to be sent out to the participants before the meeting and serve as the starting point for discussion during the planned sessions. Planck would discuss his blackbody radiation theory, while Einstein had been assigned his quantum theory of specific heat. Accorded the honor of giving the final talk, there was no room on the proposed agenda for a discussion of his light-quanta – better known these days as photons.

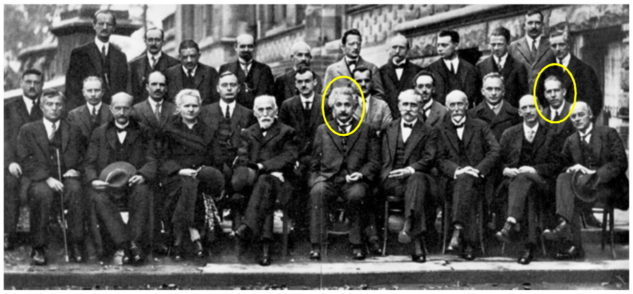
http://manjitkumar-reviewsarticles.blogspot.com/2011/12/solvay-1911.html

Particle-Wave Duality

1927 Solvay Conference, held in Belgium, was attended by the world's most notable physicist to discuss the new quantum theory.

How many do you recognize?

The major argument was between **Bohr** and **Einstein**, which continued until Einstein's death in 1955.



http://www.youtube.com/watch?v=8GZdZUouzBY

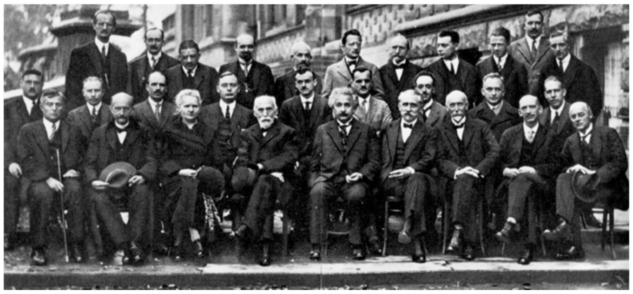
Einstein was very unhappy about this apparent randomness in nature. His views were summed up in his famous phrase, ***'God does not play dice'.*** He seemed to have felt that the uncertainty was only provisional: but that there was an underlying reality, in which particles would have well defined positions and speeds, and would evolve according to deterministic laws, in the spirit of Laplace. This reality might be known to God, but the quantum nature of light would prevent us seeing it, except through a glass darkly.

Bohr's famous response was. ***'Einstein, stop telling God what to do'.***

http://www.youtube.com/watch?v=7GTCus7KTb0

Particle-Wave Duality

1927 Solvay Conference, held in Belgium, was attended by the world's most notable physicist to discuss the new quantum theory.



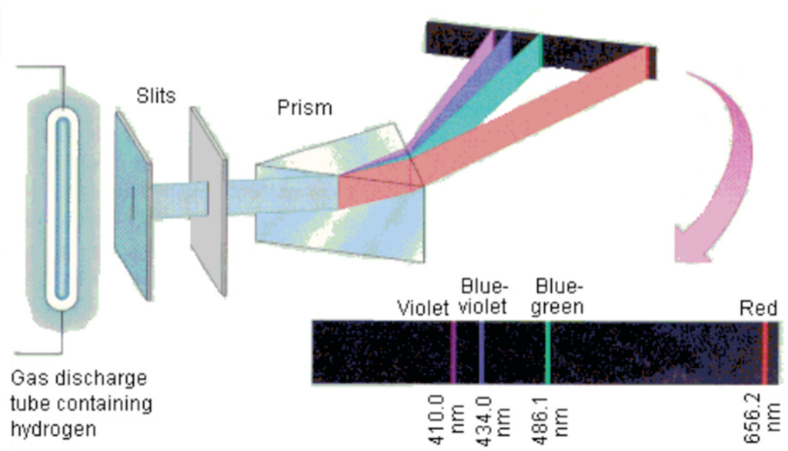
Look at Steven Hawking's lecture: http://www.hawking.org.uk/does-god-play-dice.html

Einstein's thought experiments: The Einstein-Bohr Debates, Bell inequality

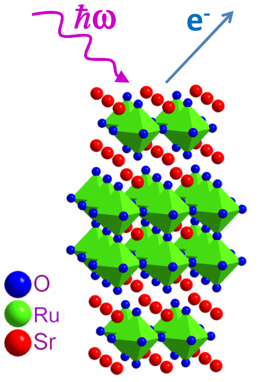
Could be a good term paper!

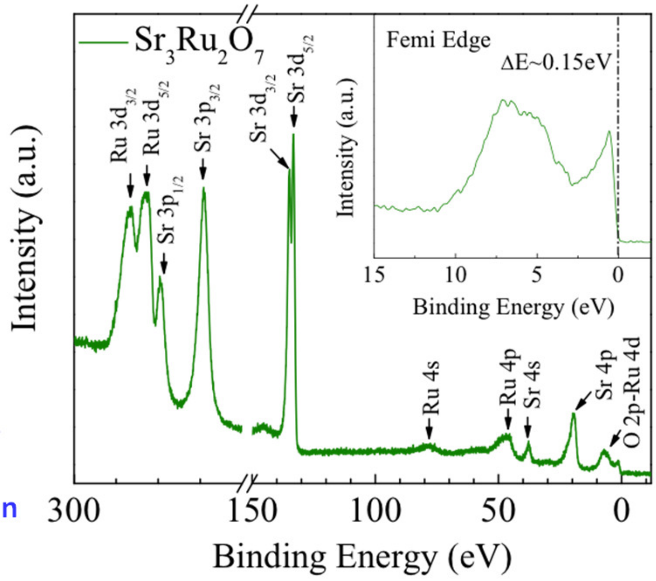
http://www.youtube.com/watch?v=7GTCus7KTb0

Ch. 5: Quantization of Atomic Energy Levels
Atomic Spectra



Atomic core-level from XPS (from downstairs)





The spectra provide the signature of chemical elements in a material !

Atomic Spectra: Hydrogen

Experimentalist had measured very carefully the emission and absorption spectra of H. They found an empirical formula to describe their observations.

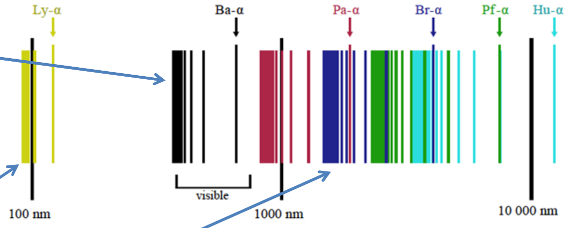
In 1885 Balmer described the visible spectrum: Balmer Series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right): \; n = 3, 4, 5, .$$

$R = 0.0110 \; nm$

In general

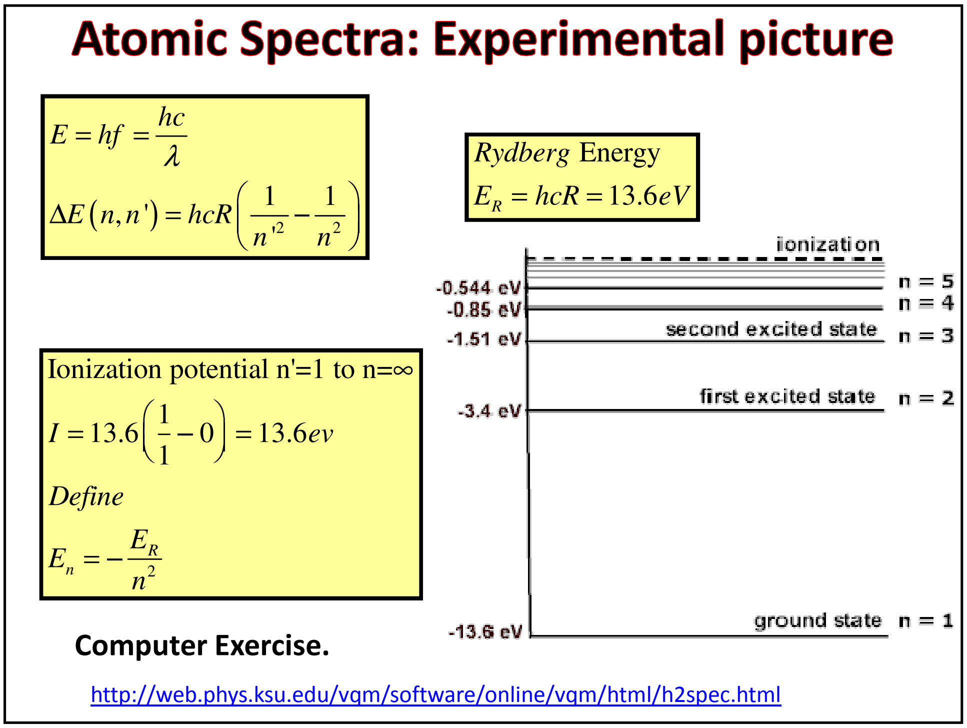
$$\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$



Lyman is n’=1

Paschen is n’=3

$$\boxed{\begin{gathered} E = \frac{hc}{\lambda} \\ E(n,n') = hcR\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \end{gathered}}$$



Atomic Spectra: Experimental picture

$$E = hf = \frac{hc}{\lambda}$$

$$\Delta E(n,n') = hcR\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)$$

Rydberg Energy

$E_R = hcR = 13.6eV$

Ionization potential n'=1 to n=∞

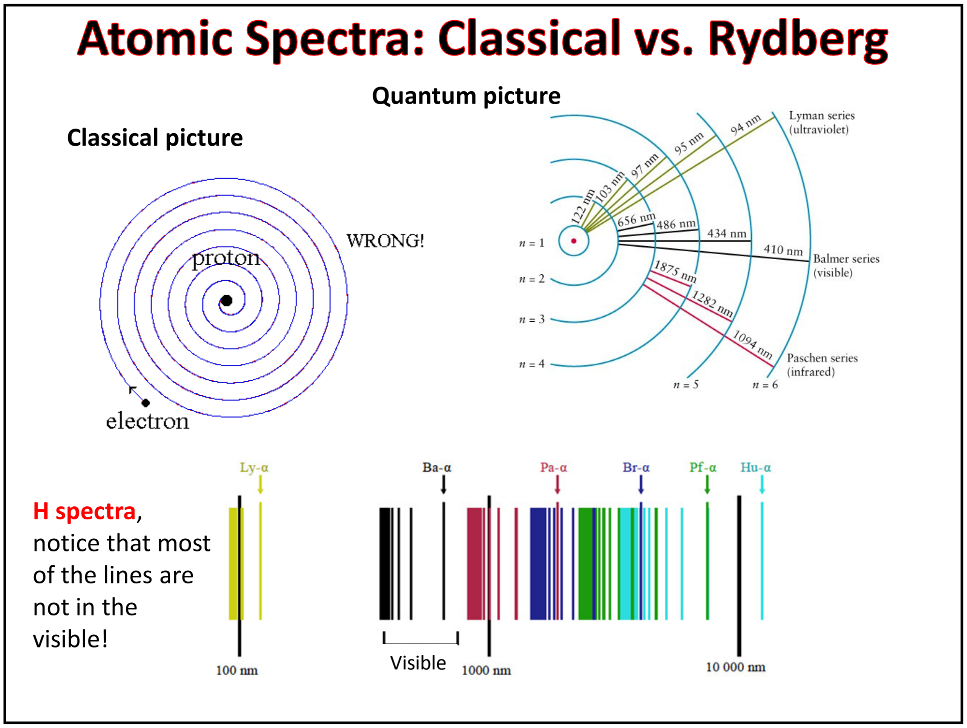
$$I = 13.6\left(\frac{1}{1}-0\right) = 13.6ev$$

Define

$$E_n = -\frac{E_R}{n^2}$$

Computer Exercise.

http://web.phys.ksu.edu/vqm/software/online/vqm/html/h2spec.html



Atomic Spectra: Classical vs. Rydberg

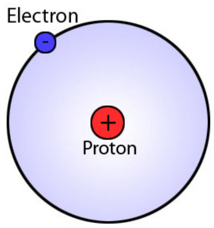
Quantum picture

Classical picture

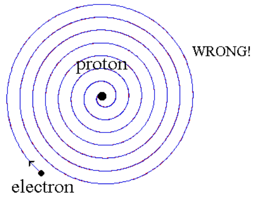
H spectra, notice that most of the lines are not in the visible!

Bohr Model: 1913

Rutherford's model of the atom appeared in 1911. The nucleus is fm is size and the atom in nm in size. Mostly free space.



Rutherford was aware that his model created terrible problems. Unlike planets revolving around the sun electrons moving around a proton MUST radiate EM radiation and consequently collapse into the nucleus.



Bohr in 1913 solved this problem by a simple postulate. ***Electrons can only exist is quantized orbits,*** called **stationary orbits** or **stationary states.**

Bohr did not explain why the electron does not radiate—he just said they don't.

Bohr Model: 1913

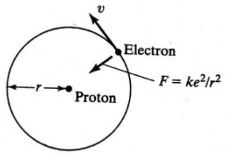
Bohr immediately realized that he had a way to explain the emission and absorption spectra. His atom had quantized energy states, E_1, E_2, E_3, …… Light could be emitted or absorbed when a transition occurred.

$$E_n \to E_{n'}$$
$$If\ \ E_n > E_{n'}\ \ Emission$$
$$If\ \ E_n < E_{n'}\ \ Absorption$$

Bohr's Model: The angular momentum L of each quantum state is quantized.

$$\boxed{\begin{array}{l} L_n = n\hbar = n\dfrac{h}{2\pi} \\ \text{Classically} \quad L = mvr \end{array}}$$

Classical model



$$\boxed{\begin{array}{l} F = \dfrac{ke^2}{r^2} = ma \\ m\dfrac{v^2}{r} = \dfrac{ke^2}{r^2} \\ \text{centripetal acc.} \end{array}}$$

$$\boxed{\begin{array}{l} E_K = \dfrac{mv^2}{2} = \dfrac{ke^2}{2r} \\ U = -\dfrac{ke^2}{r} \\ E_K = -\dfrac{U}{2} \end{array}}$$

Bohr Quantization Condition

$$L_n = n\hbar = n\frac{h}{2\pi}$$

Classically L=mvr

$$mrv = n\hbar$$

$$\text{and } mv^2 = \frac{ke^2}{r}$$

$$m\left(\frac{n\hbar}{mr}\right)^2 = \frac{ke^2}{r}$$

$$r_n = \frac{n^2\hbar^2}{ke^2 m} = n^2 a_B$$

$a_B = 0.0529$ nm : Bohr Radius

$$E_n = -\frac{ke^2}{r_n} = -\frac{ke^2}{2a_B}\frac{1}{n^2} \quad (n\ 1,2,\ 3,\ \ldots)$$

$$E_R = \frac{ke^2}{2a_B} = \frac{m\left(ke^2\right)^2}{2\hbar^2} = 13.6\ eV \quad \text{(Rydberg energy)}$$

$$E_n = -\frac{E_R}{n^2}$$

Velocity

$$v_n = \frac{n\hbar}{mr_n}$$

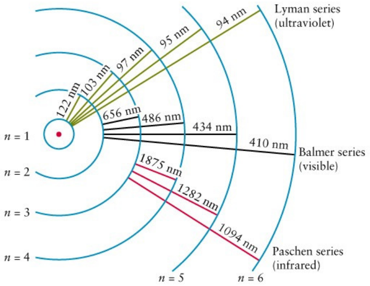
$$v_n = \frac{n\hbar\left(mke^2\right)}{m\left(n^2\hbar^2\right)} = \frac{ke^2}{n\hbar}$$

$$v_n = \frac{2.19x10^6\, m/s}{n}$$

Bohr: H spectra

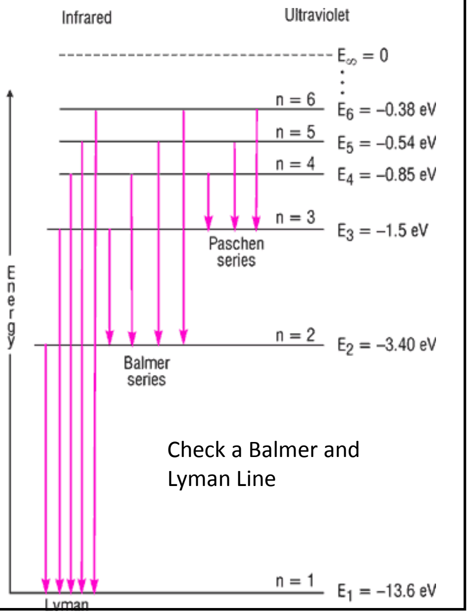
$$\Delta E = hf = E_R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) : \text{n'<n}$$

Emission



$$f = \frac{c}{\lambda} = \frac{E_R}{h}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

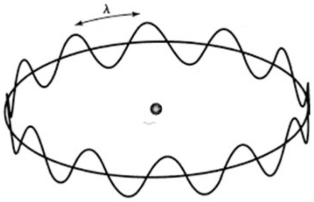
$$\lambda(n,n') = \frac{ch}{E_R}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)^{-1} = 91.2nm\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)^{-1}$$



Check a Balmer and Lyman Line

Bohr Model: What did he really do?

In **1926 De Broglie** described the wave nature of a particle. He started with the Bohr quantized orbits. He said that if the electron is in a stable orbit than **$2\pi r = n\lambda$.**



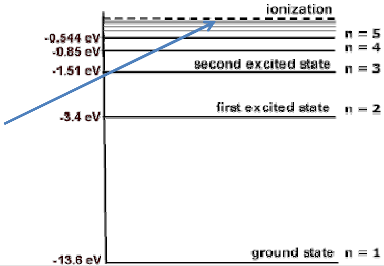
Putting this into **Bohr's quantization** gives the relationship for the wavelength of a particle.

$$L = mvr = n\hbar$$

$$\text{So } p = mv = \frac{n\hbar}{r} = \frac{n\hbar}{\frac{n\lambda}{2\pi}} = \frac{h}{\lambda}$$

Bohr explained this using the **Correspondence Principle.**
"In the limit of large orbits and large energies, quantum calculations must agree with classical calculation."

Another way of stating the **Correspondence Principle.** *"In the region of very large quantum numbers (n is this case) quantum and classical calculations must yield the same results."*



Bohr Model: Correspondence Principle

Look at a transition from n to n-1 for large n

$$hf\left(n \rightarrow n-1\right) = \frac{m\left(ke^2\right)^2}{2\hbar^2}\left(\frac{1}{\left(n-1\right)^2} - \frac{1}{n^2}\right) = \frac{m\left(ke^2\right)^2}{2\hbar^2}\left(\frac{n^2-\left(n-1\right)^2}{n^2\left(n-1\right)^2}\right)$$

$$f\left(n \rightarrow n-1\right) = \frac{c}{\lambda} = \frac{m\left(ke^2\right)^2}{4\pi\hbar^3}\left(\frac{2n-1}{n^2\left(n-1\right)^2}\right) \approx \frac{m\left(ke^2\right)^2}{2\pi\hbar^3 n^3}$$

Now look at the classical definition of f: $f = \frac{v}{2\pi r}$

Lets use our quantized definitions for r and v

$$r = \frac{n^2\hbar^2}{mke^2} \text{ and } v = \frac{n\hbar}{mr}$$

$$f = \frac{v}{2\pi r} = \frac{\left(\frac{n\hbar}{mr}\right)}{2\pi r}$$

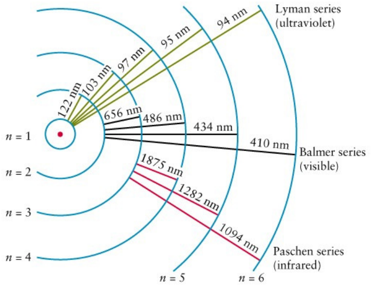
$$f = \frac{n\hbar}{2\pi m r^2} = \frac{n\hbar}{2\pi m\left(\frac{n^2\hbar^2}{mke^2}\right)^2} = \frac{m\left(ke^2\right)^2}{2\pi\left(n\hbar\right)^3}$$

QED

Bohr Model: Real Numbers for H

For a hydrogen atom, use the Bohr model to calculate the following.

(a) The orbit radius as a function of n.
(b) The velocity (can we use non-relativistic equations)?
(c) The angular momentum.
(d) The period of the orbit.
(e) The kinetic energy.
(f) The potential energy.
(g) The total energy.



Bohr Model: Real Numbers for H

For a hydrogen atom, use the Bohr model to calculate the following.

(a) The orbit radius as a function of n.
(b) The velocity, can we use non-relativistic equations?
(c) The angular momentum.
(d) The period of the orbit..

UNITS

$\hbar = 6.58\times10^{-16}\,\mathrm{eV\cdot s}$

$\hbar c = 197\ \mathrm{eV\cdot nm}$

$ke^2 = 1.44\ \mathrm{eV\cdot nm}$

$\frac{ke^2}{\hbar c} = \frac{1}{137}$ Fine-structure constant

$mc^2 = 0.511\ \mathrm{MeV}$

(a) The radius vs n.

$$r_n = \frac{n^2\hbar^2}{ke^2 m}$$

The radius vs n.

$$r_n = \frac{n^2(\hbar c)^2}{ke^2 mc^2} = \frac{n^2(137)197\ \mathrm{eV\cdot nm}}{0.511\times10^6\ \mathrm{eV}} = n^2 0.5282\ \mathrm{nm}$$

(b) The velocity vs n.

$$v_n = \frac{n\hbar}{mr_n} = \frac{ke^2}{n\hbar}$$

The velocity: $v_n = \frac{ke^2}{n\hbar} = \frac{ke^2 c}{n\hbar c} = \frac{(1.44\ \mathrm{eV\cdot nm})c}{n(197\ \mathrm{eV\cdot nm})} = 0.0073\frac{c}{n}$

(c) $L_n = n\hbar$

$$L_n = n\hbar = n\left(6.58\times10^{-16}\mathrm{eV\cdot s}\right):\ \ L_n c = n\hbar c = n\left(197\ \mathrm{eV\cdot nm}\right)$$

(d) $T = \frac{2\pi r_n}{v_n} = \frac{2\pi n^3\hbar^3}{m\left(ke^2\right)^2}$

$$T = \frac{2\pi n^3\hbar}{(mc^2)}\left(\frac{\hbar c}{ke^2}\right)^2 = \frac{2\pi n^3\left(6.58\times10^{-16}\mathrm{eV\cdot s}\right)(137)^2}{\left(0.511\times10^6 eV\right)} = n^3(1.52\times10^{-16}s)$$

Bohr Model: Real Numbers for H

For a hydrogen atom, use the Bohr model to calculate the following.

(a) The orbit radius as a function of n.
(b) The velocity, can we use non-relativistic equations?
(c) The angular momentum.
(d) The period of the orbit.
(e) The kinetic energy.
(f) The potential energy.
(g) The total energy.

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$\hbar = 6.58\times10^{-16}\,\mathrm{eV\cdot s}$
$\hbar c = 197\ \mathrm{eV\cdot nm}$
$ke^2 = 1.44\ \mathrm{eV\cdot nm}$
$\frac{ke^2}{\hbar c} = \frac{1}{137}$ Fine-structure constant
$mc^2 = 0.511\ \mathrm{MeV}$

(e) Kinetic energy $= mv_n^2/2$

$$K_n = \frac{m}{2}\left(\frac{ke^2}{n\hbar}\right)^2$$

$$K_n = \frac{mc^2}{2n^2}\left(\frac{ke^2}{\hbar c}\right)^2 = \frac{\left(0.511\times10^6 eV\right)}{2n^2(137)^2} = \frac{13.6eV}{n^2}$$

(f) Potential Energy

$$U_n = -\frac{ke^2}{r_n} = -\frac{m\left(ke^2\right)^2}{n^2(\hbar)^2}$$

$$U_n = -\frac{mc^2\left(ke^2\right)^2}{n^2(\hbar c)^2} = \frac{-0.511\times10^6 eV}{n^2(137)^2} = -\frac{27.2eV}{n^2} \quad (Rydberg)$$

(g) Total energy $= K + U = -\frac{m\left(ke^2\right)^2}{2n^2\hbar^2}$

$$E = K + U = -\frac{mc^2\left(ke^2\right)^2}{2n^2(\hbar c)^2} = -\frac{13.6eV}{n^2}$$

Bohr Model: Real Numbers for H-Check

For a hydrogen atom, use the Bohr model to calculate the following.

(a) The orbit radius as a function of n.
(b) The velocity, can we use non-relativistic equations?
(c) The angular momentum.
(d) The period of the orbit..

UNITS
$\hbar = 6.58\times10^{-16}\,\mathrm{eV\cdot s}$
$\hbar c = 197\ \mathrm{eV\cdot nm}$
$ke^2 = 1.44\ \mathrm{eV\cdot nm}$
$\frac{ke^2}{\hbar c} = \frac{1}{137}$ Fine-structure constant
$mc^2 = 0.511\ \mathrm{MeV}$

$$r_n = \frac{n^2\hbar^2}{ke^2 m}$$

$$v_n = \frac{n\hbar}{mr_n} = \frac{ke^2}{n\hbar}$$

$$T = \frac{2\pi n^3\hbar^3}{m\left(ke^2\right)^2}$$

Lets do a check.

$v_n T_n = 2\pi r_n$

$$\left(\frac{ke^2}{n\hbar}\right)\left(\frac{2\pi n^3\hbar^3}{m\left(ke^2\right)^2}\right) = 2\pi\left(\frac{n^2\hbar^2}{ke^2 m}\right)$$

$$\left(\frac{1}{n\hbar}\right)\left(\frac{n^3\hbar^3}{m}\right) = \left(\frac{n^2\hbar^2}{m}\right)$$

$1 = 1$ QED

Bohr Model: Shortcomings

1) **Bohr** did not believe in photons, so he stuck to a classical picture of excitation.

2) The angular momentum of the true quantum ground state is L=0, not L=ħ. There were attempts by Sommerfeld to fix this—elliptical orbits with no area, but experiment would show the ground state to be spherical in shape.

3) The **Bohr model** could not determine the intensity of a given line in the spectra.

4) The **Bohr model** has real trouble with larger atoms.

5) The **Bohr model** can't explain the fine structure you see in the spectra, like the Zeeman Effect..

6) The **Bohr model** violates the uncertainty principle.

Bohr Model: Refinements

1) Elliptical orbits. Bohr-Sommerfeld Model. $\int_0^T p_r dq_r = nh$

2) Use reduced mass . $m_r = \frac{m_e m_p}{m_e + m_p} = m_e \frac{1}{1 + m_e / m_p}$

Calculate the difference in the the wavelength for the first Balmer line when the reduced mass is used.

Calculate the quantized energies, velocities, radii for **positronium**.