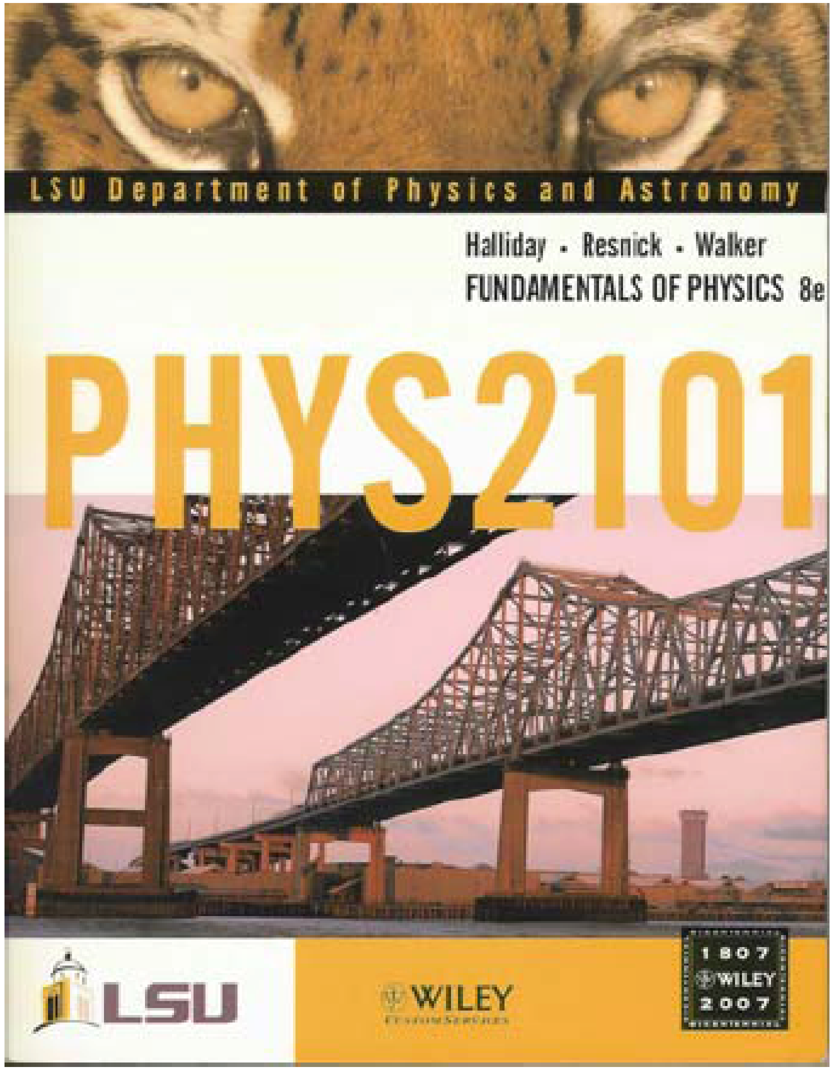


Physics 2101
Section 6
Oct. 25th:
Ch.13

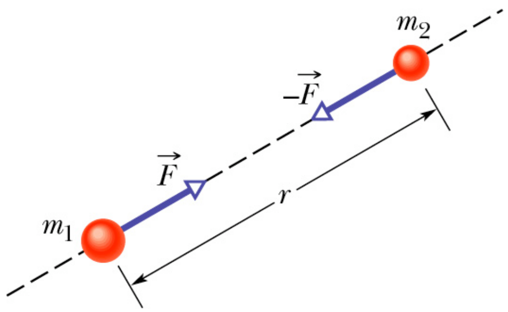
Announcement:

- There is a class on Saturday (10/27) 1:30-3:00

Lecture Notes:
http://www.phys.lsu.edu/classes/fall2012/phys2101-6/

Quick Review: Gravitation

For 2 particles the magnitude of the attractive force between them is



$$|F| = G\frac{m_1 m_2}{r^2}$$

m_1 and m_2 are masses and r is distance between them and...

$$G = 6.67\times10^{-11} N\cdot m^2/kg^2$$
$$= 6.67\times10^{-11} m^3/kg\cdot s^2$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton's third law

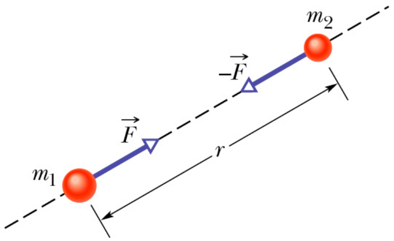
Gravitational Constant
(≠ g, ≠ 9.8 m/s²)

$$g = G\frac{m_{earth}}{r_{earth}^2}$$

Quick Review: Gravitation Notes

1) All objects -- independent of each other (Newton's 3rd Law)

2) Gravitational Force is a VECTOR - unit vector notation



$$\vec{F}_{12} = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$ Force on m_1 due to m_2 $$\hat{r}_{12} = \frac{\vec{r}_{12}}{|r_{12}|}$$

$$\vec{F}_{21} = G\frac{m_1 m_2}{r_{21}^2}\hat{r}_{21}$$ Force on m_2 due to m_1 $$\hat{r}_{21} = \frac{\vec{r}_{21}}{|r_{21}|} = -\hat{r}_{12}$$

$\longrightarrow$ $$\left|\vec{F}_{21}\right| = \left|\vec{F}_{12}\right|$$ $$\vec{F}_{21} = -\vec{F}_{12}$$

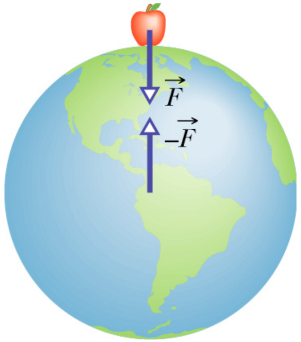
3) Principle of superposition

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n} = \sum_{i=1}^{n}\vec{F}_{1i}$$ VECTOR ADDITION!!

4) A uniform spherical shell of matter attracts an object on the outside as if all the shell's mass were concentrated at its center (note: this defines the position)

height = R_E + h

Gravitation and the earth



$$\left|F_g\right| = \left(G\frac{M_E}{R_E^2}\right)m_{apple} = m_{apple}\,g$$

Net force points towards center of earth

g differs around the earth (equator-9.780 & north pole-9.832 m/s^2)

1) Earth is not a perfect sphere - height (R_E is not constant):
- On Mount Everest (8.8 km) g=9.77 m/s^2 (0.2% smaller)
- At Equator earth bulges by 21 km

2) Earth is not uniform density: "gravity irregularities" (10^{-6}-10^{-7})g
gravimeters can measure down to 10^{-9}g

2) Earth is rotating: centripetal force makes apparent weight change

At poles:

$$W - mg = 0$$
$$W = mg$$

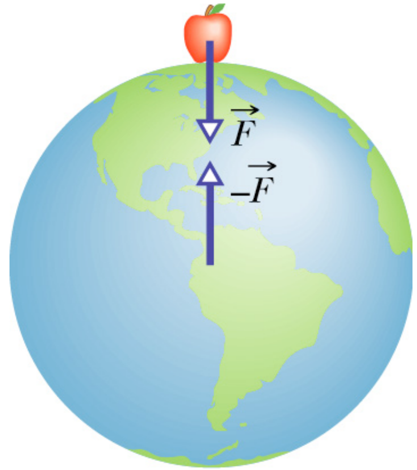
At equator:

$$W - mg = -m\left(\frac{v^2}{R_E}\right)$$
$$W = m\left(g - \frac{v^2}{R_E}\right)$$

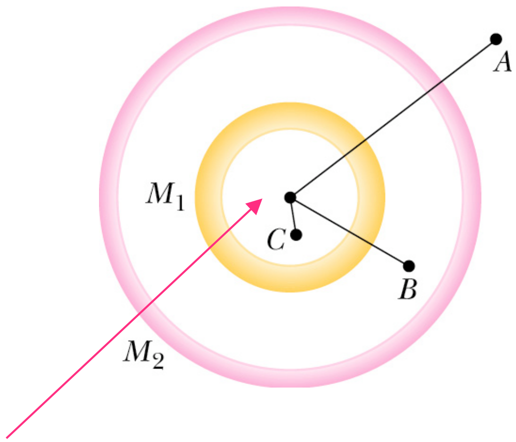
Weight is less (0.3%)

Gravity and Spheres

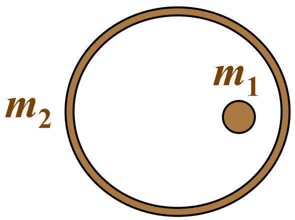
- A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

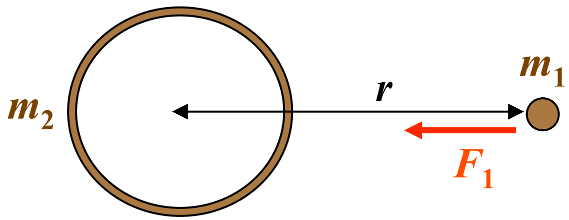


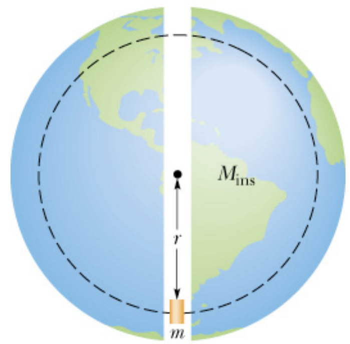
- A uniform shell of matter exerts no net gravitational force on a particle located inside it.



net vector force is zero inside







Gravitation Inside the Earth

Newton proved that the net gravitational force on a particle by a shell depends on the position of the particle with respect to the shell.

If the particle is inside the shell, the net force is zero.

If the particle is outside the shell, the force is given by: $\boxed{F_1 = G\frac{m_1 m_2}{r^2}}$.

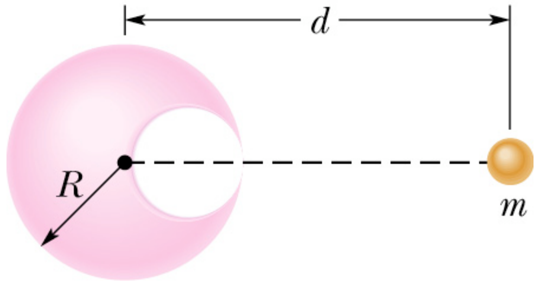
Consider a mass m inside the Earth at a distance r from the center of the Earth. If we divide the Earth into a series of concentric shells, only the shells with radius less than r exert a force on m. The net force on m is: $F = \frac{GmM_{ins}}{r^2}$.

Here M_{ins} is the mass of the part of the Earth inside a sphere of radius r :

$$M_{ins} = \rho V_{ins} = \rho \frac{4\pi r^3}{3} \rightarrow \boxed{F = \frac{4\pi G m \rho}{3} r \qquad F \text{ is linear with } r.}$$

Problem13-13

With what gravitational force does the hollowed-out sphere attract a small sphere of mass m?



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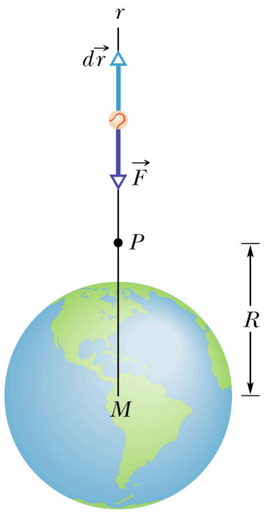
Gravitational Potential Energy

From Section 8.3 $\Rightarrow$ $-\Delta U = W_{\text{done by force}}$ $\Rightarrow$ Conservative force-path independent



$$\Delta U_g = -W_{\text{done}} = -\int_{x_i}^{x_f} \vec{F}_g \bullet d\vec{x}$$

$$W = \int_{r_i}^{r_f} \left(-G\frac{mM}{r^2}\right)dr = -GmM\int_{r_i}^{r_f}\left(\frac{1}{r^2}\right)dr$$

<u>At Earth's surface, F_g~const.</u>

$$\Delta U_g = -W_{\text{done}} = -m(-g)\int_{y_i}^{y_f} dy = mg\Delta y$$

If we define U = 0 at ∞, then the work done by taking mass m from R to ∞

$$U_\infty - U(r) = -W = -GmM\left[0-\left(-\frac{1}{r}\right)\right]$$

$$\boxed{U(r) = -\frac{GmM}{r}}$$

$$F(r) = -\frac{dU(r)}{dr}$$

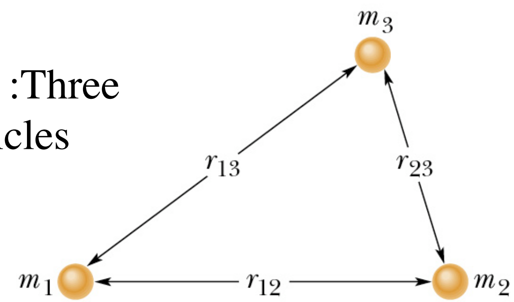
$$-\frac{d}{dr}\left(-\frac{GmM}{r}\right) = -\frac{GmM}{r^2}$$

Note:

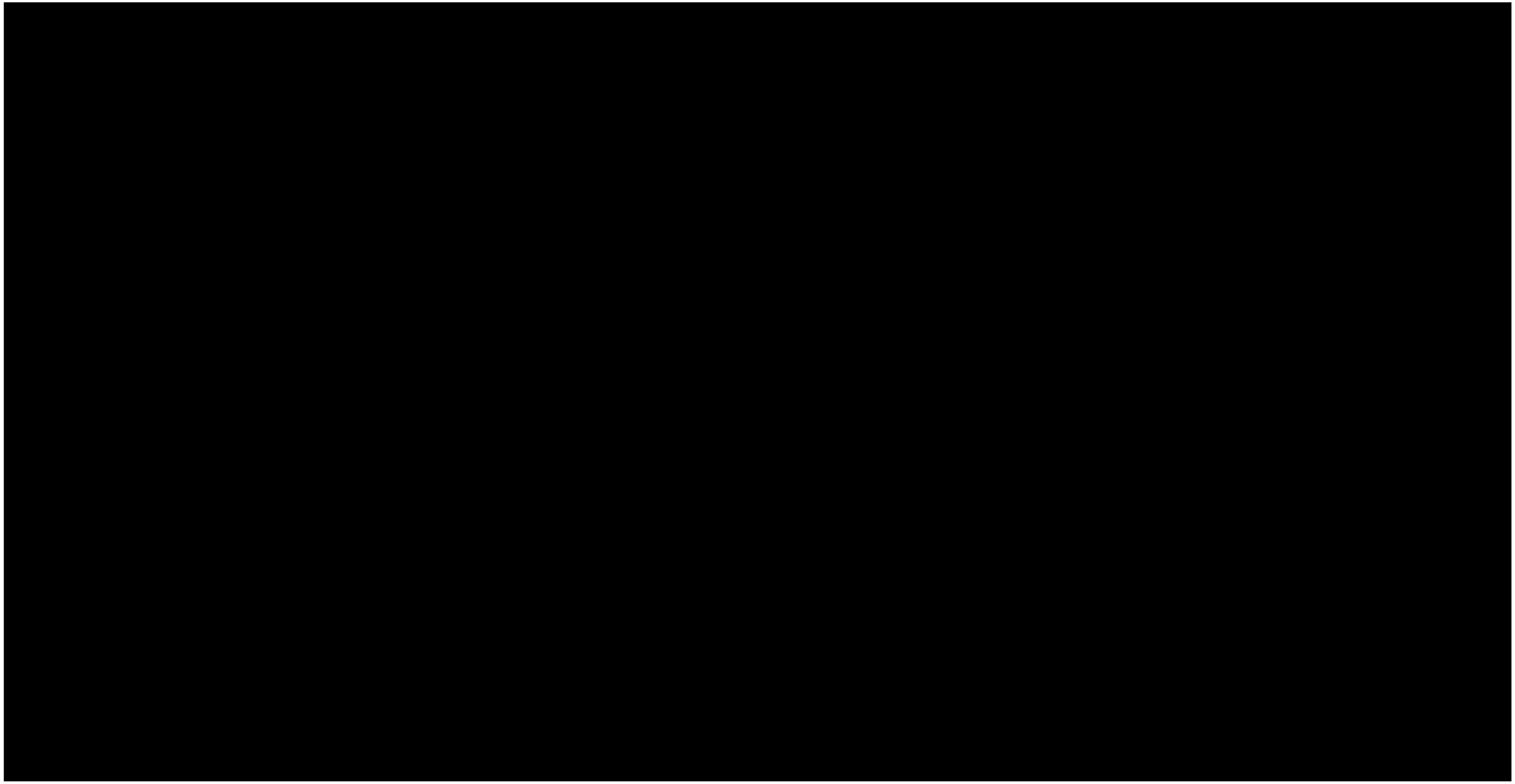
1) As before, Grav. Pot. Energy decreases as separation decreases (more negative)
2) Path independent
3) MUST HAVE AT LEAST TWO PARTICLES TO POTENTIAL ENERGY (& force)
4) Knowing potential, you can get force….

Gravitational Potential Energy

System :Three particles



What is the gravitational Potential Energy of the three-particle system?



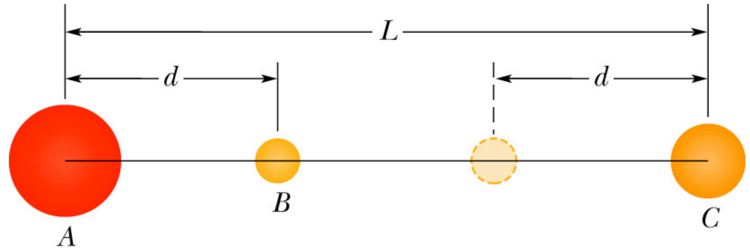
Example

Gravitational Potential Energy

Three spheres with mass m_A, m_B, and m_C. You move sphere B from left to right.

How much work is done by the gravitational force?

How much work do you do on sphere B ?



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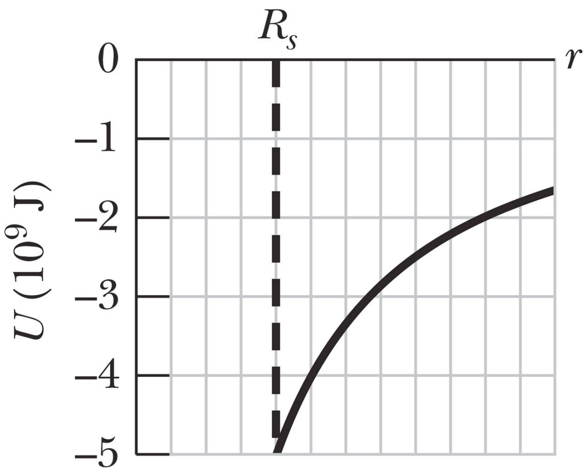
Gravitational Potential Energy

The figure gives the potential energy U(r) of a projectile, plotted outward from the surface of a planet of radius R_s. If the projectile is launched radically outward from the surface with a mechanical energy of -2.0×10^{-9} J, what are (a) its kinetic energy at radius r= 1.25 R_s and (b) its turning point in terms of R_s?



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Escape Speed

Escape speed: minimum speed (v_{escape}) required to send a mass m, from mass M and position R, to infinity, while coming to rest at infinity.

At infinity: $E_{mech} = 0$ because U = 0 and KE = 0
Thus any other place we have:

$$E_{mech} = \left(KE + U_g\right) = 0 \quad \Rightarrow \quad E_{mech} = \left(\frac{1}{2}mv^2 - \frac{GmM}{R}\right) = 0 \Rightarrow$$

$$\boxed{v_{escape} = \sqrt{\frac{2GM}{R}}}$$

Earth = 11.2 km/s (25,000 mi/hr)
Escape speed: Moon = 2.38 km/s
Sun = 618 km/s

Some Escape Speeds

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

Problem

A projectile is fired vertically from the Earth's surface with an initial speed of 10 km/s (22,500 mi/hr)

Neglecting air drag, how far above the surface of Earth will it go?

$$R_E = 6380 \cdot km$$

$$GM_E = 4 \times 10^{14} \cdot m^3 / s^2$$

[redacted]

Satellites 13-8: Weather, Spy, Moon

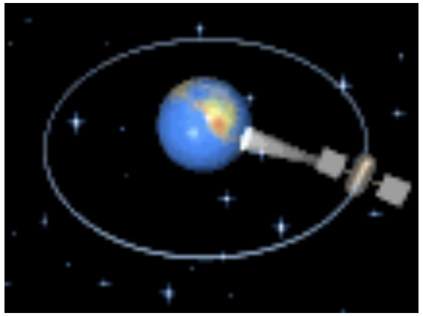
Geosynchronous Satellite: One that stays above same point on the earth (only at equator)

TV, weather, communications……

How high must it be?

Only force is gravity:

$$-\frac{Gm_{sat}M_E}{r_{sat}^2} = -m_{sat}\frac{v^2}{r_{sat}}$$



for synchronous orbit, period of satellite and earth must be the same

$$v = \frac{2\pi r}{T} \qquad T = 1\ day$$

$$-\frac{Gm_{sat}M_E}{r_{sat}^2} = -m_{sat}\frac{\left(\frac{2\pi r_{sat}}{T}\right)^2}{r_{sat}} \qquad \Rightarrow \quad r_{sat}^3 = \left(\frac{GM_E}{4\pi^2}\right)T^2$$

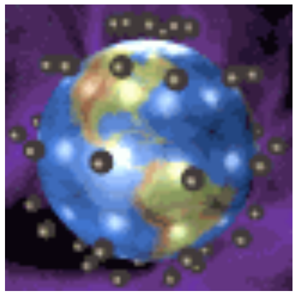
NOTE: $r^3 \propto T^2$

knowing T = 86,400 s , $\Rightarrow \quad r_{sat} = 42{,}300\ km$

subtracting radius of earth: $\Rightarrow$ height above earth surface $= 35{,}000\ km \sim 6R_E$

Geosynchronous Satellite = 22,500 miles high

Spy Satellite (polar orbit) = 400 miles high



What is period?

Space Shuttle 186 miles high

Satellites: Orbits and Energy

Newton's equation: F = ma
F is gravitational force
a is centripetal acceleration

$$\frac{GmM}{r^2} = m\frac{v^2}{r}$$

$$-\frac{1}{2}U_g = \frac{GmM}{2r} = m\frac{v^2}{2} = \tfrac{1}{2}mv^2 = KE$$

$$-\frac{1}{2}U_g = KE$$

$$KE_s = \frac{GMm}{2r}$$

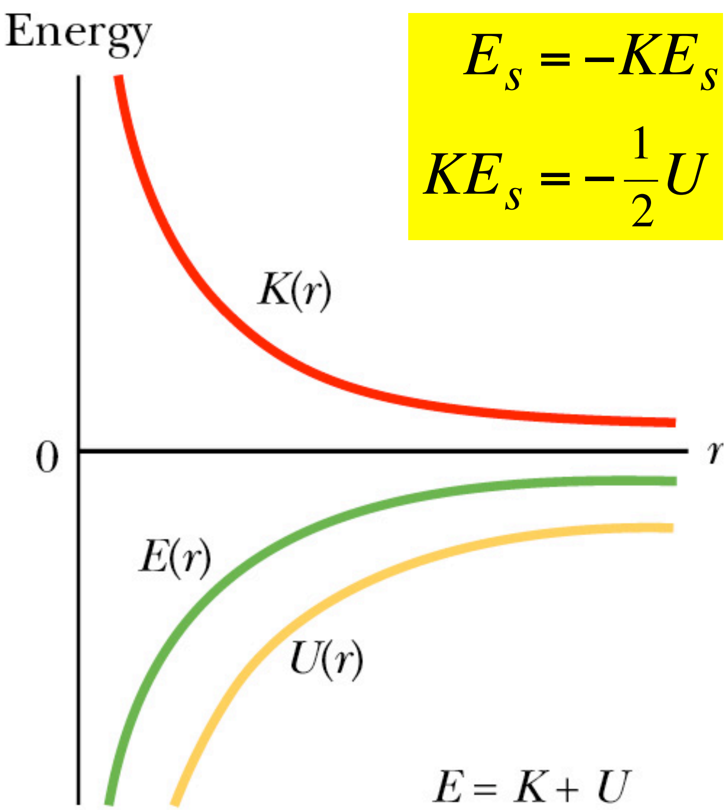
Mechanical Energy

$$E = U + KE = U + \left(-\tfrac{1}{2}U\right) = \tfrac{1}{2}U$$

$$E_s = -\frac{GMm}{2r}$$

$$E_s = -KE_s$$

Satellites: Energy graph



$$E_s = -KE_s$$

$$KE_s = -\frac{1}{2}U$$

<u>Mechanical Energy</u>

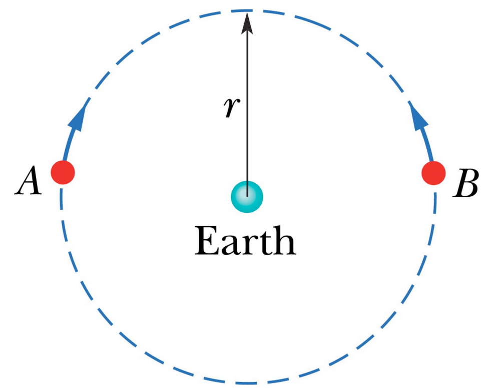
$$E_{mech} = U + KE$$

$$= -\frac{GMm}{2r}$$

$$KE = \frac{GMm}{2r}$$

$$U = -\frac{GMm}{r}$$

Satellites: Orbits and Energy



Problem: Two satellites, A and B, both of mass m=125 kg, move in the same circular orbit of radius r= 7.87 x 10^6 m around the Earth but in opposite senses of rotation and therefore on a collision course.

(a) Find the total mechanical energy $E_A + E_B$ of the two satellites + Earth before the collision.

(b) If the collision is completely inelastic so that the wreckage remains as on piece , find the total mechanical energy immediately after the collision.

© Just after the collision, is the wreckage falling directly toward the Earth's center or orbiting around the earth?

[redacted]