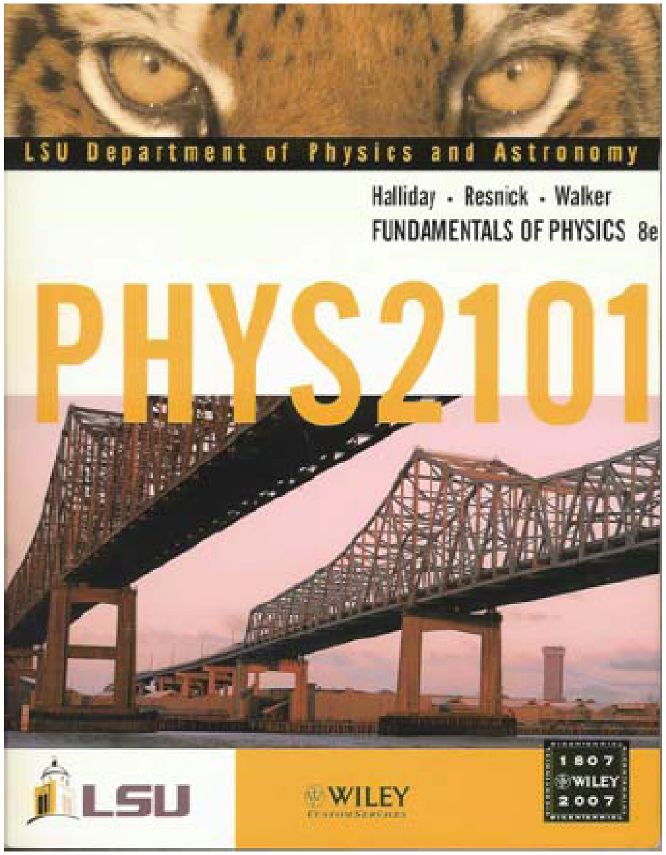


Physics 2101
Section 6
Oct. 11th: Ch. 11

Announcement:

- Exam#2 (Oct. 16th)
- Chapters 9-11
- The same locations

1.0 h (6-7pm) – Lockett 10
1.5 h (5:30-7pm) – Nicholson 109
2.0 h (5:30-7:30pm) – Nicholson 119

- Review: Monday (10/15)
  7-9 pm, Nicholson 130

Lecture Notes:
http://www.phys.lsu.edu/classes/fall2012/phys2101-6/

Quick Review: Newton's 2nd law for rotation

$$\left\{\vec{F}_{net} = \sum \vec{F}_i = m\vec{a}\right\} \qquad \vec{\tau}_{net} = \sum \vec{\tau}_i = \vec{r} \times \vec{F} = \vec{r} \times m\,\vec{a} = m\,\vec{r} \times (\vec{r} \times \vec{\alpha}) = I\vec{\alpha}$$

Work and Rotational Kinetic Energy

NET Work done ON system

$$W = \Delta KE_{rot} = \tfrac{1}{2} I\left(\omega_f^2 - \omega_i^2\right) \qquad \left\{W = \Delta KE_{trans} = \tfrac{1}{2} m\left(v_f^2 - v_i^2\right)\right\}$$

Rotational work, fixed axis rotation

$$W = \int_{\theta_1}^{\theta_2} \tau_{net}\, d\theta \qquad \left\{W = \int_{x_1}^{x_2} F\,dx \quad 1 - D\ motion\right\}$$

(if torque is const)

$$= \tau\left(\theta_f - \theta_i\right)$$

Power, fixed axis rotation

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\omega \qquad \left\{P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F} \bullet \vec{x}\right) = \vec{F} \bullet \vec{v}\right\}$$

Quick Review: Rotational Work and Energy

We can compare linear variables with rotational variables

Linear		Rotational
x	⟷	θ
v	⟷	ω
a	⟷	α
Δt	⟷	Δt
F	⟷	τ
m	⟷	I

$$s = r\theta$$

$$v_T = r\omega$$

$$a_T = r\alpha$$

The same can be done for work and energy:

For translational systems

$$W = F \cdot x$$

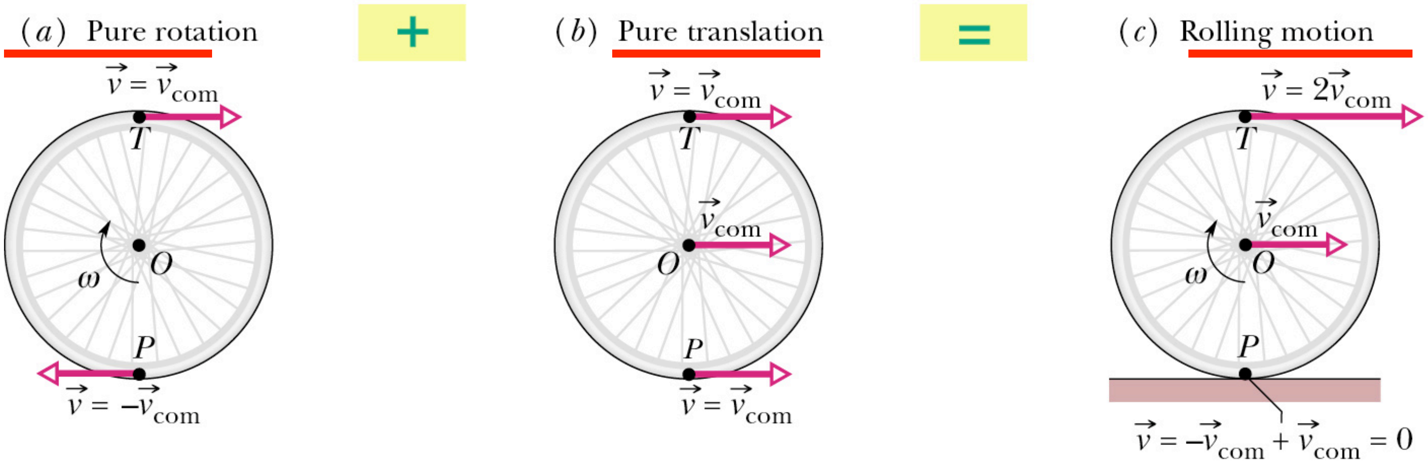
$$KE = \frac{1}{2} m v^2$$

For rotational systems

$$W = \tau \cdot \theta$$

$$KE = \frac{1}{2} I \omega^2$$

Quick Review: Rolling



All points on wheel move with same ω. All points on outer rim move with same linear speed $v = v_{com}$.

$$\vec{v} = \vec{\omega} \times \vec{r}$$

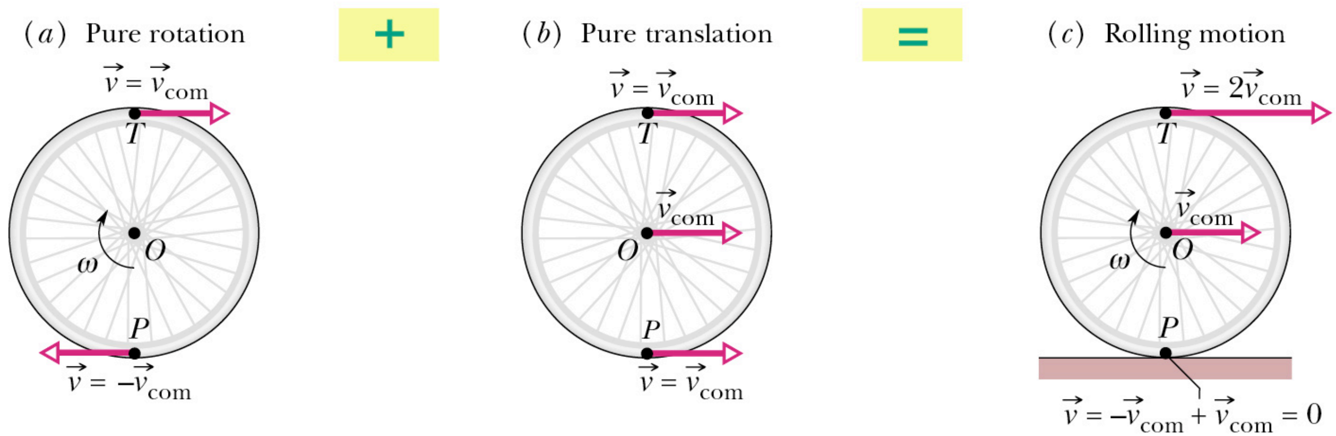
All points on wheel move to the right with same linear velocity v_{com} as center of wheel

Combination of “pure rotation” and “pure translation”

Note at point P: vector sum of velocity = 0 (point of stationary contact)

at point T: vector sum of velocity = $2v_{com}$ (top moves twice as fast as com)

Quick Review: Kinetic Energy of Rolling



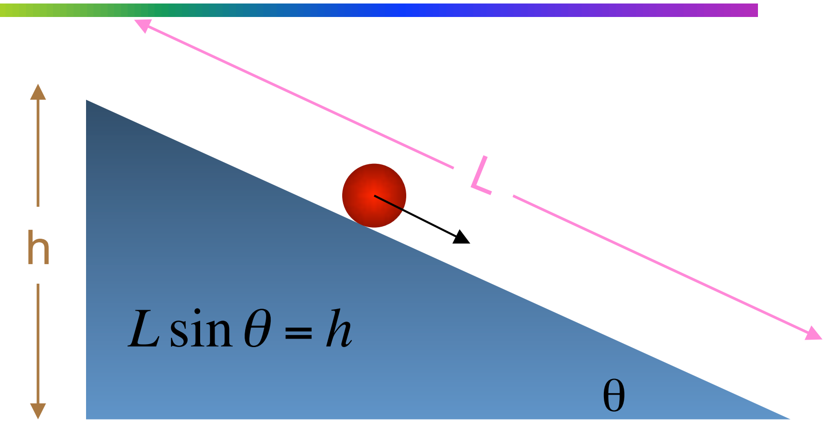
$$\frac{1}{2} I_{com}\,\omega^2 + \frac{1}{2} M v^2_{com} = KE_{tot}$$

Note: rotation about COM and translation of COM combine for total KE

Remember: $v_{com} = \omega r$

Example: Rolling down a ramp with no friction

Object, with mass **m** and radius **r**, roles from top of incline plane to bottom. What is **v**, **a**, and **Δt** at bottom



$$\Delta E_{mech} = 0$$

AT BOTTOM $$\Delta KE_{tot} = -\Delta U$$

$$\left(KE_{rot+trans,COM}\right)_{final} - (0)_{init} = -\left[(0)_{final} - (mgh)_{init}\right]$$

$$\tfrac{1}{2} m v_{COM}^2 + \tfrac{1}{2} I_{com}\omega^2 = mgL\sin\theta$$

$$\tfrac{1}{2} m v_{COM}^2 + \tfrac{1}{2} I_{COM}\left(\frac{v}{r}\right)^2 = mgL\sin\theta$$

$$v_{COM}^2\left(m + \frac{I_{COM}}{r^2}\right) = 2mgL\sin\theta$$

$$\left|v_{COM}\right| = \sqrt{\frac{2gL\sin\theta}{\left(1 + \frac{I}{mr^2}\right)}}$$

AT BOTTOM

Using 1-D kinematics

$$v^2 = v_0^2 + 2aL$$

$$a = \frac{v^2}{2L} = \frac{g\sin\theta}{\left(1 + \frac{I}{mr^2}\right)}$$

AND

$$t = \sqrt{\frac{2L}{g\sin\theta}\left(1 + \frac{I}{mr^2}\right)}$$

Compare Different Objects

Assuming same work done (same change in U), objects with larger rotational inertial have larger KE_{rot} and during rolling, their KE_{trans} is smaller.

$$KE_{tot} = KE_{trans} + KE_{rot} = KE_{trans}\left(1 + \frac{I_{com}}{mr^2}\right)$$

$$|v_{COM}| = \sqrt{\frac{2gL\sin\theta}{\left(1 + \frac{I_{com}}{mr^2}\right)}}$$

Roll a hoop, disk, and solid sphere down a ramp - what wins?

Moment of inertia large → small

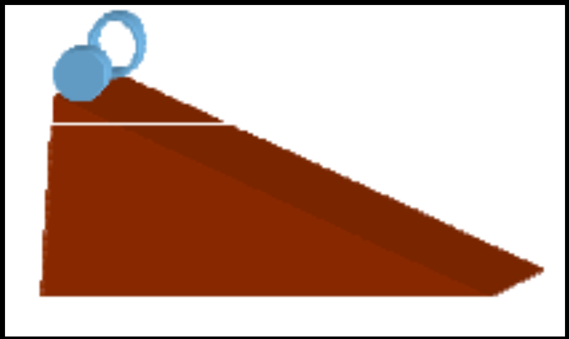
Object	Rotational Inertia, I_{com}	Fraction of Energy in Translation	Fraction of Energy in Rotation	
Hoop	$1mr^2$	0.5	0.5	slowest
Disk	$\frac{1}{2}mr^2$	0.67	0.33	
Sphere	$\frac{2}{5}mr^2$	0.71	0.29	
sliding block (no friction)	0	1	0	fastest

$$\Delta t_{bottom} = \sqrt{\frac{2L}{g\sin\theta}\left(1 + \frac{I}{mr^2}\right)}$$

Question

A ring and a solid disc, both with radius *r* and mass *m*, are released from rest at the top of a ramp. Which one gets to the bottom first?

1. *Solid disc*
2. *Ring (hoop)*
3. *both reach bottom at same time*



Question #2

Two solid disks of equal mass, but different radii, are released from rest at the top of a ramp. Which one arrives at the bottom first?

1. The smaller radius disk.
2. The larger radius disk.
3. Both arrive at the same time.

The equation for the speed of the a disk at the bottom of the ramp is $\sqrt{\frac{4}{3} g l \sin\theta}$

Notice, it does not depend on the radius or the mass of the disk!!

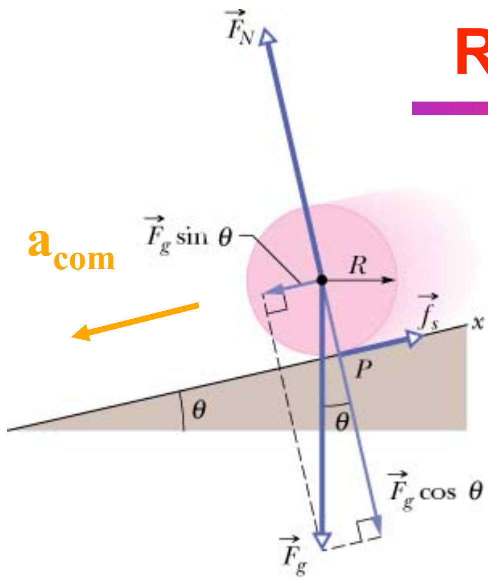
Using 1-D kinematics

$$v^2 = v_0^2 + 2aL$$

$$a = \frac{v^2}{2L} = \frac{g \sin\theta}{\left(1 + \frac{I}{mr^2}\right)}$$

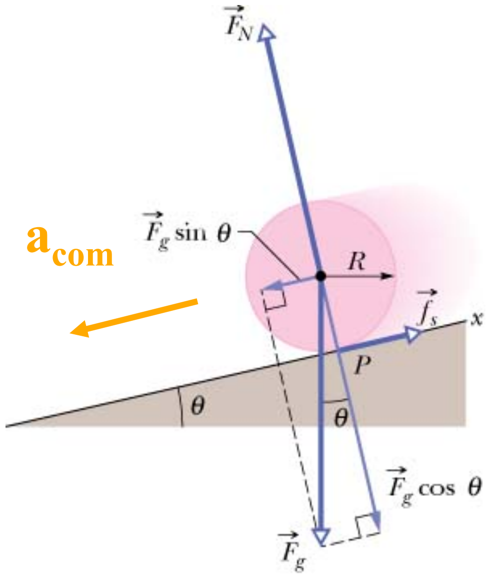
AND

$$t = \sqrt{\frac{2L}{g \sin\theta}\left(1 + \frac{I}{mr^2}\right)}$$



Rolling Down a Ramp with a Frictional Force

Consider a round uniform body of mass M and radius R rolling down an inclined plane of angle θ. We will calculate the acceleration a_{com} of the center of mass along the x-axis using Newton's second law for the translational and rotational motion.



$$|a_{com}| = \frac{g\sin\theta}{1+\dfrac{I_{com}}{MR^2}}$$

Cylinder

$$I_1 = \frac{MR^2}{2}$$

$$a_1 = \frac{g\sin\theta}{1+I_1/MR^2}$$

$$a_1 = \frac{g\sin\theta}{1+MR^2/2MR^2}$$

$$a_1 = \frac{g\sin\theta}{1+1/2}$$

$$a_1 = \frac{2g\sin\theta}{3} = (0.67)g\sin\theta$$

Hoop

$$I_2 = MR^2$$

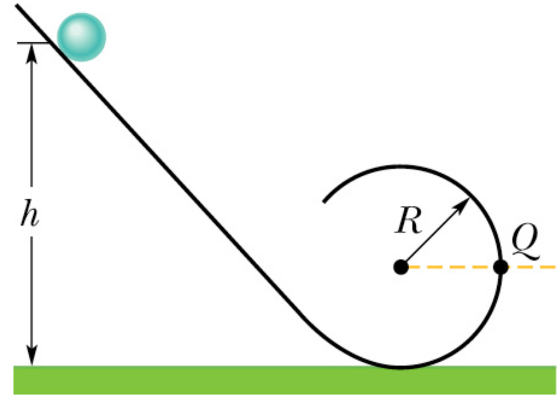
$$a_2 = \frac{g\sin\theta}{1+I_2/MR^2}$$

$$a_2 = \frac{g\sin\theta}{1+MR^2/MR^2}$$

$$a_2 = \frac{g\sin\theta}{1+1}$$

$$a_2 = \frac{g\sin\theta}{2} = (0.5)g\sin\theta$$

Sample Problem

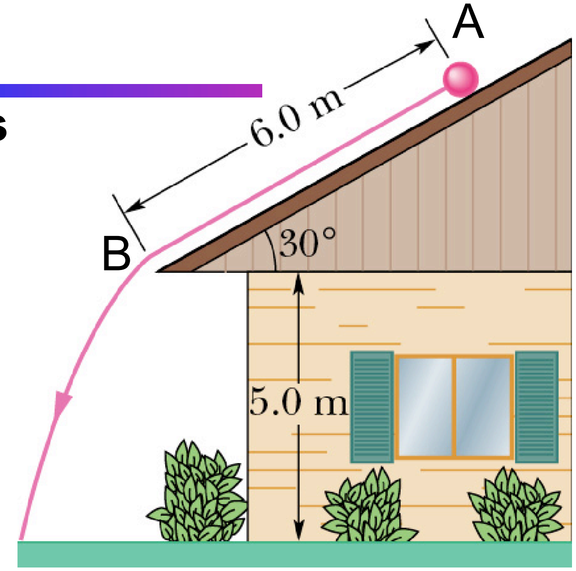


A solid cylinder starts from rest at the upper end of the track as shown. What is the angular speed of the cylinder about its center when it is at the top of the loop?

[redacted]

Sample Problem #2

A solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance of 6 m down a house roof that is inclined at 30º.



Where does it hit?

[redacted]

Forces of Rolling

1) If object is rolling with a_{com}=0 (i.e. no net forces), then $v_{com}=\omega R$ = constant (smooth roll)

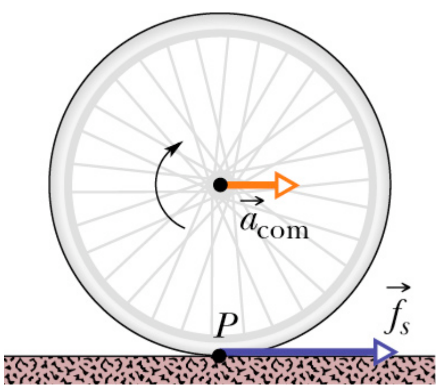
…**if constant speed**, it has no tendency to slide at point of contact - **no frictional forces**

$a_{com} = 0 \rightarrow \alpha = 0 \rightarrow \tau = 0 \rightarrow f = 0$

$a_{com} = \alpha R$ $\qquad I_{com}\alpha = \tau_{net}$ $\qquad \tau_{net} = |R||f|\sin\theta$

2) If object is rolling with $a_{com} \neq 0$ (i.e. there are net forces) and no slipping occurs,
then $\alpha \neq 0 \Rightarrow \tau \neq 0$

… static friction needed to supply torque !

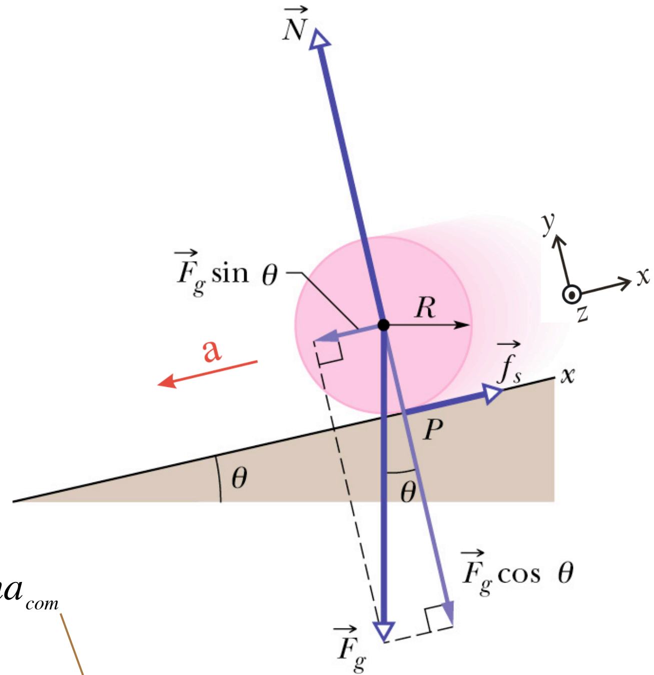


kinetic friction - during sliding ($a_{com} \neq \alpha R$)

static friction - smooth rolling ($a_{com} = \alpha R$)

Rolling Down a Ramp: Acceleration by f_s

- Acceleration downwards
- Friction provides torque
- Static friction points upwards



Newton's 2nd Law

Linear version

$$\hat{x}: \quad f_s - F_g \sin\theta = -ma_{com}$$

$$\hat{y}: \quad N - F_g \cos\theta = 0$$

Angular version

$$\hat{z}: \quad I_{com}\alpha = \tau = Rf_s$$

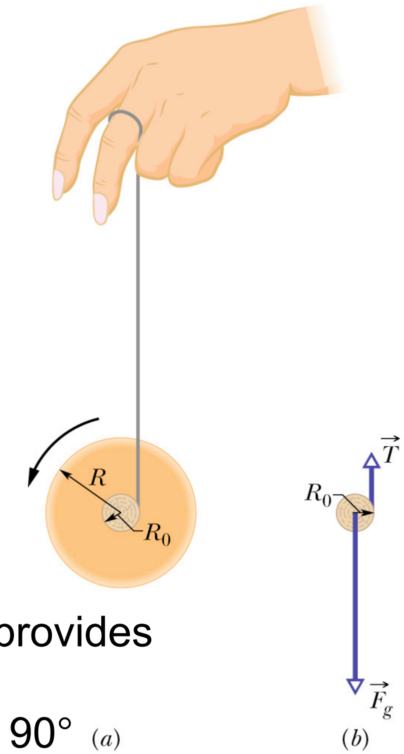
$$\rightarrow a_{com} = \alpha R$$

$$\alpha = \frac{a_{com}}{R} = \frac{Rf_s}{I_{com}}$$

$$a_{com} = \frac{R^2\left(mg\sin\theta - ma_{com}\right)}{I_{com}}$$

$$a_{com} = \frac{g\sin\theta}{1 + I_{com}/mR^2}$$

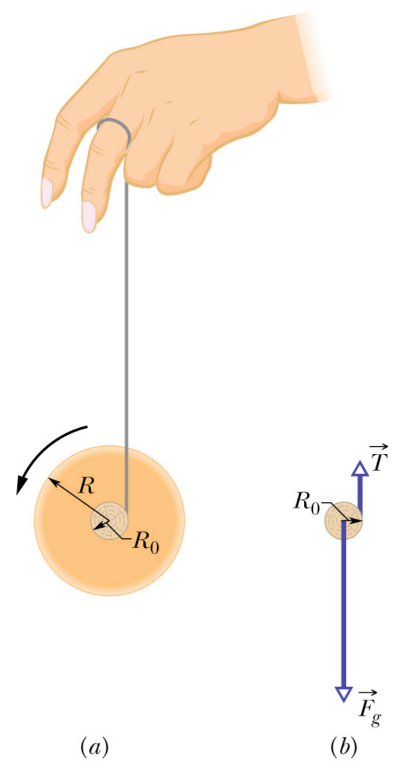
Yo-Yo



- Tension provides torque
- Here θ = 90°
- Axle: R ⇒ R_0

$$a_{com} = \frac{g}{1 + I_{com}/mR_0^2}$$

Problem



$$a_{com} = \frac{g}{1 + I_{com}/mR_0^2}$$

downwards

A yo-yo has a rotational inertia of $\mathbf{I_{com}}$ and mass of **m**. Its axle radius is $\mathbf{R_0}$ and string's length is **h**. The yo-yo is thrown so that its initial speed down the string is $\mathbf{v_0}$.

a) How long does it take to reach the end of the string?

1-D kinematics given a_{com}

$$-h = \Delta y = -v_0 t - \tfrac{1}{2} a_{com} t^2 \;\Rightarrow\; solve\ for\ t\ (quadradic\ equation)$$

b) As it reaches the end of the string, what is its <u>total</u> KE?

Conservation of mechanical energy

$$KE_f = KE_i + U = \tfrac{1}{2} m v_{com,0}^2 + \tfrac{1}{2} I_{com}\left(\frac{v_{com,0}}{R_0}\right)^2 + mgh$$

c) As it reaches the end of the string, what is its linear speed?

1-D kinematics given a_{com}

$$-|v_{com}| = -v_0 - a_{com} t \;\Rightarrow\; solve\ for\ |v_{com}|$$

d) As it reaches the end of the string, what is its translational KE?

Knowing $|v_{com}|$

$$KE_{trans} = \frac{1}{2} m v_{com}^2$$

e) As it reaches the end of the string, what is its angular speed?

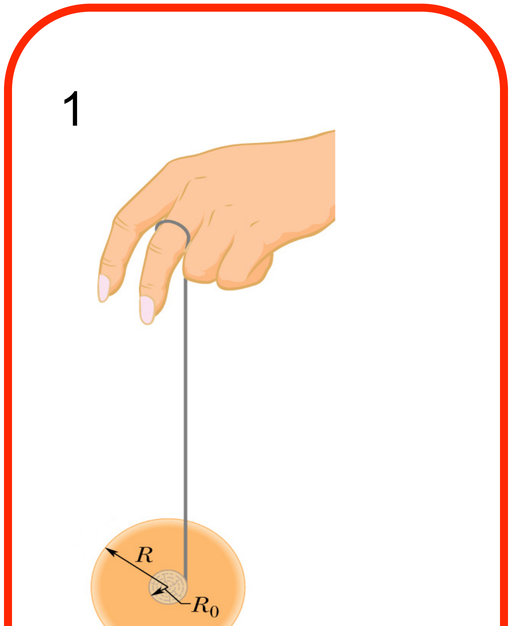
Knowing $|v_{com}|$

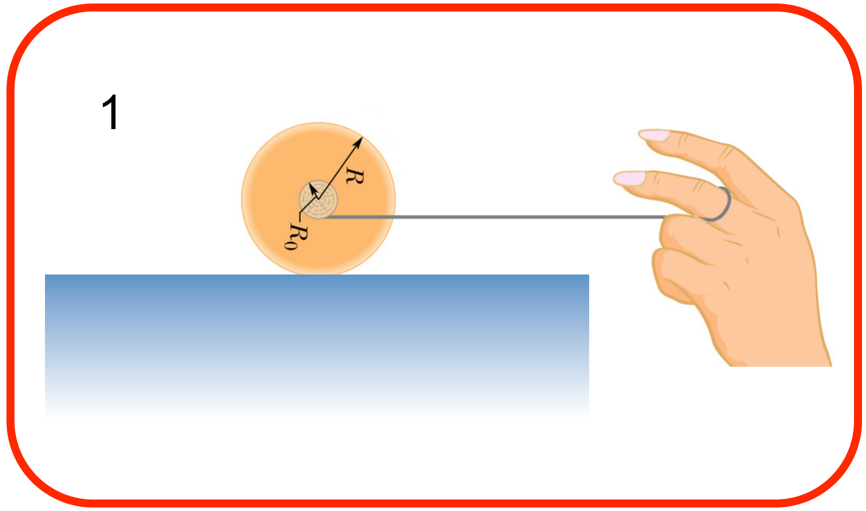
$$\omega = \frac{v_{com}}{R}$$

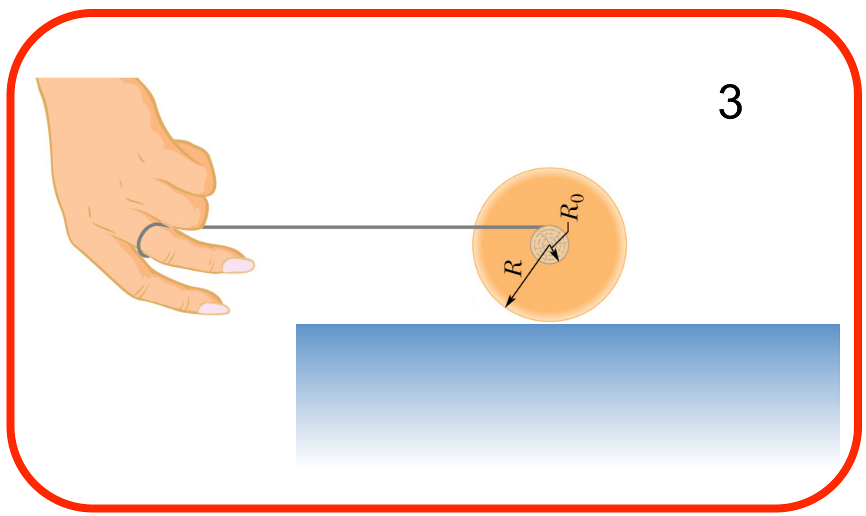
f) As it reaches the end of the string, what is its rotational KE?

Two ways: $KE_{rot} = \dfrac{1}{2} I_{com}\omega^2 \quad or \quad KE_{rot} = K_{Ef,tot} - KE_{trans}$

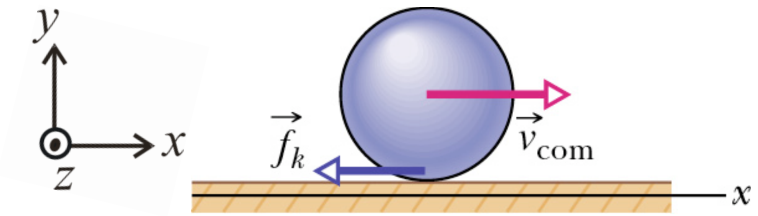
Which way will it roll??







Problem 11-13



NON-smooth rolling motion

*A bowler throws a bowling ball of radius **R** along a lane. The ball slides on the lane, with initial speed $v_{com,0}$ and initial angular speed $\omega_0 = 0$. The coefficient of kinetic friction between the ball and the lane is μ_k. The kinetic frictional force f_k acting on the ball while producing a torque that causes an angular acceleration of the ball. When the speed v_{com} has decreased enough and the angular speed ω has increased enough, the ball stops sliding and then rolls smoothly.*

a) *[After it stops sliding] What is the v_{com} in terms of ω?*

Smooth rolling means $v_{com} = R\omega$

b) *During the sliding, what is the ball's linear acceleration?*

From 2nd law: (linear)

$$\hat{x}: \; -f_k = ma_{com} \qquad \hat{y}: \; N - mg = 0$$

But $f_k = \mu_k N = \mu_k mg$

So $a_{com} = -f_k/m = -\mu_k g$

c) *During the sliding, what is the ball's angular acceleration?*

From 2nd law: (angular)

$$\vec{\tau} = Rf_k(-\hat{z}) \qquad I\alpha(-\hat{z}) = \vec{\tau} = Rf_k(-\hat{z})$$

But $f_k = \mu_k N = \mu_k mg$

So $I\alpha = Rf_k = R(\mu_k mg)$

$$\alpha = R\mu_k mg / I$$

d) *What is the speed of the ball when smooth rolling begins?*

When does $v_{com} = R\omega$?

$$v_{com} = v_0 + a_{com} t \qquad v_{com} = v_0 - \mu_k g t$$

From kinematics: $\omega = \omega_0 + \alpha t$

$$t = \omega/\alpha = I\omega / R\mu_k mg$$

$$v_{com} = v_0 - \mu_k g \left(\frac{I\left(v_{com}/R\right)}{R\mu_k mg} \right)$$

$$v_{com} = \frac{v_0}{\left(1 + I/mR^2\right)}$$

e) *How long does the ball slide?*

$$t = \frac{v_0 - v_{com}}{\mu_k g}$$