

## Physics 2101 for Section 6 Sept. 28 ${ }^{\text {th }}$ : Ch. 9

## Class Website:

http://www.phys.Isu.edu/classes/fall2012/phys2101-6/

## Quick Review: the Center of Mass

$$
\begin{aligned}
x_{c o m}= & \frac{1}{M} \sum_{i=1}^{N} m_{i} x_{i} \longrightarrow x_{\text {com }}=\frac{1}{M} \int x d m \longrightarrow x_{\text {com }}=\frac{1}{V} \int x d V \\
y_{\text {com }} & =\frac{1}{M} \sum_{i=1}^{N} m_{i} y_{i} \longrightarrow y_{\text {com }}=\frac{1}{M} \int y d m \\
z_{\text {com }}= & \frac{1}{M} \sum_{i=1}^{N} m_{i} z_{i} \longrightarrow z_{\text {com }}=\frac{1}{M} \int z d m \\
M & =\sum_{i=1}^{N} m_{i} \longrightarrow \rho=\frac{d m}{d V}=\frac{M}{V} \quad \begin{array}{l}
\text { Here "mass density" } \\
\text { replaces mass }
\end{array}
\end{aligned}
$$

(1) Center of mass of a symmetric object always lies on an axis of symmetry.
(2) Center of mass of an object does NOT need to be on the object.

## Quick Review: <br> Newton's $2^{\text {nd }}$ Law for a System of Particles



$$
M \vec{a}_{\mathrm{com}}=\vec{F}_{\mathrm{net}}
$$

$$
\begin{aligned}
& F_{\mathrm{net}, x}=M a_{\mathrm{com}, x} \\
& F_{\mathrm{net}, y}=M a_{\mathrm{com}, y} \\
& F_{\mathrm{net}, z}=M a_{\mathrm{com}, z}
\end{aligned}
$$

## Linear Momentum of a Particle



$$
\vec{p}=m \vec{v}
$$

Linear Momentum :
Linear momentum $\vec{p}$ of a particle of mass $m$ and velocity $\vec{v}$ is defined as $\vec{p}=m \vec{v}$.
The SI unit for linear momentum is the kg.m/s.

## Linear Momentum of a System of Particles

Total Linear Momentum of N particles:

$$
\begin{aligned}
& \text { omentum } \quad \vec{P}_{\text {tot }}=\vec{p}_{1}+\vec{p}_{2}+\vec{p}_{3}+\ldots+\vec{p}_{N}=M \vec{v}_{\text {com }} \\
& \text { cles: } \\
& \frac{d \vec{P}_{\text {tot }}}{d t}=M \frac{d \vec{v}_{c o m}}{d t}=M \vec{a}_{c o m}=\vec{F}_{\text {net,ext mass } M} \quad \begin{array}{l}
\text { velocity of COM }
\end{array}
\end{aligned}
$$

## Conservation of Linear Momentum

If External force is zero (isolated, closed system)...

$$
\begin{gathered}
0=\vec{F}_{\text {net }, \text { ext }}=\frac{d \vec{P}}{d t} \\
\vec{P}=\text { const } \\
\Delta \vec{P}=0
\end{gathered}
$$

## Question <br> Question 9-1

Two objects have the same momentum.

1. Their velocities must have the same magnitude and direction.
2. Their velocities have the same magnitude but direction can differ.
3. Their velocities can differ in magnitude but must have the same direction.
4. Their velocities can differ in magnitude and direction.

## Example

A man of mass $m$ climbs to a rope ladder suspended below a balloon of mass M . The balloon is stationary with respect to the ground.
a) If the man begins to climb the ladder at speed $v$ with respect to the ladder, in what direction will the balloon move?
b) What is the state of motion after the man stops climbing?


## 9. 6 Impulse \& Collisions

An isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time.


We study two types:

1) Inelastic (KE lost to $\boldsymbol{E}_{\text {therm }}$ )
2) Elastic (total KE=const)
[interaction through conservative forces]

However in both we assume closed and isolated systems


## Collision



When a collision occurs, forces act.
They can be:

- Internal Forces - objects acting on each other
- External forces - an external force exerted on the objects
$\left.\begin{array}{ccc}\text { Object } 1 & \left(\boldsymbol{F}_{\text {ext }, l}+\overline{\boldsymbol{F}}_{12}\right) \Delta t=\boldsymbol{p}_{1 f}-\boldsymbol{p}_{1 i} \\ & \text { external } & \text { internal } \\ \text { force } & \text { force } & \text { change in } \\ \text { momentum }\end{array}\right]$

$$
(\text { External Forces } \boldsymbol{+} \text { Internal Forces }) \Delta t=\boldsymbol{P}_{f}-\boldsymbol{P}_{i}
$$

## But internal

forces always $\overline{\boldsymbol{F}}_{12}=-\overline{\boldsymbol{F}}_{21}$ so (External Forces) $\Delta t=\boldsymbol{P}_{f}-\boldsymbol{P}_{i}$ cancel

## Impulse

## Collision



## Impulse

Change in Momentum is equal to Impulse acting on it

$$
\begin{aligned}
d \vec{p}_{R} & =\vec{F}_{L \rightarrow R}(t) d t \\
\int_{\vec{p}_{i}}^{\vec{p}_{f}} d \vec{p}_{R} & =\int_{t_{i}}^{t_{f}} \vec{F}_{L \rightarrow R}(t) d t
\end{aligned}
$$

$\vec{J} \equiv \int^{\prime \prime} \vec{F}_{\text {net }}(t) d t \quad$ Impulse
What changes momentum of each?
$\vec{p}_{f}-\vec{p}_{i}=\Delta \vec{p}=\vec{J} \quad$ Vector! Must satisfy for each direction!

## Impulse-momentum theorem

## Collision and Impulse

$\int_{t_{i}}^{t_{f}} d \vec{p}=\int_{t_{i}}^{t_{f}} \vec{F}(t) d t \quad \int_{t_{i}}^{t_{f}} d \vec{p}=\vec{p}_{f}-\vec{p}_{i}=\Delta \vec{p}=$ change in momentum

(a)
$\vec{J}=\int \vec{F}(t) d t \quad$ The magnitude of $\vec{J}$ is equal to the area

(b)

$$
\Delta p=J
$$

$$
J=F_{\mathrm{ave}} \Delta t \quad F_{\mathrm{ave}} \text { versus } t \text { plot (fig. b). }
$$

## Problem 9-30

A toy car of mass $m$ moves along the $x$ axis where the force $\mathrm{F}_{\mathrm{x}}$ is shown in the figure as a function on time. What does the momentum look like as a function of time (assuming the car is at rest at $\mathrm{t}=0$ )?


$$
J_{x}=\Delta P_{x}=\int F_{x} d t
$$

## Concept Questions

## Checkpoint 1: A paratrooper whose chute fails to open lands in snow; he is hurt slightly. Had he landed on bare ground, the stopping time would have been 10 times shorter and the collision lethal. Does the presence of the snow increase, decrease, or leave unchanged the values of (a) the paratrooper's change in momentum, (b) the impulse stopping the paratrooper, and (c) the force stopping the paratrooper?

Checkpoint 2: The figure shows an overhead view of a ball bouncing from a vertical wall without any change in speed. Consider the change $\Delta \vec{p}$ in the ball's linear momentum.
a) Is $\Delta p_{x}$ positive, negative or zero?
b) Is $\Delta p_{y}$ positive, negative, or zero?
c) What is the direction of $\Delta \vec{p}$
d) What direction is the Impulse?


## Example

The National Transportation Safety Board is testing the crash-worthiness of a new car. The 2300 kg vehicle, moving at $15 \mathrm{~m} / \mathrm{s}$, is allowed to collide with a bridge abutment, which stops it in 0.56 s . What is the magnitude of the average force that acts on the car during the impact?


## Superman

It is well known that bullets and other missiles fired at Superman simply bounce off his chest. Suppose that a gangster sprays Superman's chest with 10 g bullets at the rate of 100 bullets/min, the speed of each bullet being 700 $\mathrm{m} / \mathrm{s}$. Suppose too that the bullets rebound straight back with no change in speed. What is the magnitude of the average force in 1.0 minute on Superman's chest from the stream of bullets?


## Example: Baseball Hitting the Floor

A ball with a mass of 150 g strikes the floor with a speed of $5.2 \mathrm{~m} / \mathrm{s}$ and rebounds with only $50 \%$ of its initial $K E$.
a) What is the speed of the ball immediately after rebounding?

$$
\begin{aligned}
& K E_{f}=(50 \%) K E_{i} \\
& \frac{1}{2} m v_{f}^{2}=\frac{\frac{1}{2} m v_{i}^{2}}{2} \\
& v_{f}=\sqrt{\frac{1}{2} v_{i}^{2}}=\frac{1}{\sqrt{2}} v_{i} \quad=\frac{1}{\sqrt{2}}(5.2 \mathrm{~m} / \mathrm{s}) \cong 3.7 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

b) What is the magnitude of the impulse on the ball from the floor?

If the positive direction is up then the initial

$$
\begin{aligned}
\overline{\boldsymbol{J}} & =\vec{p}_{f}-\vec{p}_{i} \\
& =m\left(\vec{v}_{f}-\vec{v}_{i}\right) \\
& =m(3.7 \mathrm{~m} / \mathrm{s}-(-5.2 \mathrm{~m} / \mathrm{s}))=(0.15 \mathrm{~kg})(8.9 \mathrm{~m} / \mathrm{s}) \\
& =1.33 \cdot \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}=1.33 \cdot \mathrm{~N} \cdot \mathrm{~s}
\end{aligned}
$$

c) If the ball was in contact with the floor for 7.6 ms , what was the magnitude of the average force on the ball from the floor during this time interval?

$$
\begin{aligned}
\boldsymbol{J} & =\vec{p}_{f}-\vec{p}_{i} \\
& =\overline{\boldsymbol{F}} \Delta t
\end{aligned}
$$

$$
\begin{aligned}
\overline{\boldsymbol{F}} \Delta t & =\boldsymbol{J} \\
\overline{\boldsymbol{F}} & =\frac{\boldsymbol{J}}{\Delta t}=\frac{1.33 \cdot N s \hat{x}}{0.0076 \cdot s}=175 \cdot N \hat{x}
\end{aligned}
$$

## Conservation of Linear Momentum

$$
\begin{aligned}
& \text { If } \vec{F}_{n e t}=\frac{d \vec{P}}{d t}=\frac{d \vec{p}_{1}}{d t}+\ldots+\frac{d \vec{p}_{n}}{d t}=0 \\
& \Rightarrow \Delta \vec{P}^{d t} \vec{P}_{f}-\vec{P}_{i}=\left(\vec{P}_{1 f}+\ldots+\vec{P}_{n f}\right)-\left(\vec{P}_{1 i}+\ldots+\vec{P}_{n i}\right)=0
\end{aligned}
$$

- Conservation of Linear Momentum

If system is closed and isolated, the total linear momentum
$\vec{P}$ cannot change.

## What about energy?

## Collisions: Elastic vs. Inelastic

## Elastic collision : TOTAL KE is conserved ( $\sim$ Conservative forces )

AND if system is closed and isolated, the total linear momentum $P$ cannot change (whether the collision is elastic or inelastic !).

$$
\begin{aligned}
& \Delta K E=K_{f}-K_{i}=\left(K_{1 f}+\ldots+K_{n f}\right)-\left(K_{1 i}+\ldots+K_{n i}\right)=0 \\
& \Delta \vec{P}=\vec{P}_{f}-\vec{P}_{i}=\left(\vec{P}_{1 f}+\ldots+\vec{P}_{n f}\right)-\left(\vec{P}_{1 i}+\ldots+\vec{P}_{n i}\right)=0
\end{aligned}
$$

## Inelastic collision : KE is not conserved ( $\sim$ thermal energy )

However, if system is closed and isolated, the total linear momentum $P$ cannot change (whether the collision is elastic or inelastic !).

$$
\begin{aligned}
& \Delta K E=K_{f}-K_{i}=\left(K_{1 f}+\ldots+K_{n f}\right)-\left(K_{1 i}+\ldots+K_{n i}\right) \neq 0 \\
& \Delta \vec{P}=\vec{P}_{f}-\vec{P}_{i}=\left(\vec{P}_{1 f}+\ldots+\vec{P}_{n f}\right)-\left(\vec{P}_{1 i}+\ldots+\vec{P}_{n i}\right)=0
\end{aligned}
$$

## Concept Questions

A 9-kg object is at rest. Suddenly, it explodes and breaks into two pieces. The mass of one piece is 6 kg and the other is a $3-\mathrm{kg}$ piece. Which one of the following statements concerning these two pieces is correct?
a) The speed of the $6-\mathrm{kg}$ piece will be one eighth that of the $3-\mathrm{kg}$ piece.
b) The speed of the 3-kg piece will be one fourth that of the $6-\mathrm{kg}$ piece.
c) The speed of the $6-\mathrm{kg}$ piece will be one forth that of the $3-\mathrm{kg}$ piece.
d) The speed of the 3-kg piece will be one half that of the $6-\mathrm{kg}$ piece.
e) The speed of the 6-kg piece will be one half that of the 3-kg piece.

## Concept Questions

A sled of mass $m$ is coasting at a constant velocity on the ice covered surface of a lake. Three birds, with a combined mass $0.5 m$, gently land at the same time on the sled. The sled and birds continue sliding along the original direction of motion. How does the kinetic energy of the sled and birds compare with the initial kinetic energy of the sled before the birds landed?
a) The final kinetic energy is one half of the initial kinetic energy.
b) The final kinetic energy is one third of the initial kinetic energy.
c) The final kinetic energy is one quarter of the initial kinetic energy.
d) The final kinetic energy is one ninth of the initial kinetic energy.
e) The final kinetic energy is equal to the initial kinetic energy.

## Velocity of COM

In a closed, isolated system the COM velocity ( $\vec{v}_{\text {com }}$ ) of the system is CONSTANT. Why?

$$
\begin{aligned}
& \vec{F}_{\text {net }}=\frac{d \vec{P}_{\text {ot }}}{d t}=0 \square \begin{array}{c}
\vec{P}_{\text {tot }}=M \vec{v}_{\text {com }}=\left(m_{1}+m_{2}\right) \vec{v}_{\text {com }} \\
\vec{P}_{\text {tot }}=\vec{p}_{1 i}+\vec{p}_{2 i}
\end{array} \\
& \vec{v}_{c o m}=\frac{\vec{P}_{\text {tot }}}{\left(m_{1}+m_{2}\right)}=\frac{\vec{p}_{1 i}+\vec{p}_{2 i}}{\left(m_{1}+m_{2}\right)}=\frac{\vec{p}_{1 f}+\vec{p}_{2 f}}{\left(m_{1}+m_{2}\right)}
\end{aligned}
$$

Constant !!


Exploding things from Chapt. 9:

### 9.9 1D Inelastic Collisions

## Inelastic collision : KE is not conserved ( $\sim$ thermal energy )

However, if system is closed and isolated, the total linear momentum $P$ cannot change (whether the collision is elastic or inelastic !).


$$
\begin{aligned}
\vec{P}_{\text {before }} & =\vec{P}_{\text {affer }} \quad 1-\mathrm{D} \\
\left(m_{1} v_{1 i}+m_{2} v_{2 i}\right) & =\left(m_{1} v_{1 f}+m_{2} v_{2 f}\right)
\end{aligned}
$$

Only COLM: Conservation of Linear Momentum

Special Case:
Completely
Inelastic
Collision
(hit-'n-stick)


## Pure Inelastic Collision -- Hit-'n-stick



$$
\begin{aligned}
& v_{c o m}=V=\frac{1 m_{1}(1)}{\left(m_{1}+3 m_{1}\right)} \\
& V=v_{c o m}=\frac{1}{4} \cdot \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
\begin{aligned}
\vec{P}_{\text {before }} & =\vec{P}_{\text {affer }} \quad 1-\mathrm{D} \\
\left(m_{1} v_{1 i}+m_{2} v_{2 i}\right) & =\left(m_{1} v_{1 f}+m_{2} v_{2 f}\right) \\
\left(m_{1} v_{1 i}+0\right) & =V_{f}\left(m_{1}+m_{2}\right)
\end{aligned}
$$

Special case: $\mathrm{v}_{2 \mathrm{i}}=0 \& \mathrm{~m}_{2}=3 \mathrm{~m}_{1}$ take $\mathrm{v}_{\mathrm{ii}}=1 \mathrm{~m} / \mathrm{s}$

Lab frame: $\mathrm{v}_{\mathrm{lf}}=-1 / 2 \mathrm{v}_{\mathrm{li}} \& \mathrm{v}_{2 \mathrm{f}}=\mathrm{v}_{\mathrm{lf}}$


## Inelastic Collisions?



## Inelastic Collision

Ballistic pendulum: A bullet of mass $m$ and initial velocity $\mathrm{v}_{0}$ collides and sticks to a pendulum of mass $M$ supported on a rope of length $L$. How high does the pendulum go before coming to rest?


Conservation of momentum Kinetic Energy to Potential Energy
$p_{i}=m v_{0}=p_{f}=(m+M) v$
$\frac{(M+m) \nu^{2}}{2}=(M+m) g h$

$$
v=\frac{m v_{0}}{m+M}
$$

$$
\frac{\left(\frac{m v_{0}}{m+M}\right)^{2}}{2 g}=h
$$

