Chapter 15 Questions

1. (Question 15.3) Which of the following describe $\phi$ for the SHM of Fig. 15-20a:

(a) $-\pi < \phi < -\pi/2$.
(b) $\pi < \phi < 3\pi/2$.
(c) $-3\pi/2 < \phi < -\pi$.

Figure 15-20a (figure above) shows the displacement of a particle undergoing simple harmonic motion as function of time, $x = x_m \cos(\omega t + \phi)$. At $t = 0$ the particle’s position is below equilibrium position (negative) so at that moment $\cos \phi < 0$. That means that $\phi$ should be between $\pi/2$ and $3\pi/2$.

The particle’s velocity, $v = -x_m \omega \sin(\omega t + \phi)$, is positive at $t = 0$ (by inspecting the graph), which gives that $\sin \phi < 0$. That means that $\phi$ should be between $\pi$ and $2\pi$.

The overlap of these two conditions is $\pi < \phi < 3\pi/2$. That angle can be, also, represented as negative angle by subtracting $2\pi$, which gives another way to answer the same question, $-\pi < \phi < -\pi/2$. So, the correct answers are both (a) and (b).

The other way to answer the question is to draw $x = x_m \cos \omega t$ in the same plot as $x = x_m \cos(\omega t + \phi)$ and read the phase difference between, for example, the closest crests of the two functions. The phase difference, $\phi$, is between $-\pi/2$ and $-\pi$ for the closest crest to the right of $x = x_m \cos \omega t$ or between $\pi$ and $3\pi/2$ for the closest crest to the left of $x = x_m \cos \omega t$. See the plot below.

2. (a) What is the phase constant for the harmonic oscillation with the acceleration function $a(t)$ given in the figure below, if the position function $x(t)$ has the form $x = x_m \cos(\omega t + \phi)$? (b) What is the angular frequency? (c) What is $x_m$?

Using: $a(t) = -a_m \cos(\omega t + \phi)$ and

$slope(t) = \frac{da}{dt} = a_m \omega \sin(\omega t + \phi)$ at $t = 0$

and the initial conditions given:

$a(0) = +2 \text{ cm/s}^2, \quad slope(0) > 0, \quad \text{with } a_m = 8 \text{ cm/s}^2$

we have:

$\phi = \cos^{-1} \left( - \frac{a(0)}{a_m} \right)$

$= \cos^{-1} \left( - \frac{2 \text{ m/s}^2}{8 \text{ m/s}^2} \right) = \begin{cases} +1.82 \text{ rad} = -4.46 \text{ rad} \\ 2\pi - 1.82 = +4.46 \text{ rad} \end{cases}$

To choose between $\phi = +1.82 \text{ rad}$ and $+4.46 \text{ rad}$ we need to check for which $\phi$ is the slope at $t=0$ positive: $\sin(1.82 \text{ rad}) > 0$ while $\sin(4.46 \text{ rad}) < 0$, so the phase constant is $\phi = +1.82 \text{ rad} = -4.46 \text{ rad}$.
To relate the phase constant $\phi$ to the time along the $t$ axis one can use: $t = \frac{\phi}{2\pi} T = \frac{1.82 \text{ rad}}{2\pi \text{ rad}} (12.0 \text{ s}) = 3.47 \text{ s}$.

(b) $\omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{12.0 \text{ s}} = 0.524 \text{ rad/s}$.

(c) Using $a_m = x_m \omega^2$ we have: $x_m = \frac{a_m}{\omega^2} = \frac{8 \text{ cm/s}^2}{(0.524 \text{ rad/s})^2} = 29.1 \text{ cm} = 0.291 \text{ m}$. 
3. A 1.2 kg **square** block, with edge length of 20.0 cm is supported by a massless rod, 56.0 cm long, which is pivoted at at its end.

Find the period of small angle oscillations.

\[ T = 2\pi \sqrt{\frac{I_P}{mgh}} \] where \( h = L + d/2 \)

\[ I_P = I_{com} + m(L + d/2)^2 = \frac{1}{12} m(d^2 + d^2) + m(L + d/2)^2 = m\left(\frac{d^2}{6} + (L + d/2)^2\right) \]

\[ m\left[\frac{d^2}{6} + (L + d/2)^2\right] = (1.2 \text{ kg})\left[\frac{(0.2 \text{ m})^2}{6} + (0.56 + 0.1)^2\right] = 0.53 \text{ kg m}^2 \]

\[ T = 2\pi \sqrt{\frac{0.53 \text{ kg m}^2}{(1.2 \text{ kg})(9.8 \text{ m/s}^2)(0.56 + 0.1)\text{ m}}} = 1.64 \text{ s} \]