
The Structure, Stability, and Dynamics of Self-Gravitating Systems

Virial Equations

In studying the structure and stability of self-gravitating systems, we often will find it useful to draw upon

The 2nd-Order Tensor Virial Equation (Tensor Virial Theorem)

$$(1/2) \mathbf{D}^2 \mathbf{I}_{ij} = 2 \mathbf{T}_{ij} + \mathbf{\Pi}_{ij} + \mathbf{W}_{ij},$$

[Equation I.E.1]

=
EFE, p.23, Eq. (51)
BT87, p.213, Eq. (4-78)



which defines in a fundamental way the relationship between various global properties of isolated astrophysical systems. (Click on any one of these tensor variables to obtain its formal definition and click on the "origin" button to see a derivation of the equation.) A somewhat more familiar relation, usually referred to as the

Scalar Virial Theorem

$$(1/2) \mathbf{D}^2 \mathbf{I} = 2 \mathbf{E}_{\text{rot}} + 3 (\gamma - 1) \mathbf{U} + \mathbf{E}_{\text{grav}},$$

[Equation I.E.2]

results immediately from the trace of the 2nd-order tensor virial equation. This expression relates the second time-derivative of a system's moment of inertia to its gravitational potential energy \mathbf{E}_{grav} , its total internal energy \mathbf{U} , and its global kinetic energy (such as the energy tied up in rotation).

As we explain elsewhere in this H_Book, the 2nd-order tensor virial equation is not independent of the principal governing equations. Indeed, it is derived by taking the "first moment" of the Euler equation and integrating over the entire volume of the physical system under consideration. In his now classical analysis of ellipsoidal figures of equilibrium, Chandrasekhar has demonstrated how even higher order moments of Euler's equation can be used to examine the global stability of self-gravitating systems. Generally speaking, however, we shall not find it necessary to draw upon relationships that extend beyond the 2nd order tensor virial equation, although we may occasionally allude to the results that have been derived from them.