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# The Structure, Stability, and Dynamics of Self-Gravitating Systems

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## Supplemental Relations

Apart from the independent variables  $t$  and  $\mathbf{x}$ , the [principal governing equations](#) involve the vector velocity  $\mathbf{v}$ , and the four scalar variables  $\Phi$ ,  $\rho$ ,  $P$ , and  $\epsilon$ . Because the variables outnumber the equations by one, one (additional) supplemental relationship between the physical variables must be specified in order to close the set of equations.

Also, in order to complete the unique specification of a particular physical problem, either a steady-state flow field or initial conditions must be specified, depending on whether one is studying a time-independent ([structure](#)) or time-dependent ([stability](#) or [dynamics](#)) problem, respectively.

Throughout this H\_Book, the following strategy will be adopted in order to complete the physical specification of each examined system:

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- For [time-dependent](#) problems, we will
    - [adopt](#) an equation of state and
    - [specify](#) initial conditions.
  
  - For [time-independent](#) problems, we will
    - [adopt](#) a structural relationship between  $P$  and  $\rho$  and
    - [specify](#) a steady-state flow-field.
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## Time-Dependent Problems

### Equation of State

The equation of state that generally will be adopted for *time-dependent* problems is one that describes an ideal gas. As the [accompanying discussion](#) illustrates, the ideal gas equation of state can assume a variety of different forms. Throughout this H\_Book, we frequently will use [Form B](#) of the ideal gas equation of state to supplement the [principal governing equations](#):

$$P = (\gamma - 1) \epsilon \rho,$$

[Equation II.A.4]



where the ratio of specific heats  $\gamma$  is assumed to be independent of both  $\mathbf{x}$  and  $t$ . Simultaneously, [Form A](#) of the ideal gas equation of state [\[II.A.3\]](#) provides a relationship between the gas temperature  $T$  and the state variables  $P$  and  $\rho$ .

See [Tassoul \(1978\)](#) -- specifically the discussion associated with his Chapter 4, Eq. 13 -- for a more general statement related to a proper specification of the supplemental, equation of state relationship.

### Initial Conditions

For *time-dependent* problems, the [principal governing equations](#) must be supplemented further through the specification of initial

conditions. Frequently throughout this H\_Book, we will select as initial conditions a specification of the functions  $\rho(\mathbf{x}, t=0)$ ,  $P(\mathbf{x}, t=0)$ , and  $\mathbf{v}(\mathbf{x}, t=0)$  that itself defines a static or [steady-state](#), equilibrium [structure](#).

## Time-Independent Problems

### Barotropic Structure

For time-*in* dependent problems, a structural relationship between  $P$  and  $\rho$  is required to close the system of ([principal governing](#)) equations.

[Tassoul \(1978\)](#) refers to it as a "geometrical" rather than a "structural" relationship; see the discussion associated with his Chapter 4, Eq. 14.

Generally throughout this H\_Book, we will assume that all time-independent configurations can be described as barotropic structures. That is, we will assume that the gas pressure  $P$  is only a function of the gas density  $\rho$  throughout such structures. More specifically, we generally will adopt one of the following two *analytically* prescribable  $P(\rho)$  relationships.

In *Polytropic Structures* ,

$$P = K_n \rho^{1 + 1/n} ,$$

[Equation II.C.1]



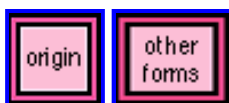
where the polytropic index  $n$  and the coefficient  $K_n$  are assumed to be independent of both  $\mathbf{x}$  and  $t$  .

In *Zero-Temperature, White Dwarf Structures* ,

$$P = a [ \chi ( 2\chi^3 - 3 ) ( \chi^2 + 1 )^{1/2} + 3 \sinh^{-1}\chi ] ,$$

where:  $\chi = ( \rho / b )^{1/3}$ ,

[Equation II.C.2]



and the coefficients  $a = ( 6.002 \times 10^{22} \text{ dyne cm}^{-2} )$  and  $b / \mu = ( 9.736 \times 10^{22} \text{ g cm}^{-3} )$  are assumed to be independent of both  $\mathbf{x}$  and  $t$

Q

What is the relationship, if any, between the polytropic index  $n$  and the ratio of specific heats  $\gamma$  in an adiabatic system?

A

It is worth noting that, for any barotropic equation of state, it is possible to define a fluid enthalpy  $H$  such that

$$H \equiv \int (1/\rho) dP,$$

[Equation III.F.17]

and

$$\nabla H = (1/\rho) \nabla P.$$

[Equation III.F.16]

Because enthalpy has the same dimensional units as the gravitational potential  $\Phi$  (*i.e.*, energy per unit mass), it often proves to be a useful variable through which to describe the equilibrium structure of self-gravitating systems. It should be clear immediately, for example, that the above expression [III.F.16] can be used to simplify considerably the form of the right-hand-side of Euler's equation [I.A.1].

Q

Assuming that  $H = 0$  at  $\rho = 0$ , show that the analytical expression for the enthalpy  $H(\rho)$  of a polytropic gas is,

$$H = (1 + n) K_n \rho^{(1/n)},$$

[Equation III.F.18]

and that, in terms of the specific entropy of the gas  $s$ ,

$$K_n = (1/n) \exp[ s/c_v ].$$

[Equation III.F.19]

A

## Accompanying Discussions in this Chapter

- **Ideal Gas Relations**
- **Equations of State**
- **Specific Heats**
- **Polytropic Relations**

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