The Structure, Stability, and Dynamics of Self-Gravitating Systems

Principal Governing Equations

According to Landau and Lifshitz' (1975) elequent discussion of the broad subject of *Fluid Mechanics* [1], the state of a moving fluid is determined by five quantities: the three components of the velocity $\mathbf{v}$ and, for example, the pressure P and the density ρ. For our discussions of astrophysical fluid systems throughout this Hypertext Book [H_Book], we will add to this the gravitational potential Φ. Accordingly, a complete system of equations of fluid dynamics should be six in number. For an ideal fluid these are ...

Euler's Equation

(The Vector Equation of Motion)

$$\mathbf{Dv} = - (1/\rho)\nabla P - \nabla\Phi$$

[Equation I.A.1]

other forms

Equation of Continuity

(Mass Conservation)

$$\mathbf{D}\rho + \rho\ \nabla\cdot\mathbf{v} = 0$$

[Equation I.B.1]

other forms

Adiabatic Form of the First Law of Thermodynamics

(Specific Entropy Conservation)

$$\mathbf{D}\varepsilon + P\ \mathbf{D}(1/\rho) = 0$$

[Equation I.C.1]

other forms

Poisson Equation

$$\nabla^2\Phi = 4\pi G\rho$$

[Equation I.D.1]

These equations relate the time (t) and spatial ($\mathbf{x}$) variation of the variables

$\mathbf{v}$	=	velocity,
P	=	gas pressure,
Φ	=	gravitational potential,
ρ	=	mass density,
ε	=	specific internal energy,

to one another in a physically consistent fashion.[2] By restricting our discussions to physical systems that are governed by this set of equations, for the most part **we will be considering the structure, stability and dynamical behavior of compressible, inviscid fluid systems that are self-gravitating.** We will assume that no electromagnetic forces act on the fluid (*eg.,* the effects of magnetic fields on an ionized plasma fluid will not be considered) and, in the absence of dynamically generated shocks, we will assume that all compressions and rarefactions occur adiabatically.

To complete the description of any specific astrophysical system, this set of differential equations must be supplemented by additional relations which (at least in the context of this H_Book) usually will be algebraic expressions motivated by the specific physics that is relevant to the chosen system.

Footnotes

[1]Text in green is taken verbatim from Chapter I, §2 of Landau and Lifshitz's Fluid Mechanics (1975).

[2]If you need to be reminded what the dimensional units of any of these variables is, what the definition or form is of the various mathematical operators, or what the numerical value is of a particular physical constant in the above equations, click on the relevant highlighted text symbol and you'll find the answer on the page of definitions that accompanies this H_Book.

[This PDF document created: *08/10/98*]