Quantum Optical Computing, Imaging, and Metrology

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Quantum (Optical) Sensors
Ballroom I: WED 12:00N & 8:50PM

12:00 Christoph Wildfeuer & A. Migdall
NIST & Louisiana State University
“Techniques for enhancing the
precision of measurements with photon
number resolving detectors”

12:20 Petr M. Anisimov
Louisiana State University
“Squeezing the Vacuum and beating
Heisenberg: Limits? Where we’re going
we do not need any limits!”

12:40 E. E. Mikhailov & I. Novikova
College of William and Mary
“Generation of squeezed vacuum with
hot and ultra-cold Rb atoms”

20:50 Geoff Pryde
Griffith University, AUS
“Optimal multiphoton phase
sensing and measurement”

21:10 J. C. F. Matthews & J. L. O’Brien
University of Bristol, UK
“Entangled Multi-Photon States in
Waveguide for Quantum Metrology”

21:30 Kevin McCusker & Paul Kwiat
University of Illinois
“Efficient Quantum Optical State
Engineering”

21:50 Xi Wang
Texas A&M University
“Self-Implemented Heterodyne CARS
by Using Its Intrinsic Background”


Bottom Inset C. Brignac, R.Cross, B.Gard, D.J.Lum, Keith Motes, G.M.Raterman, C.Sabottke,
Outline

1. Nonlinear Optics vs. Projective Measurements
2. Quantum Imaging vs. Precision Measurements
3. Showdown at High NOON!
4. Mitigating Photon Loss
6. Super Resolution with Classical Light
7. Super-Duper Sensitivity Beats Heisenberg!
8. A Parody on Parity
Optical Quantum Computing: Two-Photon CNOT with Kerr Nonlinearity

The Controlled-NOT can be implemented using a Kerr medium:

\[ |0\rangle = |H\rangle \text{ Polarization} \]
\[ |1\rangle = |V\rangle \text{ Qubits} \]

\( R \) is a \( \pi/2 \) polarization rotation, followed by a polarization dependent phase shift \( \pi \).

Unfortunately, the interaction \( \chi^{(3)} \) is extremely weak*:

\[ 10^{-22} \text{ at the single photon level} \] — This is not practical!

Two Roads to Optical Quantum Computing

I. Enhance Nonlinear Interaction with a Cavity or EIT — Kimble, Walther, Lukin, et al.

II. Exploit Nonlinearity of Measurement — Knill, LaFlamme, Milburn, Nemoto, et al.
Linear Optics can be Used to Construct $2 \times \text{CSIGN} = \text{CNOT}$ Gate and a Quantum Computer:

$$\psi_{\text{in}} \rightarrow \psi_{\text{out}}$$

$\alpha |0\rangle + \beta |1\rangle + \gamma |2\rangle \rightarrow \alpha |0\rangle + \beta |1\rangle - \gamma |2\rangle$

Knill E, Laflamme R, Milburn GJ
NATURE 409 (6816): 46-52 JAN 4 2001

PRL 89 (13): Art. No. 137901 SEP 23 2002
WHY IS A KERR NONLINEARITY LIKE A PROJECTIVE MEASUREMENT?
Projective Measurement Yields Effective Nonlinearity!

A Revolution in Nonlinear Optics at the Few Photon Level: No Longer Limited by the Nonlinearities We Find in Nature!

\[ Q = \frac{\pi \hbar}{2} (5\hat{n} - \hat{n}^2) \]

KLM CSIGN: Self Kerr

\[ Q = \frac{\pi \hbar}{2} (3 + a_b^\dagger (1 - \hat{n}_b) + (1 - \hat{n}_b) a_b) \hat{n}_a \]

Franson CNOT: Cross Kerr
Quantum Metrology

MACH-ZEHNDER INTERFEROMETER

$|N\rangle_A |0\rangle_B + e^{iN\varphi} |0\rangle_A |N\rangle_B$

N-Photon Detector

A

$|N\rangle_A |0\rangle_B$

magic BS

N-XOR Gates

B

$|0\rangle_B$

N-XOR Gates

† We call the state of the form $|N, \phi\rangle + |\phi, N\rangle$ the NOON state, and the High NOON state a large $N$.

Shot noise

Heisenberg

$\varphi = kx$

$\Delta\varphi: 1/\sqrt{N} \rightarrow 1/N$
Sub-Shot-Noise Interferometric Measurements
With Two-Photon NOON States

Low NOON

\[ |2\rangle|0\rangle + e^{i2\varphi}|0\rangle|2\rangle \]
FROM QUANTUM INTERFEROMETRY TO QUANTUM LITHOGRAPHY

\[ |N\rangle_A |0\rangle_B + e^{iN\varphi} |0\rangle_A |N\rangle_B \]

N-Photon Detector

Magic BS

N-XOR Gates

Mirror

\[ \langle \psi | a^\dagger N a^N |\psi \rangle \]

Super-Resolution

Oscillates in REAL Space!

\[ \frac{1 + \cos \varphi}{2} \quad \text{uncorrelated} \]

\[ \frac{1 + \cos N\varphi}{2} \quad \text{correlated} \]

\[ \varphi = kx \]

\[ \varphi \rightarrow N\varphi \]

\[ \lambda \rightarrow \lambda/N \]
Quantum Lithography Experiment

Quantum lithography: setup

- Milena D’Angelo, Maria V. Chekhova, and Yanhua Shih, PRL 87, 013602 (2001)

Two-photon source: Degenerate Collinear type-II SPDC

Double-slit VERY close to the crystal \( \Rightarrow \Delta \phi \ll b/D \)

\[ |\psi\rangle = e^{i(\Delta \phi) \cdot \mathbf{b}} |0\rangle \]

\( \Delta \phi \) - scattering angle inside the crystal; \( b \) - distance between slits; \( D \) - distance between input face of crystal and double slit

Low N00N

\[ |2\rangle |0\rangle + e^{i2\phi} |0\rangle |2\rangle \]
Quantum Imaging: Super-Resolution

$\lambda / N$

$N=1$ (classical)
$N=5$ (N00N)
Quantum Metrology: Super-Sensitivity

\[ \Delta \phi = \frac{\Delta \hat{P}}{\left| \frac{d \langle \hat{P} \rangle}{d \phi} \right|} \]

\( \Delta \phi_{N=1} = \frac{1}{\sqrt{N}} \) (classical)

\( \Delta \phi_{N=5} = \frac{1}{N} \) (N00N)

Shotnoise Limit:
\[ \Delta \phi_{\text{shot}} = \frac{1}{\sqrt{N}} \]

Heisenberg Limit:
\[ \Delta \phi_{\text{Heis}} = \frac{1}{N} \]
Showdown at High-NOON!

How do we make \textbf{High-NOON}!?

\[ |N,0\rangle + |0,N\rangle \]

With a large cross-Kerr nonlinearity!* \( H = \kappa a^\dagger a b^\dagger b \)

\[ |1\rangle \quad \quad MZI-2 \]
\[ |0\rangle \quad \quad BS3 \quad BS4 \quad D1 \]
\[ |N\rangle \quad \quad KERR \]
\[ |0\rangle \quad \quad BS1 \quad BS2 \quad D2 \]

This is not practical! — need \( \kappa = \pi \) but \( \kappa = 10^{-22} \)!

FIRST LINEAR-OPTICS BASED HIGH-N00N GENERATOR PROPOSAL

Success probability approximately 5% for 4-photon output.

N00N State Experiments

1990’s
2-photon
Rarity, (1990)
Ou, et al. (1990)
Shih (1990)
Kuzmich (1998)
Shih (2001)

6-photon
Super-resolution
Only!
Resch,...,White
PRL (2007)
Queensland

2004
3, 4-photon
Super-resolution only

2007–2010
4-photon
Super-sensitivity &
Super-resolution
Nagata,...,Takeuchi,
Science (04 MAY)
Hokkaido & Bristol;
J.C.F.Matthews,
A.Politi, Damien
Bonneau, J.L.O'Brien,
arXiv:1005.5119

Mitchell,...,Steinberg
Nature (13 MAY)
Toronto

Walther,...,Zeilinger
Nature (13 MAY)
Vienna
Quantum LIDAR

“DARPA Eyes Quantum Mechanics for Sensor Applications” — Jane’s Defence Weekly

Winning LSU Proposal

Nonclassical Light Source

Delay Line

Detection

Forward problem solver

\[ \delta \phi = f(\| \Psi_{in} \|, \phi; \text{loss A, loss B}) \]

Feedback Loop: Genetic Algorithm

\[ \text{INPUT: } \begin{array}{c}
\text{find } \\
\min(\delta \phi)
\end{array} \]

\[ N: \text{photon number} \]

\[ \text{loss A} \]

\[ \text{loss B} \]

\[ |\Psi_{in}\rangle = \sum_{i} c_{i} |N-i,i\rangle \]

\[ \text{OUTPUT: } \begin{array}{c}
\text{min(} \delta \phi \text{)} \\
|\Psi_{in}^{\text{opt}}\rangle = \sum_{i} d_{i}^{\text{opt}} |N-i,i\rangle, \phi_{\text{opt}}
\end{array} \]
Loss in Quantum Sensors


Visibility:
\[ |\psi\rangle = (|10,0\rangle + |0,10\rangle)/\sqrt{2} \]

Sensitivity:
\[ |\psi\rangle = (|10,0\rangle + |0,10\rangle)/\sqrt{2} \]
Super-Lossitivity
Gilbert, G; Hamrick, M; Weinstein, YS; JOSA B 25 (8): 1336-1340 AUG 2008

$$\Delta \varphi = \frac{\Delta \hat{P}}{|d\langle \hat{P} \rangle / d\varphi|}$$

$$dP_N / d\varphi$$

$$e^{i\varphi} \rightarrow e^{iN\varphi}$$

$$e^{-\alpha L} \rightarrow e^{-N\alpha L}$$

$$dP_1 / d\varphi$$

3dB Loss, Visibility & Slope — Super Beer’s Law!
Q: Why do N00N States “Suck” in the Presence of Loss?

A: Single Photon Loss = Complete “Which Path” Information!

\[ |N\rangle_A |0\rangle_B + e^{iN\varphi} |0\rangle_A |N\rangle_B \rightarrow |0\rangle_A |N - 1\rangle_B \]
Towards A Realistic Quantum Sensor


Try other detection scheme and states!

M&M state: $|\psi\rangle = \frac{(|m,m\rangle + |m',m\rangle)}{\sqrt{2}}$

M&M Visibility
$|\psi\rangle = \frac{(|20,10\rangle + |10,20\rangle)}{\sqrt{2}}$

N00N Visibility
$|\psi\rangle = \frac{(|10,0\rangle + |0,10\rangle)}{\sqrt{2}}$

M&M’ Adds Decoy Photons
Towards A Realistic Quantum Sensor


M&M state: $|\psi\rangle = (|m,m\rangle + |m',m\rangle)/\sqrt{2}$

\[ M&M \text{ state: } |\psi\rangle = (|m,m\rangle + |m',m\rangle)/\sqrt{2} \]

Lost photons

A Few Photons Lost Does Not Give Complete "Which Path"

\[ \delta \phi \]

\[ \begin{array}{c|c|c|c|c|c} \phi & -3 & -2 & -1 & 1 & 2 & 3 \\ \hline 0.5 & & & & & & \\ 1.0 & & & & & & \\ 1.5 & & & & & & \\ 2.0 & & & & & & \\ \end{array} \]
We show that coherent light coupled with photon number resolving detectors — implementing parity detection — produces super-resolution much below the Rayleigh diffraction limit, with sensitivity at the shot-noise limit.
Quantum Metrology with Two-Mode Squeezed Vacuum: Parity Detection Beats the Heisenberg Limit
PRL 104, 103602 (2010)
PM Anisimov, GM Raterman, A Chiruvelli, WN Plick, SD Huver, H Lee, JP Dowling

We show that super-resolution and sub-Heisenberg sensitivity is obtained with parity detection. In particular, in our setup, dependence of the signal on the phase evolves \(<n>\) times faster than in traditional schemes, and uncertainty in the phase estimation is better than \(1/<n>\).

\[\text{SNL} \equiv 1/\sqrt{\langle \hat{n} \rangle} \quad \text{HL} \equiv 1/\langle \hat{n} \rangle \quad \text{TMSV} \equiv 1/\sqrt{\langle \hat{n} \rangle \langle \hat{n} + 2 \rangle} \quad \text{HofL} \equiv 1/\sqrt{\langle \hat{n}^2 \rangle}\]
Parity detection in quantum optical metrology without number-resolving detectors

William N Plick, Petr M Anisimov, Jónáthán P Døwlíng, Hwang Lee, and Girish S Agarwal

Abstract. We present a method for directly obtaining the parity of a Gaussian state of light without recourse to photon-number counting. The scheme uses only a simple balanced homodyne technique and intensity correlation. Thus interferometric schemes utilizing coherent or squeezed light and parity detection may be practically implemented for an arbitrary photon flux.
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