Quantum Optical Metrology, Imaging, and Computing

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On The «Georgy Zhukov» Between Valaam and St. Petersburg, Russia
Not to be confused with: Quantum Meteorology!

Outline

1. Nonlinear Optics vs. Projective Measurements
2. Quantum Imaging vs. Precision Measurements
3. Showdown at High NOON!
4. Mitigating Photon Loss
6. Super Resolution with Classical Light
7. Super-Duper Sensitivity Beats Heisenberg!
Optical Quantum Computing: Two-Photon CNOT with Kerr Nonlinearity

The Controlled-NOT can be implemented using a Kerr medium:

\[ |0\rangle = |H\rangle \text{ Polarization} \]
\[ |1\rangle = |V\rangle \text{ Qubits} \]

\[ R \text{ is a } \pi/2 \text{ polarization rotation, followed by a polarization dependent phase shift } \pi. \]

Unfortunately, the interaction \( \chi^{(3)} \) is extremely weak*: \( 10^{-22} \) at the single photon level — This is not practical!

Two Roads to
Optical Quantum Computing

I. Enhance Nonlinear Interaction with a Cavity or EIT — Kimble, Walther, Lukin, et al.

II. Exploit Nonlinearity of Measurement — Knill, LaFlamme, Milburn, Franson, et al.
Linear Optics can be Used to Construct $2 \times \text{CSIGN} = \text{CNOT}$ Gate and a Quantum Computer:

$\alpha |0\rangle + \beta |1\rangle + \gamma |2\rangle \rightarrow \alpha |0\rangle + \beta |1\rangle - \gamma |2\rangle$

Knill E, Laflamme R, Milburn GJ
*NATURE* 409 (6816): 46-52 JAN 4 2001

*PRL* 89 (13): Art. No. 137901 SEP 23 2002
WHY IS A KERR NONLINEARITY LIKE A PROJECTIVE MEASUREMENT?

LOQC
KLM

Photon-Photon XOR Gate

Cavity QED EIT

Photon-Photon Nonlinearity

Projective Measurement

Kerr Material

???
A Revolution in Nonlinear Optics at the Few Photon Level: No Longer Limited by the Nonlinearities We Find in Nature!

\[ Q = \frac{\pi \hbar}{2} (5\hat{n} - \hat{n}^2) \]

KLM CSIGN: Self Kerr

\[ Q = \frac{\pi \hbar}{2} (3 + a_b^\dagger (1 - \hat{n}_b) + (1 - \hat{n}_b) a_b) \hat{n}_a \]

Franson CNOT: Cross Kerr

Quantum Metrology

We call the state of the form $|N, \phi > + |\phi, N >$ the NOON state, and the High NOON state a large $N$.

Shot noise

Heisenberg

Oscillates $N$ times as fast
Sub-Shot-Noise Interferometric Measurements With Two-Photon NOON States


Low NOON

\[ |2\rangle|0\rangle + e^{i2\varphi} |0\rangle|2\rangle \]
FROM QUANTUM INTERFEROMETRY TO QUANTUM LITHOGRAPHY

| \[ N \rangle_A |0\rangle_B + e^{iN\varphi} |0\rangle_A |N\rangle_B |

N-Photon Detector

\[ \langle \psi \mid a^\dagger_N a^N \mid \psi \rangle \]

Super-Resolution

[Equation for uncorrelated and correlated states]

Oscillates in REAL Space!

AN Boto, DS Abrams, CP Williams, JPD, PRL 85 (2000) 2733
Quantum Interferometric Optical Lithography: Exploiting Entanglement to Beat the Diffraction Limit

Agedi N. Boto, Pieter Kok, Daniel S. Abrams, Samuel L. Braunstein, Colin P. Williams, and Jonathan P. Dowling

WHAT'S NEXT

New York Times

Quantum Leap May Transform Chips

By IAN AUSTEN

physics : Fine lines

PHILIP BALL nature

Yoked Photons Break a Light Barrier

Discovery Could Mean Faster Computer Chips
Quantum Lithography Experiment

Quantum lithography: setup

- Milena D’Angelo, Maria V. Chekhnova, and Yanhua Shih,
PRL 87, 013602 (2001)

Two-photon source: Degenerate Collinear type-II
SPDC

Double-slit VERY close to the crystal ⇒ Δφ << b/D

→ |ψ⟩ = ϵ(a_s†a_i† + b_s†b_i†) |0⟩

Δφ-scattering angle inside the crystal; b-distance between slits; D-distance between input face of crystal and double slit

Low N00N

|2⟩|0⟩ + e^{i2φ}|0⟩|2⟩

|10⟩+|02⟩

|0⟩+|10⟩

R_c(θ) = sinc^2[(2πa/λ) θ] × cos^2[(2πb/λ) θ]

I(θ) = sinc^2[(πa/λ) θ] × cos^2[(πb/λ) θ]
Quantum Imaging: Super-Resolution

$\frac{\lambda}{N}$

$N=1$ (classical)
$N=5$ (N00N)
Quantum Metrology: Super-Sensitivity

\[ \Delta \phi = \frac{\Delta \hat{P}}{\left| d \langle \hat{P} \rangle / d\phi \right|} \]

- Shotnoise Limit: \( \Delta \phi_1 = \frac{1}{\sqrt{N}} \)
- Heisenberg Limit: \( \Delta \phi_N = \frac{1}{N} \)

\[ \frac{dP_N}{d\phi} \]

\[ \frac{dP_1}{d\phi} \]
Showdown at High-NOON!

How do we make High-NOON!? 

\[ |N,0\rangle + |0,N\rangle \]

With a large cross-Kerr nonlinearity!* \( H = \kappa a\dagger a b\dagger b \)

This is not practical! — need \( \kappa = \pi \) but \( \kappa = 10^{-22} \) !

First linear-optics based high-N00N generator proposal.

Success probability approximately 5% for 4-photon output.

Scheme conditions on the detection of one photon at each detector.

Entangled photons conspire to create interference patterns that would normally be associated with a wavelength much smaller than that of the individual photons — beating the diffraction limit.

It would be more interesting if \( |N,0\rangle \) states could be generated with \( N>2 \) but using photons produced by light sources that have a wavelength of at least \( \lambda/2 \). The existence of such states — dubbed ‘high NOON’ states by Jonathan Dowling — would be an unambiguous demonstration that the diffraction limit has been beaten. This is exactly what Mitchell et al.\(^2\) and Walther et al.\(^3\) have achieved, with \( |N,0\rangle \) states for \( N=3 \) and \( N=4 \), respectively.

\[
|N::0\rangle_{a,b} \equiv \frac{1}{\sqrt{2}} \left( |N,0\rangle_{a,b} + 0,N\rangle_{a,b} \right)
\]
N00N State Experiments

1990’s
2-photon
Rarity, (1990)
Ou, et al. (1990)
Shih (1990)
Kuzmich (1998)
Shih (2001)

6-photon
Super-resolution
Only!
Resch, …, White
PRL (2007)
Queensland

2004
3, 4-photon
Super-resolution
only

2007
4-photon
Super-sensitivity &
Super-resolution

Mitchell, …, Steinberg
Nature (13 MAY)
Toronto

Walther, …, Zeilinger
Nature (13 MAY)
Vienna

Nagata, …, Takeuchi,
Science (04 MAY)
Hokkaido & Bristol
Quantum LIDAR

“DARPA Eyes Quantum Mechanics for Sensor Applications”  
— Jane’s Defence Weekly

Nonclassical Light Source

Delay Line

Detection

Noise

Target

INPUT

\[ |\Psi_{in}\rangle = \sum_{\substack{i \geq 0 \\ i \leq N}} |i\rangle |N-i,i\rangle \]

\[ \delta \phi = f(|\Psi_{in}\rangle, \phi; \text{loss A, loss B}) \]

FEEDBACK LOOP: Genetic Algorithm

\[ \min(\delta \phi) \]

\[ |\Psi_{opt}\rangle = \sum_{i=0}^{N} c_{i}^{opt} |N-i,i\rangle, \phi_{opt} \]

Nonclassical Light Source

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Loss in Quantum Sensors


Visibility:
$|\psi\rangle = (|10,0\rangle + |0,10\rangle)/\sqrt{2}$

Sensitivity:
$|\psi\rangle = (|10,0\rangle + |0,10\rangle)/\sqrt{2}$

Lost photons

N00N 3dB Loss ---
N00N No Loss —
SNL---
HL—
Super-Lossitivity

Gilbert, G; Hamrick, M; Weinstein, YS; JOSA B 25 (8): 1336-1340 AUG 2008

\[
\Delta \varphi = \frac{\Delta \hat{P}}{|d\langle \hat{P} \rangle / d\varphi|}
\]

\[
dP_N / d\varphi
\]

3dB Loss, Visibility & Slope — Super Beer’s Law!
Q: Why do N00N States Do Poorly in the Presence of Loss?

A: Single Photon Loss = Complete “Which Path” Information!

\[ |N\rangle_A |0\rangle_B + e^{iN\phi} |0\rangle_A |N\rangle_B \rightarrow |0\rangle_A |N - 1\rangle_B \]
Towards A Realistic Quantum Sensor


Try other detection scheme and states!

M&M state:

\[ |\psi\rangle = \frac{(|m,m\rangle + |m',m\rangle)}{\sqrt{2}} \]

M&M Visibility

\[ |\psi\rangle = \frac{(|20,10\rangle + |10,20\rangle)}{\sqrt{2}} \]

N00N Visibility

\[ |\psi\rangle = \frac{(|10,0\rangle + |0,10\rangle)}{\sqrt{2}} \]

M&M’ Adds Decoy Photons
Towards A Realistic Quantum Sensor


M&M Generator

$|\psi\rangle = \frac{1}{\sqrt{2}}(|m,m\rangle + |m',m\rangle)$

M&M state:

Lost photons

Lost photons

A Few Photons Lost Does Not Give Complete "Which Path"

N00N State ---
M&M State —

N00N SNL ---
M&M SNL ---

M&M HL —
M&M HL —
We optimize two-mode, entangled, number states of light in the presence of loss in order to maximize the extraction of the available phase information in an interferometer. Our approach optimizes over the entire available input Hilbert space with no constraints, other than fixed total initial photon number.
Here we take the optimal state, outputted by the code, at each loss level and project it on to one of three known states, NOON, M&M, and "Spin" Coherent.

The conclusion from this plot is that the optimal states found by the computer code are NOON states for very low loss, M&M states for intermediate loss, and "spin" coherent states for high loss.
We show that coherent light coupled with photon number resolving detectors — implementing parity detection — produces super-resolution much below the Rayleigh diffraction limit, with sensitivity at the shot-noise limit.
We show that super-resolution and sub-Heisenberg sensitivity is obtained with parity detection. In particular, in our setup, dependence of the signal on the phase evolves $<n>$ times faster than in traditional schemes, and uncertainty in the phase estimation is better than $1/<n>$.

$$\text{SNL} \equiv 1 / \sqrt{\langle \hat{n} \rangle} \quad \text{HL} \equiv 1 / \langle \hat{n} \rangle \quad \text{TMSV} \equiv 1 / \sqrt{\langle \hat{n} \rangle \langle \hat{n} + 2 \rangle} \quad \text{HofL} \equiv 1 / \sqrt{\langle \hat{n}^2 \rangle}$$
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