
Problem 2. Barton Problem 1.9, p.39.


Problem 5. Barton Problem 1.12, p.39. (Hint: Assume $n = 3$.)

Problem 6. An infinitely thin and infinitely long positively charged wire, with charge per unit length $\lambda$, is placed on and parallel to the $z$-axis. Compute the electrostatic potential $\Phi(r)$ everywhere in space outside the wire. Use this result to compute the electric field $E(r)$ everywhere in space outside the wire. Assume a reference potential of a constant $\Phi(r_0) = \Phi_0$ on a cylinder of a constant radius $r_0$ around the wire. Please use SI units.

(a) First try to compute the potential using the PHYS2102 approach of integrating the contributions along the wire from the differential charges as per the prescription:

$$\Phi(r) = \frac{1}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{\lambda dz}{\sqrt{r^2 + z^2}}.$$

What goes wrong and why? (Hint: Only potential differences have meaning. This approach assumes a particular reference potential value at infinity. Is this value consistent with the set up of the problem?)

(b) Now use a suitably defined delta function in cylindrical coordinates to describe the charge density $\rho(r)$. Mind your units! Then directly integrate Poisson’s equation:

$$\nabla^2 \Phi(r) = -\rho(r)/\varepsilon_0.$$

(c) The electric field then is computed from $E = -\nabla \Phi$.

(d) Check your calculation by computing the electric field directly by using Gauss’s law with a cylindrical Gaussian surface of radius of radius $r$ centered on the $z$-axis. (This will double-check your constants.)