Exam 2: Physics 2113 Fall 2014

6:00PM TUE 14 OCT 2014

Name (Last, First): ____________________________

Section #___________________________

Instructor’s name: __________________________

Answer all 3 problems & all 4 questions.

Be sure to write your name. Please read the questions carefully.

You may use only scientific or graphing calculators. In particular you may not use the calculator app on your phone or tablet!

You may detach and use the formula sheet provided at the back of this test. No other reference materials are allowed.

You may not answer or use cell phones during the exam. Please note that the official departmental policy for exams is as follows: “During your test, the only electronic device you may have with you at your seat is a scientific or graphing calculator. You may not have your cell phone, tablet, smartphone, PDA, pager, digital camera, computer, or any other device capable of taking pictures or video, sending text messages, or accessing the Internet. This means not just on your person, but close enough to you that you could reach it during the test. Any student found with such a device during a test will be assumed to be violating the LSU Honor Code and will be referred to the Dean of Students for Judicial Affairs.” The simplest remedy is to bring nothing to this test but the calculator, and leave your backpack or purse at home. If you have brought your cell phone or tablet with you, please leave it at the front of the room under the watchful eye of your instructor.

Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and to determine if your thinking is correct.

On problems that are not multiple choice, be sure to show all of your work since no credit will be given for an answer without explanation or work. These will be graded in full, and you are expected to show all relevant steps that lead to your answer. Please use complete sentences where explanations are asked for. For numerical answers that require units you must give the correct units for full credit.

YOU GET 60 min (1 hr)
1. (Question) [9 points] A potential difference V is connected across a device with resistance R, causing current i through the device. Circle the correct changed rate at which electrical energy is converted to the thermal energy, assuming the original rate to be P.

\[ P = R \cdot i^2 = \frac{V^2}{R} = i \cdot V \]

(i) [3 points] V is doubled with R unchanged.

\[ P_{\text{new}} = \frac{V_{\text{new}}^2}{R} = \left(\frac{2V}{R}\right)^2 = 4P \]

(ii) [3 points] i is tripled with R unchanged

\[ P_{\text{new}} = i_{\text{new}}^2 \cdot R = (3i)^2 \cdot R = 9P \]

(iii) [3 points] R is doubled with i unchanged

\[ P_{\text{new}} = i^2 \cdot R_{\text{new}} = i^2 (2R) = 2P \]

2. (Question) [9 points] A parallel-plate capacitor of capacitance C is charged using a battery of potential V between its terminals. Then the battery is disconnected and the size of the plates (A) of the isolated capacitor is increased. Circle T if the statement is true and F if the statement is false.

(i) [3 points] The capacitance of the capacitor has increased

\[ C_{\text{new}} = \varepsilon_0 \frac{A_{\text{new}}}{d} \Rightarrow A_{\text{new}} > A \Rightarrow C_{\text{new}} > C \]

(ii) [3 points] The charge on the plates of the capacitor has increased

\[ \text{no battery} \Rightarrow \text{charge stayed constant} \]

(iv) [3 points] The potential difference across the capacitor has decreased

\[ \frac{q \cdot CV}{C} = V = \frac{3}{2} \quad C_{\text{increased}} \Rightarrow V_{\text{decreased}} \]
3. (Question) [9 points]

The figure shows a uniform electric field, pointing to the right. The equipotential surfaces are indicated with dashed lines. There are four points indicated on the figure (1, 2, 3, and 4). We define $V = 0$ at point 3 ($V_3 = 0$).

a) [3 points] What is the electric potential at the three other points? Circle the right answer for each $V$.

i) $V_1 < 0$  \hspace{1cm} \text{ii)} $V_2 < 0$  \hspace{1cm} \text{iii)} $V_4 < 0$

\hspace{1cm} $V_1 = 0$  \hspace{1cm} $V_2 = 0$  \hspace{1cm} $V_4 = 0$

\hspace{1cm} $V_1 > 0$  \hspace{1cm} $V_2 > 0$  \hspace{1cm} $V_4 > 0$

\hspace{1cm} \text{• the electric potential decreases along the direction of electric field \text{ (ii) and (iii)} }

b) [3 points] What is the work done by the electric field on an electron if it is moved from point 2 to point 4? Circle the correct answer.

Positive  \hspace{1cm} \text{Zero}  \hspace{1cm} \text{Negative}  \hspace{1cm} \text{Not enough information to tell}

\[ W_E = -\Delta U = -q(\Delta V) \]

\[ < 0 \quad < 0 \]

\[ \Rightarrow W_E < 0 \]

c) [3 points] What is the work done by the electric field on an electron if it is moved from point 1 to point 3? Circle the correct answer.

Positive  \hspace{1cm} \text{Zero}  \hspace{1cm} \text{Negative}  \hspace{1cm} \text{Not enough information to tell}

\[ \Delta V = 0 = V_3 - V_1 \]

\text{because } V_1 = V_3 \quad (\text{equipotential surface})
4. (Question) [9 points] The figure below shows three cylindrical copper conductors along with their face areas and lengths.

\[ R_a = \rho \frac{L}{A} \]

\[ R_b = \rho \frac{1.5L}{A/2} = 3R_a \]

\[ R_c = \rho \frac{L/2}{A/2} = R_a \]

\[ b > a = c \]

b) [3 points] Rank them according to the current through them, greatest first, when the same potential difference \( V \) is placed across their lengths.

\[ i_a = \frac{V}{R_a} \]

\[ i_b = \frac{V}{R_b} = \frac{V}{3R_a} = \frac{1}{3} i_a \]

\[ i_c = \frac{V}{R_c} = \frac{V}{R_a} = i_a \]

\[ a = c > b \]

c) [3 points] Rank them according to their current density, greatest first, when the same potential difference \( V \) is placed across their lengths.

\[ j_a = \frac{i_a}{A} \]

\[ j_b = \frac{i_{b}}{A_b} = \frac{\frac{1}{3} i_a}{A/2} = \frac{2}{3} j_A \]

\[ j_c = \frac{i_c}{A/2} = 2 j_A \]

\[ c > a > b \]
5. **(Problem) [21 points]**

The figure shows an arrangement of 4 charged particles at the corners of a square. Particles are initially infinitely far apart and at rest. Use that \( q = 2.2 \times 10^{-12} \text{C} \) and \( a = 0.3 \text{ m} \).

\[ a = 0.3 \text{ m} \]

(a) **[7 points]** Find the electric potential in the center of the square. Show the calculations or state why.

\[ V_c = 0 \]

because there are 2 +q and 2 -q charges, of the same magnitude and the same distance from center.

(b) **[7 points]** Find how much work is required to set up the arrangement.

\[ W_{\text{ext}} = \Delta U = U_f - U_i = U_f = k \frac{q_1q_2}{r_{12}} + k \frac{q_1q_3}{r_{13}} + k \frac{q_1q_4}{r_{14}} + k \frac{q_2q_3}{r_{23}} \]

\[ + k \frac{q_2q_4}{r_{24}} + k \frac{q_3q_4}{r_{34}} = \frac{k q_2^2}{a} \left( -1 + \frac{1}{\sqrt{2}} - 1 - 1 + \frac{1}{\sqrt{2}} - 1 \right) \]

\[ = \frac{(8.99 \times 10^9)(2.2 \times 10^{-12})}{0.3} (-4 + \frac{2}{\sqrt{2}}) = -3.75 \times 10^{-13} \text{ J} \]

(c) **[7 points]** How much work would be required to bring an electron from infinite distance to the center of the square?

\[ W = q \Delta V = q (V_f^o - V_i^o) = 0 \]
6. (Problem) **[21 points]** In the figure, a potential difference \( V = 5.0 \text{ V} \) is applied across a capacitor arrangement with capacitances \( C_1 = 3.0 \mu\text{F}, \ C_2 = 2.0 \mu\text{F}, \text{ and } C_3 = 1.0 \mu\text{F}. \)

\[ C_1 \text{ and } C_2 \ldots \text{ in series} \]

\[ C_{12} \text{ and } C_3 \text{ in parallel} \]

\[ C_{12} \parallel C_3 = \frac{1}{C_{eq}} \]

\[ C_{eq} = C_3 + \frac{C_2}{C_1 + C_2} = \frac{1}{\mu\text{F}} + \frac{(3)(2)}{3 + 2} \mu\text{F} = 2.2 \mu\text{F} \]

\[ C_{12} = 1.2 \mu\text{F} \quad \Rightarrow \quad V_{12} = C_{12} V = 1.2 \mu\text{F}(5\text{V}) = 6.0 \mu\text{C} \]

\[ \phi_1 = \phi_2 = \phi_{12} = C_{12} V_{12} = 6.0 \mu\text{C} \]

\[ \phi_2 = \phi_{12} - \phi_3 = C_{eq} V - C_3 V = (2.2 \mu\text{F} - 1 \mu\text{F})(5\text{V}) = 6.0 \mu\text{C} \]

\[ U_3 = \frac{C_3 V_3^2}{2} = \frac{(1 \times 10^{-6})(5)^2}{2} = 12.5 \times 10^{-6} \text{ J} \]

\[ V_2 = V \]

\[ C_3 = 1 \mu\text{F} = 1 \times 10^{-6} \text{ F} \]
7. (Problem) [22 points] In the figure, a nonconducting spherical shell of inner radius \(a = 2.00 \text{ cm}\) and outer radius \(b = 2.40 \text{ cm}\) has (within its thickness) a positive volume charge density \(\rho = \frac{A}{r}\), where \(A\) is a constant and \(r\) is the distance from the center of the shell. In addition, a small ball of charge \(q = 45.00 \text{ nC}\) is located in the center.

\[
q = 4.5 \times 10^{-8} \text{ C} \\
A = 10 \text{ nC/cm}^2 = 10 \times 10^{-9} \times 10 = 10 \times 10^{-5} \frac{A}{\text{m}^2}
\]

\(a)\) [7 points] If \(A = 10 \text{ nC/cm}^2\), what is the value of the electric field at \(r = 1.90 \text{ cm}\)?

Inside the spherical shell, electric field is only due to the point charge \(q\) (electric field due to the spherical shell is zero within the shell).

\[
|\mathbf{E}| = k \frac{q}{r^2} = \left(8.99 \times 10^9\right) \frac{4.5 \times 10^{-8}}{(1.9 \times 10^{-2})^2} = 1.12 \times 10^6 \text{ N/C}
\]

\(b)\) [7 points] If \(A = 10 \text{ nC/cm}^2\), what is the value of the electric field at \(r = 2.20 \text{ cm}\), ignoring the contribution of the charge \(q\) (just compute the effect of the shell)?

To find electric field at \(a < r < b\), we'll use Gauss' law, due to shell:

Along the Gaussian surface \(E \| dA\) and \(|\mathbf{E}| = \text{const because } r = \text{const}\)

\[
|\mathbf{E}| = \frac{\mathbf{E} \cdot dA}{dA} = \frac{\mathbf{E} \cdot dA}{\mathbf{dA}} = |\mathbf{E}| \frac{4\pi}{r} r^2 \quad \text{left side}
\]

\(\text{Gauss law: } \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\varepsilon_0}\)

\[
|\mathbf{E}| = \frac{\int A}{r^2} r^2 dr = \int A r^2 dr = 4\pi A \int r^2 dr = 4\pi A \frac{r^2 - a^2}{2}
\]

\[
|\mathbf{E}| = \frac{4\pi A}{2\varepsilon_0} \frac{r^2 - a^2}{2} \quad \text{...Electric field at } a < r < b \text{ due to the shell}
\]

\[
|\mathbf{E}| = \frac{A}{2\varepsilon_0} \left(1 - \frac{a^2}{r^2}\right)
\]

\(c)\) [8 points] What value of \(A\) would make the field within the shell \((a < r < b)\) constant?

\[
|\mathbf{E}|_{\text{net}} = |\mathbf{E}_{\text{pc}}| + |\mathbf{E}_{\text{sh}}| = \frac{q}{4\pi \varepsilon_0} \frac{L}{r^2} + \frac{A}{2\varepsilon_0} \frac{a^2}{r^2} = \text{const}
\]

\[
|\mathbf{E}|_{\text{net}} = \text{constant only if} \quad \frac{q}{4\pi \varepsilon_0} \frac{L}{r^2} = \frac{A a^2}{2 \varepsilon_0 r^2}
\]

\[
\Rightarrow A = \frac{q}{2 \pi a^2} = \frac{4.5 \times 10^{-8}}{2\pi (0.02)^2} = 1.79 \times 10^{-5} \frac{C}{\text{m}^2} = 1.79 \frac{nC}{\text{cm}^2}
\]