Lecture 41: FRI 01 MAY
Final Exam Review

3. Find x.

Here it is X O
The Grading

• Midterms  -  100 points each
• Final Exam  -  200 points
• Homework   -  75 points
• In Class PP  -  25 points

TOTAL: 600 points

• Numerical Grade: Total Points / 6

• A: 90–100    B: 75–89    C: 60–74    D: 50–59    F: <50
Final Exam

• 10:00AM-12:00N **SAT 09 MAY**

• **LOCKETT 6**

• ≥ 50 % of Exam From HW01-14

• 100 PTS: CH 13, 21–30

• 100 PTS: CH 30–33
Final Exam

- 2 Problems from CHS30–33
- 4 Multiple Choice Questions from CHS30–33
- 10 Multiple Choice Questions from CHS13, 21–29.
- No partial credit for multiple choice questions.
- Must show your work to get partial credit on problems.
What do you need to make on the final to get an A, B, C, etc.?

A: 90–100   B: 75–89   C: 60–74   D: 50–59   F: <50

Solve this simple equation for x:

\[
\left[ mt_1 + mt_2 + mt_3 + \left( \frac{hw}{450} \right) \times 75 + \left( \max \left[ \left( \frac{icppc - icppx}{icpp} \right), 0 \right] \right) \times 25 + x \right] / 6 = y
\]

Where \( mt_1 = \text{exam1} \), \( mt_2 = \text{exam2} \), \( mt_3 = \text{exam3} \), \( hw = \text{total points on your hws01–14 (out of 450)} \), \( icppc = \text{checks} \), \( icppx = \text{X’s} \), \( icpp = \text{number of times you were called on} \), \( \max \) is the binary maximum function, and \( y \) is your desired cutoff number, \( y = 90, 75, 60, \) or 50. Then \( x \) is the score out of 200 you need on the final to make that cutoff grade \( y \). This assumes no curve.
Example: John D’oh wants to know what he needs to make on the final in order to get an A = 90 in this class.

\[ mt_1 = 81 \]
\[ mt_2 = 70 \]
\[ mt_3 = 90 \]
\[ hw = 435 \]
\[ icppc = 3 \]
\[ icppx = 0 \]
\[ icpp = 3 \]

\[
\max [(icppc - icppx) / icpp, 0] = \max [1, 0] = 1
\]
\[
\{(81+70+90) + [(435/450) \times 75] + (1 \times 25) + x\} = 90
\]
Solve: \( x = 201.5 / 200 > 100\%

It is very likely impossible for John to get an A as he’d need better than a perfect score on the final. How good does he need to do to avoid a C = 74?

\[
\{(81+70+90) + [(435/450) \times 75] + (1 \times 25) + x\} = 75
\]
Solve: \( x = 111.5 / 200 = 56\%

John is extremely unlikely to get an A, and is unlikely to get a C, so the most probable outcome is that John will get a B in this class.
- **Induction:**

  Magnetic Flux: \( \Phi = \int \vec{B} \cdot d\vec{A} \)

  Faraday’s law: \( \mathcal{E} = -\frac{d\Phi}{dt} \)  
  \( (= -N \frac{d\Phi}{dt} \) for a coil with \( N \) turns)

  Induced Electric Field: \( \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt} \)

  Motional emf: \( \mathcal{E} = BLv \)

  Definition of Self-Inductance: \( L = \frac{N\Phi}{i} \)

  Inductance of a solenoid: \( L = \mu_0 n^2 Al \)

  EMF (Voltage) across an inductor: \( \mathcal{E}_L = -L \frac{di}{dt} \)

  RL Circuit: Rise of current: \( i = \frac{\mathcal{E}}{R} \left( 1 - e^{-\frac{t}{\tau_L}} \right) \)  
  Time constant \( \tau_L = L/R \)

  Decay of current: \( i = i_0 e^{-\frac{tR}{L}} \)

  Magnetic Energy: \( U_B = \frac{1}{2} Li^2 \)  
  Magnetic energy density: \( u_B = \frac{B^2}{2\mu_0} \)
Lenz’s Law
Induction and Inductance

- Faraday’s law: \( \mathcal{E} = -\frac{d\Phi_B}{dt} \) or \( \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \)

- Inductance: \( L = N \frac{\Phi}{i} \)
  - For a solenoid: \( L = \mu_0 n^2 A l = \mu_0 N^2 A l \)

- Inductor EMF: \( \mathcal{E}_L = -L \frac{di}{dt} \)

- RL circuits: \( i(t) = \left(\frac{\mathcal{E}}{R}\right)(1 - e^{-tR/L}) \) or \( i(t) = i_0 e^{-tR/L} \)

- RL Time Constant: \( \tau = L/R \) Units: [s]

- Magnetic energy: \( U = L i^2/2 \) Units: [J]

- Magnetic energy density: \( u = B^2/2m_0 \) Units: [J/m^3]
Changing B-Flux Induces EMF
\[
\tau_L = \frac{L}{R}
\]

\[
i_{up}(t) = \frac{\mathcal{E}}{R} \left( 1 - e^{-t/\tau_L} \right)
\]

\[
U_B(t) = \frac{1}{2} Li^2
\]

\[
= \frac{1}{2} L \frac{\mathcal{E}^2}{R^2} \left( 1 - e^{-t/\tau_L} \right)^2
\]

To find \( \mathcal{E}_L \) walk the loop: \(+\mathcal{E} + V_R + \mathcal{E}_L = 0\)

\[
\mathcal{E}_L = -\mathcal{E} - V_R = -\mathcal{E} - (-iR)
\]

\[
= -\mathcal{E} + \left[ \frac{\mathcal{E}}{R} \left( 1 - e^{-t/\tau_L} \right) \right] R
\]

\[
\mathcal{E}_L = -\mathcal{E} e^{-t/\tau_L}
\]

\[
i_{dn}(t) = \frac{\mathcal{E}}{R} e^{-t R / L}
\]

\[
U_B(t) = \frac{1}{2} Li^2
\]

\[
= \frac{1}{2} L \frac{\mathcal{E}^2}{R^2} e^{-2t R / L
\]

Walk the loop!

\[
\mathcal{E}_L = -V_R = -iR
\]

\[
\mathcal{E}_L = -\mathcal{E} e^{-t / \tau_L}
\]
Make or Break?
At $t=0$ all $L$'s make breaks so throw out all loops with a break and solve circuit.
At $t=\infty$ all $L$'s make solid wires so replace $L$'s with wire and solve circuit.
Checkpoints/Questions

Magnitude of induced emf/current?

Magnitude/direction of induced current?

Magnitude/direction of magnetic field inducing current?

Given $\int E \cdot ds$, direction of magnetic field?

Current inducing $E_L$?

Given $B$, $dB/dt$, magnitude of electric field?

Largest current?

Largest $L$?

Current through the battery?
Time for current to rise 50% of max value?

R, L or 2R, L or R, 2L or 2R, 2L?
When the switch is closed, the inductor begins to get fluxed up, and the current is
\[ i = (E/R)(1 - e^{-t/\tau}) \]
When the switch is opened, the inductors begins to deflux.

The current in this case is then
\[ i = (E/R) \ e^{-t/\tau} \]
31 ELECTROMAGNETIC OSCILLATIONS AND
ALTERNATING CURRENT 826

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- LC circuits:
  Electric energy in a capacitor: $U_E = \frac{q^2}{2C} = \frac{CV^2}{2}$. Magnetic energy in an inductor: $U_B = \frac{Li^2}{2}$
  LC circuit oscillations: $q = Q \cos(\omega t + \phi)$  \( (i = \frac{dq}{dt}, \quad q = Cv) \quad \omega = \frac{1}{\sqrt{LC}} \quad T = \frac{2\pi}{\omega} \quad f = \frac{1}{T} \)

- Series RLC circuit: $q(t) = Q e^{-\frac{Rt}{2L}} \cos(\omega't + \phi)$ where $\omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}$

- Transformers:
  Transformation of voltage: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$  \( \text{Turns ratio: } \frac{N_p}{N_s} \quad \text{Energy conservation: } I_pV_p = I_sV_s \)

Fig. 31-5 A series RLC circuit. As the charge contained in the circuit oscillates back and forth through the resistance, electromagnetic energy is dissipated as thermal energy, damping (decreasing the amplitude of) the oscillations.
$U_B = \frac{Li^2}{2},$

$U_E = \frac{q^2}{2C},$

LC Circuits
Capacitor initially charged. Initially, current is zero, energy is all stored in the E-field of the capacitor.

Energy Conservation: \( U_{\text{tot}} = U_B + U_E \)

A current gets going, energy gets split between the capacitor and the inductor.

\[
U_B = \frac{1}{2} L t^2 \quad U_E = \frac{1}{2} \frac{q^2}{C}
\]

Capacitor discharges completely, yet current keeps going. Energy is all in the B-field of the inductor all fluxed up.

The magnetic field on the coil starts to deflux, which will start to recharge the capacitor.

\[
U_{\text{tot}} = \frac{1}{2} L t^2 + \frac{1}{2} \frac{q^2}{C}
\]

Finally, we reach the same state we started with (with opposite polarity) and the cycle restarts.
A charged capacitor and an inductor are connected in series at time $t = 0$. In terms of the period $T$ of the resulting oscillations, determine how much later the following reach their maximum value: (a) the charge on the capacitor; (b) the voltage across the capacitor, with its original polarity; (c) the energy stored in the electric field; and (d) the current.

(a) $T/2$

(b) $T$

$V_c = \frac{q}{C}$

(c) $T/2$

(d) $T/4$
Electric Oscillators: the Math

Energy as Function of Time

Voltage as Function of Time

\[ q = q_0 \cos(\omega t + \varphi_0) \]

Amplitude = ?

\[ i(t) = q'(t) = -q_0 \omega \sin(\omega t + \varphi_0) \]

\[ i'(t) = q''(t) = -\omega^2 q_0 \cos(\omega t + \varphi_0) \]

\[ U_B = \frac{1}{2} L \left[ i \right]^2 = \frac{1}{2} L \left[ q_0 \omega \sin(\omega t + \varphi_0) \right]^2 \]

\[ V_L = L i'(t) = -L \omega^2 q_0 \cos(\omega t + \varphi_0) \]

\[ U_E = \frac{1}{2} \left[ \frac{q}{C} \right]^2 = \frac{1}{2} \left[ q_0 \cos(\omega t + \varphi_0) \right]^2 \]

\[ V_C = \frac{1}{C} \left[ q(t) \right] = \frac{1}{C} \left[ q_0 \cos(\omega t + \varphi_0) \right] \]
Example

• In an LC circuit, \( L = 40 \text{ mH}; C = 4 \text{ \(\mu\)F} \)
• At \( t = 0 \), the current is a maximum;
• When will the capacitor be fully charged for the first time?

\[
\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{16 \times 10^{-8}}} \text{ rad/s}
\]

• \( \omega = 2500 \text{ rad/s} \)
• \( T = \text{period of one complete cycle} \)
• \( T = \frac{2\pi}{\omega} = 2.5 \text{ ms} \)
• Capacitor will be charged after \( T = 1/4 \) cycle i.e at
• \( t = T/4 = 0.6 \text{ ms} \)
Example

- In the circuit shown, the switch is in position “a” for a long time. It is then thrown to position “b.”
- Calculate the amplitude $\omega q_0$ of the resulting oscillating current.

\[ i = -\omega q_0 \sin(\omega t + \phi_0) \]

- Switch in position “a”: $q=CV = (1 \text{ mF})(10 \text{ V}) = 10 \text{ mC}$
- Switch in position “b”: maximum charge on C = $q_0 = 10 \text{ mC}$
- So, amplitude of oscillating current =

\[ \omega q_0 = \frac{1}{\sqrt{(1 \text{ mH})(1 \mu F)}}(10 \mu \text{C}) = 0.316 \text{ A} \]
Damped LCR Oscillator

Ideal LC circuit without resistance: oscillations go on forever; \( \omega = (LC)^{-1/2} \)

Real circuit has resistance, dissipates energy: oscillations die out, or are “damped”

Math is complicated! Important points:

- Frequency of oscillator shifts away from \( \omega = (LC)^{-1/2} \)
- Peak CHARGE decays with time constant = \( \tau_{QLCR} = 2L/R \)
- For small damping, peak ENERGY decays with time constant \( \tau_{ULCR} = L/R \)

\[ U_{\text{max}} = \frac{Q^2}{2C} e^{-\frac{Rt}{L}} \]
A series RLC circuit has inductance $L = 12\ \text{mH}$, capacitance $C = 1.6\ \mu\text{F}$, and resistance $R = 1.5\ \Omega$ and begins to oscillate at time $t = 0$.

(a) At what time $t$ will the amplitude of the charge oscillations in the circuit be 50% of its initial value? (Note that we do not know that initial value.)

Solving for $t$ and then substituting given data yield

$$t = -\frac{2L}{R} \ln 0.50 = -\frac{(2)(12 \times 10^{-3}\ \text{H})(\ln 0.50)}{1.5\ \Omega}$$

$$= 0.0111\ \text{s} \approx 11\ \text{ms}.$$  \text{(Answer)}

(b) How many oscillations are completed within this time?

The time for one complete oscillation is the period $T = \frac{2\pi}{\omega}$, where the angular frequency for LC oscillations is given by Eq. 31-4 ($\omega = \frac{1}{\sqrt{LC}}$).

**Calculation:** In the time interval $\Delta t = 0.0111\ \text{s}$, the number of complete oscillations is

$$\frac{\Delta t}{T} = \frac{\Delta t}{2\pi \sqrt{LC}}$$

$$= \frac{0.0111\ \text{s}}{2\pi[(12 \times 10^{-3}\ \text{H})(1.6 \times 10^{-6}\ \text{F})]^{1/2}} \approx 13.$$  \text{(Answer)}

Thus, the amplitude decays by 50% in about 13 complete oscillations. This damping is less severe than that shown in Fig. 31-3, where the amplitude decays by a little more than 50% in one oscillation.

$$\frac{R^2}{4L^2} \ll \frac{1}{LC} \quad 10^3 \ll 10^8! \quad \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \approx \sqrt{\frac{1}{LC}} = \omega$$
In the time interval $\Delta t = 0.0111$ s, the number of complete oscillations is

$$\frac{\Delta t}{T} = \frac{\Delta t}{2\pi \sqrt{LC}}$$

$$= \frac{0.0111 \text{ s}}{2\pi [(12 \times 10^{-3} \text{ H})(1.6 \times 10^{-6} \text{ F})]^{1/2}} \approx 13.$$  

(Answer)

Thus, the amplitude decays by 50% in about 13 complete oscillations. This damping is less severe than that shown in Fig. 31-3, where the amplitude decays by a little more than 50% in one oscillation.
32 MAXWELL’S EQUATIONS; MAGNETISM OF MATTER 861

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• Maxwell’s Equations:

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{d\Phi_{\text{enc}}}{\varepsilon_0} \quad \oint \mathbf{B} \cdot d\mathbf{A} = 0 \quad \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \quad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}} \]

Displacement current: \( i_d = \varepsilon_0 \frac{d\Phi_E}{dt} \)

Magnetization: \( \mathbf{M} = \frac{\mathbf{\mu}}{\text{volume}} \)

The changing of the electric field between the plates creates a magnetic field.
Displacement “Current”

\[ \oint_C \mathbf{B} \cdot d\mathbf{s} \neq 0 \]

Maxwell proposed it based on symmetry and math — no experiment!

\[ \oint_C \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \mathbf{E} \cdot d\mathbf{A} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

Changing E-field Gives Rise to B-Field!
32.3: Induced Magnetic Fields:

Here \( B \) is the magnetic field induced along a closed loop by the changing electric flux \( \Phi_E \) in the region encircled by that loop.

\[
\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{(Maxwell's law of induction)}
\]

**Fig. 32-5** (a) A circular parallel-plate capacitor, shown in side view, is being charged by a constant current \( i \). (b) A view from within the capacitor, looking toward the plate at the right in (a). The electric field is uniform, is directed into the page (toward the plate), and grows in magnitude as the charge on the capacitor increases. The magnetic field induced by this changing electric field is shown at four points on a circle with a radius \( r \) less than the plate radius \( R \).
The figure shows graphs of the electric field magnitude $E$ versus time $t$ for four uniform electric fields, all contained within identical circular regions as in Fig. 32-5b. Rank the fields according to the magnitudes of the magnetic fields they induce at the edge of the region, greatest first.

$$B_a > B_c > B_b > B_d = 0$$

$\int \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$  \hspace{1cm} \text{(Maxwell’s law of induction)}$

$$\frac{d\Phi_E}{dt} = \frac{d}{dt} EA = A \frac{dE}{dt} \propto \text{slope}$$
Example, Magnetic Field Induced by Changing Electric Field:

A parallel-plate capacitor with circular plates of radius \( R \) is being charged as in Fig. 32-5a.

(a) Derive an expression for the magnetic field at radius \( r \) for the case \( r \leq R \):

\[
\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.
\]  

(32-6)

We shall separately evaluate the left and right sides of this equation.

**Left side of Eq. 32-6:** We choose a circular Amperian loop with a radius \( r \leq R \) as shown in Fig. 32-5b because we want to evaluate the magnetic field for \( r \leq R \)—that is, inside the capacitor. The magnetic field \( \vec{B} \) at all points along the loop is tangent to the loop, as is the path element \( d\vec{s} \). Thus, \( \vec{B} \) and \( d\vec{s} \) are either parallel or antiparallel at each point of the loop. For simplicity, assume they are parallel (the choice does not alter our outcome here). Then

\[
\oint \vec{B} \cdot d\vec{s} = \oint B \, ds \cos 0^\circ = \oint B \, ds.
\]

Due to the circular symmetry of the plates, we can also assume that \( \vec{B} \) has the same magnitude at every point around the loop. Thus, \( B \) can be taken outside the integral on the right side of the above equation. The integral that remains is \( \oint ds \), which simply gives the circumference \( 2\pi r \) of the loop. The left side of Eq. 32-6 becomes

\[
2\pi r B = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.
\]

(32-7)

Right side of Eq. 32-6: We assume that the electric field \( \vec{E} \) is uniform between the capacitor plates and directed perpendicular to the plates. Then the electric flux \( \Phi_E \) through the Amperian loop is \( EA \), where \( A \) is the area encircled by the loop within the electric field. Thus, the right side of Eq. 32-6 is \( \mu_0 \varepsilon_0 \frac{d(EA)}{dt} \).

**Combining results:** Substituting our results for the left and right sides into Eq. 32-6, we get

\[
(B)(2\pi r) = \mu_0 \varepsilon_0 \frac{d(EA)}{dt}.
\]

Because \( A \) is a constant, we write \( d(EA) \) as \( A \, dE \); so we have

\[
(B)(2\pi r) = \mu_0 \varepsilon_0 A \frac{dE}{dt}.
\]

(32-7)

The area \( A \) that is encircled by the Amperian loop within the electric field is the *full area* \( \pi r^2 \) of the loop because the loop’s radius \( r \) is less than (or equal to) the plate radius \( R \). Substituting \( \pi r^2 \) for \( A \) in Eq. 32-7 leads to, for \( r \leq R \),
Example, Magnetic Field Induced by Changing Electric Field, cont.:

A parallel-plate capacitor with circular plates of radius $R$ is being charged as in Fig. 32-5a.

Fig. 32-5
(a) 

(c) Derive an expression for the induced magnetic field for the case $r \geq R$.

**Calculation:** Our procedure is the same as in (a) except we now use an Amperian loop with a radius $r$ that is greater than the plate radius $R$, to evaluate $B$ outside the capacitor. Evaluating the left and right sides of Eq. 32-6 again leads to Eq. 32-7. However, we then need this subtle point: The electric field exists only between the plates, not outside the plates. Thus, the area $A$ that is encircled by the Amperian loop in the electric field is not the full area $\pi r^2$ of the loop. Rather, $A$ is only the plate area $\pi R^2$.

Substituting $\pi R^2$ for $A$ in Eq. 32-7 and solving the result for $B$ give us, for $r \geq R$,

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt}. \quad \text{(Answer)} \quad (32-9)$$

(b) Evaluate the field magnitude $B$ for $r = R/5 = 11.0 \text{ mm}$ and $dE/dt = 1.50 \times 10^{12} \text{ V/m} \cdot \text{s}$.

**Calculation:** From the answer to (a), we have

$$B = \frac{1}{2} \mu_0 \varepsilon_0 r \frac{dE}{dt}$$

$$= \frac{1}{2} \left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right) \left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)$$

$$\times (11.0 \times 10^{-3} \text{ m})(1.50 \times 10^{12} \text{ V/m} \cdot \text{s})$$

$$= 9.18 \times 10^{-8} \text{ T}. \quad \text{(Answer)}$$
32.4: Displacement Current:

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}} \quad \text{(Ampere–Maxwell law).} \]

Comparing the last two terms on the right side of the above equation shows that the term \(\varepsilon_0 \left( \frac{d\Phi_E}{dt} \right)\) must have the dimension of a current. This product is usually treated as being a fictitious current called the displacement current \(i_d\):

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}} \quad \text{(Ampere–Maxwell law),} \]

in which \(i_{d,\text{enc}}\) is the displacement current that is encircled by the integration loop.

The charge \(q\) on the plates of a parallel plate capacitor at any time is related to the magnitude \(E\) of the field between the plates at \(q = \varepsilon_0 AE\), by

\[ \quad \frac{dq}{dt} = i = \varepsilon_0 A \frac{dE}{dt}. \quad i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}. \]

\[ i_d = i \quad \text{(displacement current in a capacitor).} \]

\[ B = \left( \frac{\mu_0 i_d}{2\pi R^2} \right) r \quad \text{(inside a circular capacitor).} \]

\[ B = \frac{\mu_0 i_d}{2\pi r} \quad \text{(outside a circular capacitor).} \]
Example, Treating a Changing Electric Field as a Displacement Current:

A circular parallel-plate capacitor with plate radius \( R \) is being charged with a current \( i \).

(a) Between the plates, what is the magnitude of \( \oint \vec{B} \cdot d\vec{s} \), in terms of \( \mu_0 \) and \( i \), at a radius \( r = R/5 \) from their center?

**Calculations:** Because we want to evaluate \( \oint \vec{B} \cdot d\vec{s} \) at radius \( r = R/5 \) (within the capacitor), the integration loop encircles only a portion \( i_{d,\text{enc}} \) of the total displacement current \( i_d \). Let’s assume that \( i_d \) is uniformly spread over the full plate area. Then the portion of the displacement current encircled by the loop is proportional to the area encircled by the loop:

\[
\frac{\text{encircled displacement current } i_{d,\text{enc}}}{\text{total displacement current } i_d} = \frac{\text{encircled area } \pi r^2}{\text{full plate area } \pi R^2}.
\]

This gives us

\[
i_{d,\text{enc}} = i_d \frac{\pi r^2}{\pi R^2}.
\]

Substituting this into Eq. 32-18, we obtain

\[
\oint \vec{B} \cdot d\vec{s} = \mu_0 i_d \frac{\pi r^2}{\pi R^2}.
\]  (32-19)

Now substituting \( i_d = i \) (from Eq. 32-15) and \( r = R/5 \) into Eq. 32-19 leads to

\[
\oint \vec{B} \cdot d\vec{s} = \mu_0 i \frac{(R/5)^2}{R^2} = \frac{\mu_0 i}{25}.
\]  (Answer)

(b) In terms of the maximum induced magnetic field, what is the magnitude of the magnetic field induced at \( r = R/5 \), inside the capacitor?

**Calculations:** At \( r = R/5 \), Eq. 32-16 yields

\[
B = \left( \frac{\mu_0 i_d}{2\pi R^2} \right) R = \frac{\mu_0 i_d (R/5)}{2\pi R^2} = \frac{\mu_0 i_d}{10\pi R}.
\]  (32-20)

\[
B_{\text{max}} = \left( \frac{\mu_0 i_d}{2\pi R^2} \right) R = \frac{\mu_0 i_d}{2\pi R}.
\]  (32-21)

Dividing Eq. 32-20 by Eq. 32-21 and rearranging the result, we find that the field magnitude at \( r = R/5 \) is

\[
B = \frac{1}{5} B_{\text{max}}.
\]  (Answer)

We should be able to obtain this result with a little reasoning and less work. Equation 32-16 tells us that inside the capacitor, \( B \) increases linearly with \( r \). Therefore, a point \( \frac{1}{5} \) the distance out to the full radius \( R \) of the plates, where \( B_{\text{max}} \) occurs, should have a field \( B \) that is \( \frac{1}{5} B_{\text{max}} \).
The figure is a view of one plate of a parallel-plate capacitor from within the capacitor. The dashed lines show four integration paths (path $b$ follows the edge of the plate). Rank the paths according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ along the paths during the discharging of the capacitor, greatest first.

The displacement current $i_d = i$ is distributed evenly over grey area.

So rank by $i_d^{\text{enc}} = \text{amount of grey area enclosed by each loop}$.

\[
\oint \vec{B} \cdot d\vec{s} = \mu_0 i_d^{\text{enc}}
\]

\[
i_d^{\text{enc}} = \varepsilon_0 \frac{d\Phi_{E}^{\text{enc}}}{dt}
\]

\[d = c > b > a\]
Electromagnetic Waves:
Wave traveling in +x direction: \( E = E_m \sin(kx - \omega t) \quad B = B_m \sin(kx - \omega t) \)
where \( \vec{E} \perp \vec{B} \), the direction of travel is \( \vec{E} \times \vec{B} \), \( E_m/B_m = c \), \( f\lambda = c \), \( \lambda = 2\pi/k \)

Velocity of light in vacuum = \( c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \)

Energy flow: \( \tilde{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad I = \frac{1}{2c\mu_0} E_m^2 \quad E_{rms} = \frac{E_m}{\sqrt{2}} \quad I = \frac{P}{\text{Area}} \)

Radiation force and pressure: total absorption: \( F_r = \frac{IA}{c} \), \( p_r = \frac{I}{c} \) total reflection: \( F_r = \frac{2IA}{c} \), \( p_r = \frac{2I}{c} \)

Polarizing Sheets:
Unpolarized \( \to \) polarized: \( I = \frac{1}{2}I_0 \)

Polarized \( \to \) polarized: \( I = I_0 \cos^2 \theta \)

Reflection/refraction:
Law of reflection: \( \theta_i = \theta_r \)

Total internal reflection (critical angle): \( \theta_c = \sin^{-1} \frac{n_2}{n_1} \)

Polarization by reflection (Brewster's angle): \( \theta_B = \tan^{-1} \frac{n_2}{n_1} \)

[Diagram of electromagnetic waves and polarization]

The sheet's polarizing axis is tilted, so only a fraction of the intensity passes.
Mathematical Description of Traveling EM Waves

Electric Field: \( E = E_m \sin(kx - \omega t) \)

Magnetic Field: \( B = B_m \sin(kx - \omega t) \)

Wave Speed: \( c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \)

All EM waves travel at \( c \) in vacuum

Wavenumber: \( k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \)

Angular frequency: \( \omega = \frac{2\pi}{T} \)

Vacuum Permittivity: \( \varepsilon_0 \)

Vacuum Permeability: \( \mu_0 \)

Amplitude Ratio: \( \frac{E_m}{B_m} = c \)

Magnitude Ratio: \( \frac{E(t)}{B(t)} = c \)

(33-5)
Electromagnetic waves are able to transport energy from transmitter to receiver (example: from the Sun to our skin).

The power transported by the wave and its direction is quantified by the Poynting vector.

For a wave, since \( E \) is perpendicular to \( B \):

\[
| \mathbf{S} | = \frac{1}{\mu_0} \mathbf{EB} = \frac{1}{c\mu_0} \mathbf{E}^2
\]

In a wave, the fields change with time. Therefore the Poynting vector changes too!!

The direction is constant, but the magnitude changes from 0 to a maximum value.
CHECKPOINT 2

The figure here gives the electric field of an electromagnetic wave at a certain point and a certain instant. The wave is transporting energy in the negative $z$ direction. What is the direction of the magnetic field of the wave at that point and instant?

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$
EM Wave Intensity, Energy Density

A better measure of the amount of energy in an EM wave is obtained by averaging the Poynting vector over one wave cycle. The resulting quantity is called intensity. Units are also Watts/m².

\[ I = \bar{S} = \frac{1}{c \mu_0} E^2 = \frac{1}{c \mu_0} E_m^2 \sin^2 (kx - \omega t) \]

Both fields have the same energy density.

\[ u_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 (cB)^2 = \frac{1}{2} \varepsilon_0 \frac{B^2}{\varepsilon_0 \mu_0} = \frac{1}{2} \frac{B^2}{\mu_0} = u_B \]

The total EM energy density is then

\[ u = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} \]
The intensity of a wave is power *per unit area*. If one has a source that emits isotropically (equally in all directions) the power emitted by the source pierces a larger and larger sphere as the wave travels outwards: $1/r^2$ Law!

So the power per unit area decreases as the inverse of distance squared.

\[
I = \frac{P_s}{4\pi r^2}
\]
Example

A radio station transmits a 10 kW signal at a frequency of 100 MHz. Assume a spherical wave. At a distance of 1km from the antenna, find the amplitude of the electric and magnetic field strengths, and the energy incident normally on a square plate of side 10cm in 5 minutes.

\[
I = \frac{P_s}{4\pi r^2} = \frac{10 \times 10^3 \text{W}}{4\pi(1\times 10^3 \text{m})^2} = 0.8 \text{mW/m}^2
\]

\[
I = \frac{1}{2c\mu_0} E_m^2 \Rightarrow E_m = \sqrt{2c\mu_0 I} = 0.775 \text{V/m}
\]

\[
B_m = \frac{E_m}{c} = 2.58 \text{nT}
\]
Radiation Pressure

Waves not only carry energy but also momentum. The effect is very small (we don’t ordinarily feel pressure from light). If light is completely absorbed during an interval $\Delta t$, the momentum transferred $\Delta p$ is given by:

$$\Delta p = \frac{\Delta u}{c}$$

and twice as much if reflected.

Newton’s law:

$$F = \frac{\Delta p}{\Delta t}$$

Now, supposing one has a wave that hits a surface of area $A$ (perpendicularly), the amount of energy transferred to that surface in time $\Delta t$ will be:

$$\Delta U = IA\Delta t$$

therefore

$$\Delta p = \frac{IA\Delta t}{c}$$

$$F = \frac{IA}{c}$$

Radiation pressure: $p_r = \frac{I}{c}$ (total absorption), $p_r = \frac{2I}{c}$ (total reflection)

[Pa=N/m²]
CHECKPOINT 3

Light of uniform intensity shines perpendicularly on a totally absorbing surface, fully illuminating the surface. If the area of the surface is decreased, do (a) the radiation pressure and (b) the radiation force on the surface increase, decrease, or stay the same?

\[
p_r = \frac{I}{c} \quad \text{(total absorption)}
\]

The pressure \( p \) is independent of the area \( A \).

(a) The pressure remains the same.

\[
F = \frac{IA}{c} \quad \text{(total absorption)}.
\]

The force \( F \) is proportional to the area \( A \).

(b) The force decreases.
Radio transmitter:

If the dipole antenna is vertical, so will be the electric fields. The magnetic field will be horizontal.

The radio wave generated is said to be “polarized”.

In general light sources produce “unpolarized waves” emitted by atomic motions in random directions.
Unpolarized light headed toward you—the electric fields are in all directions in the plane.

(a)

This is a quick way to symbolize unpolarized light.

(b)

The sheet’s polarizing axis is vertical, so only vertically polarized light emerges.

Incident light ray

Unpolarized light

Polarizing sheet

Vertically polarized light

The sheet’s polarizing axis is vertical, so only vertical components of the electric fields pass.

The sheet’s polarizing axis is tilted, so only a fraction of the intensity passes.
When polarized light hits a polarizing sheet, only the component of the field aligned with the sheet will get through.

\[ E_y = E \cos(\theta) \]

And therefore:

\[ I = I_0 \cos^2 \theta \]
Example

Initially unpolarized light of intensity $I_0$ is sent into a system of three polarizers as shown. What fraction of the initial intensity emerges from the system? What is the polarization of the exiting light?

- Through the first polarizer: unpolarized to polarized, so $I_1 = \frac{1}{2} I_0$.
- Into the second polarizer, the light is now vertically polarized. Then, $I_2 = I_1 \cos^2(60^\circ) = \frac{1}{4} I_1 = \frac{1}{8} I_0$.
- Now the light is again polarized, but at $60^\circ$. The last polarizer is horizontal, so $I_3 = I_2 \cos^2(30^\circ) = \frac{3}{4} I_2 = \frac{3}{32} I_0 = 0.094 I_0$.
- The exiting light is horizontally polarized, and has 9% of the original amplitude.
Completely unpolarized light will have equal components in horizontal and vertical directions. Therefore running the light through first polarizer will cut the intensity in half: \[ I = I_0 / 2 \]

When the now polarized light hits second polarizing sheet, only the component of the field aligned with the sheet will get through.

\[
\begin{align*}
(a) & \quad I_0 \rightarrow \frac{1}{2} I_0 \rightarrow \frac{1}{2} I_0 \cos^2(0^\circ) = \frac{1}{2} I_0 \\
(b) & \quad I_0 \rightarrow \frac{1}{2} I_0 \rightarrow \frac{1}{2} I_0 \cos^2(60^\circ) = \frac{1}{8} I_0 \\
(c) & \quad I_0 \rightarrow \frac{1}{2} I_0 \rightarrow \frac{1}{2} I_0 \cos^2(90^\circ) = 0 \\
(d) & \quad I_0 \rightarrow \frac{1}{2} I_0 \rightarrow \frac{1}{2} I_0 \cos^2(30^\circ) = \frac{3}{8} I_0
\end{align*}
\]

First polarizer cuts intensity in half. Second cuts by \( \cos^2 \theta \).

The \( \theta \) is angle between dashed lines. More gets through when more aligned. Less gets through when less aligned.

\[ a > d > b > c = 0 \]