Lecture 33: MON 13 APR

Induction and Inductance III

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30-8 Self-Induction 806

Fender Stratocaster
Solenoid Pickup
Faraday's Experiments
In a series of experiments, Michael Faraday in England and Joseph Henry in the U.S. were able to generate electric currents without the use of batteries.

The circuit shown in the figure consists of a wire loop connected to a sensitive ammeter (known as a "galvanometer"). If we approach the loop with a permanent magnet we see a current being registered by the galvanometer.

1. A current appears only if there is relative motion between the magnet and the loop.

2. Faster motion results in a larger current.

3. If we reverse the direction of motion or the polarity of the magnet, the current reverses sign and flows in the opposite direction.

The current generated is known as "induced current"; the emf that appears is known as "induced emf"; the whole effect is called "induction."
In the figure we show a second type of experiment in which current is induced in loop 2 when the switch S in loop 1 is either closed or opened. When the current in loop 1 is constant no induced current is observed in loop 2. The conclusion is that the magnetic field in an induction experiment can be generated either by a permanent magnet or by an electric current in a coil.

Faraday summarized the results of his experiments in what is known as "Faraday's law of induction."

An emf is induced in a loop when the number of magnetic field lines that pass through the loop is changing.
Lenz’s Law

- The Loop Current Produces a B Field that Opposes the CHANGE in the bar magnet field.

- Upper Drawing: B Field from Magnet is INCREASING so Loop Current is Clockwise and Produces an Opposing B Field that Tries to CANCEL the INCREASING Magnet Field.

- Lower Drawing: B Field from Magnet is DECREASING so Loop Current is Counterclockwise and Tries to BOOST the Decreasing Magnet Field.
If the loop is pulled at a constant velocity \( v \), one must apply a constant force \( F \) to the loop since an equal and opposite magnetic force acts on the loop to oppose it. The power is \( P = Fv \).

As the loop is pulled, the portion of its area within the magnetic field, and therefore the magnetic flux, decrease. According to Faraday’s law, a current is produced in the loop. The magnitude of the flux through the loop is \( \Phi_B = BA = BLx \).

Therefore,
\[
\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv,
\]

The induced current is therefore
\[
i = \frac{BLv}{R}.
\]

The net deflecting force is:
\[
F = F_1 = iLB \sin 90^\circ = iLB. = \frac{B^2L^2v}{R}.
\]

The power is therefore
\[
P = Fv = \frac{B^2L^2v^2}{R}.
\]
The figure shows four wire loops, with edge lengths of either $L$ or $2L$. All four loops will move through a region of uniform magnetic field $\vec{B}$ (directed out of the page) at the same constant velocity. Rank the four loops according to the maximum magnitude of the emf induced as they move through the field, greatest first.

$$d = c > b = a$$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv,$$
Fig. 30-10  (a) As you pull a solid conducting plate out of a magnetic field, *eddy currents* are induced in the plate. A typical loop of eddy current is shown. (b) A conducting plate is allowed to swing like a pendulum about a pivot and into a region of magnetic field. As it enters and leaves the field, eddy currents are induced in the plate.
Example: Eddy Currents

- A non-magnetic (e.g. copper, aluminum) ring is placed near a solenoid.
- What happens if:
  - There is a steady current in the solenoid?
  - The current in the solenoid is suddenly changed?
  - The ring has a “cut” in it?
  - The ring is extremely cold?
Jumping Ring

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Another Experimental Observation

- Drop a non-magnetic pendulum (copper or aluminum) through an inhomogeneous magnetic field
- What do you observe? Why? (Think about energy conservation!)

Pendulum had kinetic energy. What happened to it? Isn’t energy conserved??

Energy is Dissipated by Resistance: $P=i^2R$. This acts like friction!!
Pendulum and Magnet

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Start Your Engines: The Ignition Coil

- The gap between the spark plug in a combustion engine needs an electric field of $\sim 10^7 \, \text{V/m}$ in order to ignite the air-fuel mixture. For a typical spark plug gap, one needs to generate a potential difference $> 10^4 \, \text{V}$!
- But, the typical EMF of a car battery is 12 V. So, how does a spark plug even work at all!? 

The “ignition coil” is a double layer solenoid:
- Primary: small number of turns -- 12 V
- Secondary: MANY turns -- spark plug

- Breaking the circuit changes the current through “primary coil”
- Result: LARGE change in flux thru secondary -- large induced EMF!

http://www.familycar.com/Classroom/ignition.htm
Start Your Engines: The Ignition Coil

The battery establishes a large current in the low-resistance primary coil.

The "points" are opened by cam action to quickly interrupt the current in the primary coil.

Many turns on the secondary coil compared to the primary coil forms a transformer with a large multiplication of voltage.

The capacitor, or "condenser" helps to handle the surge of voltage from the switch action which might otherwise cause sparking across the points.

The sudden change in magnetic field in the primary from the switching off of the current induces a very high voltage in the secondary coil by Faraday's Law.

Transformer: \( P = iV \)
30.6: Induced Electric Field:

A changing magnetic field produces an electric field.

Fig. 30-11  (a) If the magnetic field increases at a steady rate, a constant induced current appears, as shown, in the copper ring of radius r. (b) An induced electric field exists even when the ring is removed; the electric field is shown at four points. (c) The complete picture of the induced electric field, displayed as field lines. (d) Four similar closed paths that enclose identical areas. Equal emfs are induced around paths 1 and 2, which lie entirely within the region of changing magnetic field. A smaller emf is induced around path 3, which only partially lies in that region. No net emf is induced around path 4, which lies entirely outside the magnetic field.
30.6: Induced Electric Fields, Reformulation of Faraday’s Law:

Consider a particle of charge \( q_0 \) moving around the circular path. The work \( W \) done on it in one revolution by the induced electric field is \( W = E q_0 \), where \( E \) is the induced emf.

From another point of view, the
\[
W = \int \vec{F} \cdot d\vec{s} = (q_0 E)(2\pi r),
\]

Here where \( q_0 E \) is the magnitude of the force acting on the test charge and \( 2pr \) is the distance over which that force acts:
\[
\mathcal{E} = 2\pi r E.
\]

In general,
\[
W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}.
\]
\[
\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday’s law).}
\]
The figure shows five lettered regions in which a uniform magnetic field extends either directly out of the page or into the page, with the direction indicated only for region $a$. The field is increasing in magnitude at the same steady rate in all five regions; the regions are identical in area. Also shown are four numbered paths along which $\phi \vec{E} \cdot d\vec{s}$ has the magnitudes given below in terms of a quantity “mag.” Determine whether the magnetic field is directed into or out of the page for regions $b$ through $e$. 

<table>
<thead>
<tr>
<th>Path</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \vec{E} \cdot d\vec{s}$</td>
<td>mag</td>
<td>2(mag)</td>
<td>3(mag)</td>
<td>0</td>
</tr>
</tbody>
</table>

$b = \text{out}$

c = \text{out}

e = \text{into}

d = \text{into}

First loop 3

Next Loop 4

Last Loop 2
Changing B-Field Produces E-Field!

- We saw that a time varying magnetic FLUX creates an induced EMF in a wire, exhibited as a current.

- Recall that a current flows in a conductor because of electric field.

- Hence, a time varying magnetic flux must induce an ELECTRIC FIELD!

- A Changing B-Field Produces an E-Field in Empty Space!

\[
\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt}
\]

To decide direction of E-field use Lenz’s law as in current loop.
Example

• The figure shows two circular regions $R_1$ & $R_2$ with radii $r_1 = 1\, \text{m}$ & $r_2 = 2\, \text{m}$. In $R_1$, the magnetic field $B_1$ points out of the page. In $R_2$, the magnetic field $B_2$ points into the page.

• Both fields are uniform and are DECREASING at the SAME steady rate $= 1\, \text{T/s}$.

• Calculate the “Faraday” integral for the two paths shown.

\[
\oint_C \mathbf{E} \cdot d\mathbf{s} = \int_C \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} = -(\pi r_1^2)(-1\, \text{T} / \text{s}) = +3.14\, \text{V}
\]

Path II:

\[
\oint_C \mathbf{E} \cdot d\mathbf{s} = -\left[ (\pi r_1^2)(-1\, \text{T} / \text{s}) + (\pi r_2^2)(-1\, \text{T} / \text{s}) \right] = +9.42\, \text{V}
\]
Solenoid Example

- A long solenoid has a circular cross-section of radius $R$.
- The magnetic field $B$ through the solenoid is increasing at a steady rate $dB/dt$.
- Compute the variation of the electric field as a function of the distance $r$ from the axis of the solenoid.

First, let's look at $r < R$:

$$|E| (2\pi r) = (\pi r^2) \frac{dB}{dt}$$

$$E = \frac{r \ dB}{2 \ dt}$$

Next, let's look at $r > R$:

$$|E| (2\pi r) = (\pi R^2) \frac{dB}{dt}$$

$$E = \frac{R^2 \ dB}{2r \ dt}$$
Solenoid Example Cont.

\[ E_{r<R} = \frac{r \ dB}{2 \ \frac{dt}{dt}} \]

\[ \propto r \]

\[ E_{r>R} = \frac{R^2 \ dB}{2r \ \frac{dt}{dt}} \]

\[ \propto \frac{1}{r} \]

\[ B = \mu_0 ni \]

\[ \frac{dB}{dt} = \mu_0 n \frac{di}{dt} \]

Added Complication: Changing B Field Is Produced by Changing Current \( i \) in the Loops of Solenoid!
Summary

Two versions of Faradays’ law:

– A time varying magnetic flux produces an EMF:

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

– A time varying magnetic flux produces an electric field:

\[ \oint_{C} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]
An inductor (symbol \( \mathcal{L} \)) can be used to produce a desired magnetic field.

If we establish a current \( i \) in the windings (turns) of the solenoid which can be treated as our inductor, the current produces a magnetic flux \( \Phi_B \) through the central region of the inductor.

The inductance of the inductor is then

\[
L = \frac{N\Phi_B}{i} \quad \text{(inductance defined)}
\]

The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife’s petticoats. *(The Royal Institution/Bridgeman Art Library/NY)*

The SI unit of inductance is the tesla–square meter per ampere (T m\(^2\)/A). We call this the henry (H), after American physicist Joseph Henry.
Inductors are with respect to the magnetic field what capacitors are with respect to the electric field. They “pack a lot of field in a small region”. Also, the higher the current, the higher the magnetic field they produce.

**Capacitance** $C$ how much **potential** for a given charge: $Q=CV$

**Inductance** $L$ how much **magnetic flux** for a given current: $\Phi=Li$

Using Faraday’s law:

$$\epsilon = -\frac{d\Phi}{dt} = -L \frac{di}{dt}$$

Units: $[L]=\frac{\text{Tesla} \cdot \text{m}^2}{\text{Ampere}} \equiv \text{H (Henry)}$
**Inductance**

Consider a solenoid of length \( \ell \) that has \( N \) loops of area \( A \) each, and \( n = \frac{N}{\ell} \) windings per unit length. A current \( i \) flows through the solenoid and generates a uniform magnetic field \( B = \mu_0 n i \) inside the solenoid.

The solenoid magnetic flux is \( \Phi_B = NBA \).

\[
L = \mu_0 n^2 \ell A
\]

The total number of turns \( N = n\ell \rightarrow \Phi_B = \left( \mu_0 n^2 \ell A \right) i. \) The result we got for the special case of the solenoid is true for any inductor: \( \Phi_B = Li. \) Here \( L \) is a constant known as the *inductance* of the solenoid. The inductance depends on the geometry of the particular inductor.

**Inductance of the Solenoid**

For the solenoid, \( L = \frac{\Phi_B}{i} = \frac{\mu_0 n^2 \ell A i}{i} = \mu_0 n^2 \ell A. \)
**Self-Induction**

In the picture to the right we already have seen how a change in the current of loop 1 results in a change in the flux through loop 2, and thus creates an induced emf in loop 2.

If we change the current through an inductor this causes a change in the magnetic flux \( \Phi_B = Li \) through the inductor according to the equation \( \frac{d\Phi_B}{dt} = L \frac{di}{dt} \). Using Faraday's law we can determine the resulting emf known as **self-induced** emf: \( \mathcal{E} = -\frac{d\Phi_B}{dt} = -L \frac{di}{dt} \).

**SI unit for** \( L \): the henry (symbol: H)

An inductor has inductance \( L = 1 \text{ H} \) if a current change of 1 A/s results in a self-induced emf of 1 V.
CHECKPOINT 5

The figure shows an emf $\mathcal{E}_L$ induced in a coil. Which of the following can describe the current through the coil: (a) constant and rightward, (b) constant and leftward, (c) increasing and rightward, (d) decreasing and rightward, (e) increasing and leftward, (f) decreasing and leftward?

\[ \mathcal{E} = -\frac{d\Phi}{dt} = -L \frac{di}{dt} \]

If $i$ is constant (in time) then $\mathcal{E} = 0$.

The changing current changes the flux, which creates an emf that opposes the change.
The current in a L=10H inductor is decreasing at a steady rate of \( i = 5 \text{A/s} \).

If the current is as shown at some instant in time, what is the magnitude and direction of the induced EMF?

\[ \mathcal{E} = -L \frac{di}{dt} \]

- Magnitude = \((10 \text{ H})(5 \text{ A/s}) = 50 \text{ V}\)
- Current is decreasing
- Induced EMF must be in a direction that OPPOSES this change.
- So, induced EMF must be in same direction as current

(a) 50 V
(b) 50 V

Example