Lecture 30: FRI 27 MAR
Magnetic Fields Due to Currents III

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What Are We Going to Learn?
A Road Map

- Electric charge
  - Electric force on other electric charges
  - Electric field, and electric potential
- Moving electric charges: current
- Electronic circuit components: batteries, resistors, capacitors
- Electric currents ➔ Magnetic field
  - Magnetic force on moving charges
- Time-varying magnetic field ➔ Electric Field
- More circuit components: inductors.
- Electromagnetic waves ➔ light waves
- Geometrical Optics (light rays).
- Physical optics (light waves)
Ampere’s law: Closed Loops

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{\text{enclosed}} \]

The circulation of \( \mathbf{B} \) (the integral of \( \mathbf{B} \) scalar ds) along an imaginary closed loop is proportional to the net amount of current traversing the loop.

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 (i_1 - i_2) \]

Thumb rule for sign; ignore \( i_3 \)

If you have a lot of symmetry, knowing the circulation of \( \mathbf{B} \) allows you to know \( \mathbf{B} \).
Calculation of $i_{\text{enc}}$. We curl the fingers of the right hand in the direction in which the Amperian loop was traversed. We note the direction of the thumb.

All currents inside the loop parallel to the thumb are counted as positive. All currents inside the loop antiparallel to the thumb are counted as negative. All currents outside the loop are not counted. In this example: $i_{\text{enc}} = i_1 - i_2$. 
Figure 29-31 shows four circular Amperian loops \((a, b, c, d)\) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, 4 A out of the page, 9 A into the page, 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of \(\oint \vec{B} \cdot d\vec{s}\) around each, greatest first.

\[
\begin{align*}
\oint_a & \propto |4| = 4 \\
\oint_b & \propto |4 - 9| = 5 \\
\oint_c & \propto |4 - 9 + 5| = 0 \\
\oint_d & \propto |4 - 9 + 5 - 3| = 3 \\
\end{align*}
\]

\[
\oint_b > \oint_a > \oint_d > \oint_c = 0
\]
Figure 29-31 shows four circular Amperian loops \((a, b, c, d)\) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, \(4 \, \text{A} \) out of the page, \(9 \, \text{A} \) into the page, \(5 \, \text{A} \) out of the page, and \(3 \, \text{A} \) in. \( r_a = 1 \, \text{m} \)

**Fig. 29-31** Question 9. Rank the Amperian loops around each, greatest first.

\[
B_a \propto \left| \frac{4}{1} \right| = \frac{4}{1} = \frac{16}{4} = 4 \, \text{mT}
\]

\[
B_b \propto \left| \frac{4 - 9}{2} \right| = \frac{5}{2} = \frac{10}{4} = 2.5 \, \text{mT}
\]

\[
B_c \propto \left| \frac{4 - 9 + 5}{3} \right| = 0
\]

\[
B_d \propto \left| \frac{4 - 9 + 5 - 3}{4} \right| = \frac{3}{4} = 0.75 \, \text{mT}
\]

\[
B = \frac{\mu_0 i_{\text{enclosed}}}{2\pi r}
\]

\[
1/\text{r} \text{ Law too}
\]

\[
B_a > B_b > B_d > B_c = 0
\]
29.4.1. A copper cylinder has an outer radius $2R$ and an inner radius of $R$ and carries a current $i$. Which one of the following statements concerning the magnetic field in the hollow region of the cylinder is true?

a) The magnetic field within the hollow region may be represented as concentric circles with the direction of the field being the same as that outside the cylinder.

b) The magnetic field within the hollow region may be represented as concentric circles with the direction of the field being the opposite as that outside the cylinder.

c) The magnetic field within the hollow region is parallel to the axis of the cylinder and is directed in the same direction as the current.

d) The magnetic field within the hollow region is parallel to the axis of the cylinder and is directed in the opposite direction as the current.

e) The magnetic field within the hollow region is equal to zero tesla.
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a) The magnetic field within the hollow region may be represented as concentric circles with the direction of the field being the same as that outside the cylinder.

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e) The magnetic field within the hollow region is equal to zero tesla.
29.4.3. The drawing shows two long, straight wires that are parallel to each other and carry a current of magnitude $i$ toward you. The wires are separated by a distance $d$; and the centers of the wires are a distance $d$ from the $y$ axis. Which one of the following expressions correctly gives the magnitude of the total magnetic field at the origin of the $x, y$ coordinate system?

a) $\frac{\mu_0 i}{2d}$

b) $\frac{\mu_0 i}{\sqrt{2}d}$

c) $\frac{\mu_0 i}{2\pi d}$

d) $\frac{\mu_0 i}{\pi d}$

e) zero tesla
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d) $\frac{\mu_0 i}{\pi d}$

e) zero tesla
29.5: Solenoids and Toroids:

Fig. 29-16 A solenoid carrying current $i$.

Fig. 29-17 A vertical cross section through the central axis of a “stretched-out” solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid’s axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.
Solenoids: Compute the B-Field Inside

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's Law)} \]

\[ \oint \mathbf{B} \cdot d\mathbf{s} = B h + 0 + 0 + 0 \]

\[ i_{\text{enc}} = i N_h = i (N / L) h = i n h \]

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{\text{enc}} \Rightarrow B h = \mu_0 i n h \Rightarrow B = \mu_0 i n \]

\[ n = N / L \text{ is turns per unit length.} \]
29.5: Solenoids:

Fig. 29-19 Application of Ampere’s law to a section of a long ideal solenoid carrying a current $i$. The Amperian loop is the rectangle $abcd$. 

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}, \]

\[ \oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_{nh} \vec{B} \cdot d\vec{s} + \int_{nh} \vec{B} \cdot d\vec{s}. \]

$i_{enc} = i(nh)$. Here $n$ be the number of turns per unit length of the solenoid

$Bh = \mu_0 inh$

\[ B = \mu_0 in \quad \text{(ideal solenoid)}. \]
29.5.5. A solenoid carries current $I$ as shown in the figure. If the observer could “see” the magnetic field inside the solenoid, how would it appear?
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29.5.1. The drawing shows a rectangular wire loop that has one side passing through the center of a solenoid. Which one of the following statements describes the force, if any, that acts on the rectangular loop when a current is passing through the solenoid.

a) The magnetic force causes the loop to move upward.

b) The magnetic force causes the loop to move downward.

c) The magnetic force causes the loop to move to the right.

d) The magnetic force causes the loop to move to the left.

e) The loop is not affected by the current passing through the solenoid or the magnetic field resulting from it.
29.5.1. The drawing shows a rectangular current carrying wire loop that has one side passing through the center of a solenoid. Which one of the following statements describes the force, if any, that acts on the rectangular loop when a current is passing through the solenoid.

a) The magnetic force causes the loop to move upward.

b) The magnetic force causes the loop to move downward.

c) The magnetic force causes the loop to move to the right.

d) The magnetic force causes the loop to move to the left.

e) The loop is not affected by the current passing through the solenoid or the magnetic field resulting from it.
Sample Problem

The field inside a solenoid (a long coil of current)

A solenoid has length \( L = 1.23 \text{ m} \) and inner diameter \( d = 3.55 \text{ cm} \), and it carries a current \( i = 5.57 \text{ A} \). It consists of five close-packed layers, each with 850 turns along length \( L \). What is \( B \) at its center?

**KEY IDEA**

The magnitude \( B \) of the magnetic field along the solenoid’s central axis is related to the solenoid’s current \( i \) and number of turns per unit length \( n \) by Eq. 29-23 \((B = \mu_0 in)\).

**Calculation:** Because \( B \) does not depend on the diameter of the windings, the value of \( n \) for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

\[
B = \mu_0 in = (4\pi \times 10^{-7} \text{T} \cdot \text{m}/\text{A})(5.57 \text{ A}) \frac{5 \times 850 \text{ turns}}{1.23 \text{ m}}
\]

\[
= 2.42 \times 10^{-2} \text{T} = 24.2 \text{ mT}.
\]

(Answer)

To a good approximation, this is the field magnitude throughout most of the solenoid.
29.5.3. Which one of the following statements concerning the magnetic field inside (far from the surface) a long, current-carrying solenoid is true?

a) The magnetic field is zero.

b) The magnetic field is non-zero and nearly uniform.

c) The magnetic field is independent of the number of windings.

d) The magnetic field is independent of the current in the solenoid.

e) The magnetic field varies as $1/r$ as measured from the solenoid axis.
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Magnetic Field in a Toroid “Doughnut” Solenoid

\[ B = \frac{\mu_0 Ni}{2\pi r} \]

Toriod Fusion Reactor: Power NYC For a Day on a Glass of H₂O
29.5: Magnetic Field of a Toroid:

Fig. 29-20 (a) A toroid carrying a current $i$. (b) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere’s law with the Amperian loop shown.

\[
\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's Law)}
\]

\[
\oint \vec{B} \cdot d\vec{s} = B2\pi r
\]

\[i_{\text{enc}} = Ni\]

where $i$ is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and $N$ is the total number of turns. This gives

\[
B = \frac{\mu_0 iN}{2\pi r} \quad \text{(toroid)}.
\]
Solenoid Fusion Reactors
Lockheed Martin's new fusion reactor might change humanity forever
Magnetic Field of a Magnetic Dipole

A circular loop or a coil currying electrical current is a magnetic dipole, with magnetic dipole moment of magnitude \( \mu = NiA \).

Since the coil curries a current, it produces a magnetic field, that can be calculated using Biot-Savart’s law:

\[
\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\mu}{(R^2 + z^2)^{3/2}} \approx \frac{\mu_0}{2\pi} \frac{\mu}{z^3}
\]

All loops in the figure have radius \( r \) or \( 2r \). Which of these arrangements produce the largest magnetic field at the point indicated?
29.6: A Current Carrying Coil as a Magnetic Dipole:

For small $z \ll R$

$$B(z) = \frac{\mu_0 i R}{2 R^3 \left(1 + \frac{z^2}{R^2}\right)^{3/2}} \approx \frac{\mu_0 i}{2 R^2} \frac{1}{R^2} \text{ Law! Double the R one fourth the field.}$$
The figure here shows four arrangements of circular loops of radius $r$ or $2r$, centered on vertical axes (perpendicular to the loops) and carrying identical currents in the directions indicated. Rank the arrangements according to the magnitude of the net magnetic field at the dot, midway between the loops on the central axis, greatest-first.

\[ B_d > B_a > B_b = B_c = 0 \]

For small \( z \ll R \)

\[ B(z) \equiv \frac{\mu_0 i}{2R^2} \]

1 / $R^2$ Law!

Double the $R \Rightarrow$ one 4th the field.
29.6: A Current Carrying Coil as a Magnetic Dipole:

![Diagram of current carrying coil]

\[ B(z) = \frac{\mu_0 i R^2}{2 (R^2 + z^2)^{3/2}} \]

\[ z \gg R \]

\[ B(z) \approx \frac{\mu_0 i R^2}{2z^3} \]

\[ B(z) = \frac{\mu_0}{2\pi} \frac{NiA}{z^3} \]

**Fig. 29-21** A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment \( \vec{\mu} \) of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field \( \vec{B} \) within the loop.

\[ \vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3} \] (current-carrying coil).

It's a \( 1/r^3 \) law!!!
Neutron Star a Large Magnetic Dipole \[ B_{\text{surface}} = 10^{10}\text{ Tesla} \]