Lecture 22: MON 09 MAR

DC Circuits I

Ch27.1–5

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27.2: Pumping Charges:

In order to produce a steady flow of charge through a resistor, one needs a “charge pump,” a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals.

Such a device is called an emf, or electromotive force.

A common emf device is the battery, used to power a wide variety of machines from wristwatches to submarines. The emf device that most influences our daily lives is the electric generator, which, by means of electrical connections (wires) from a generating plant, creates a potential difference in our homes and workplaces.

Some other emf devices known are solar cells, fuel cells. An emf device does not have to be an instrument—living systems, ranging from electric eels and human beings to plants, have physiological emf devices.
27.3: Work, Energy, and Emf:

In any time interval $dt$, a charge $dq$ passes through any cross section of the circuit shown, such as $aa'$. This same amount of charge must enter the emf device at its low-potential end and leave at its high-potential end.

The emf device must do an amount of work $dW$ on the charge $dq$ to force it to move in this way.

We define the emf of the emf device in terms of this work:

$$\varepsilon = \frac{dW}{dq} \quad \text{(definition of } \varepsilon).$$

Fig. 27-1 A simple electric circuit, in which a device of emf $\varepsilon$ does work on the charge carriers and maintains a steady current $i$ in a resistor of resistance $R$.

An ideal emf device is one that has no internal resistance to the internal movement of charge from terminal to terminal. The potential difference between the terminals of an ideal emf device is exactly equal to the emf of the device.

A real emf device, such as any real battery, has internal resistance to the internal movement of charge. When a real emf device is not connected to a circuit, and thus does not have current through it, the potential difference between its terminals is equal to its emf. However, when that device has current through it, the potential difference between its terminals differs from its emf.
The battery operates as a “pump” that moves positive charges from lower to higher electric potential. A battery is an example of an “electromotive force” (EMF) device.

These come in various kinds, and all transform one source of energy into electrical energy. A battery uses chemical energy, a generator mechanical energy, a solar cell energy from light, etc.

The difference in potential energy that the device establishes is called the EMF and denoted by $\mathcal{E}$.

$$\mathcal{E} = iR$$
27.4: Calculating the Current in a Single-Loop Circuit, Potential Method:

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

In the figure, let us start at point \( a \), whose potential is \( V_a \), and mentally go clockwise around the circuit until we are back at \( a \), keeping track of potential changes as we move.

Our starting point is at the low-potential terminal of the battery. Since the battery is ideal, the potential difference between its terminals is equal to \( \mathcal{E} \).

As we go along the top wire to the top end of the resistor, there is no potential change because the wire has negligible resistance.

When we pass through the resistor, however, the potential decreases by \( iR \).

We return to point \( a \) by moving along the bottom wire. At point \( a \), the potential is again \( V_a \). The initial potential, as modified for potential changes along the way, must be equal to our final potential; that is,

\[
V_a + \mathcal{E} - iR = V_a.
\]

\[
\mathcal{E} - iR = 0.
\]
For circuits that are more complex than that of the previous figure, two basic rules are usually followed for finding potential differences as we move around a loop:

**RESISTANCE RULE:** For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

**EMF RULE:** For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\varepsilon$; in the opposite direction it is $-\varepsilon$. 
27.4: Calculating the Current in a Single-Loop Circuit:

The equation \( P = i^2R \) tells us that in a time interval \( dt \) an amount of energy given by \( i^2R \, dt \) will appear in the resistor, as shown in the figure, as thermal energy.

During the same interval, a charge \( dq = i \, dt \) will have moved through battery B, and the work that the battery will have done on this charge, is

\[
\frac{dW}{dq} = \mathcal{E} \, dq = \mathcal{E} \, i \, dt.
\]

From the principle of conservation of energy, the work done by the (ideal) battery must equal the thermal energy that appears in the resistor:

\[
\mathcal{E} \, i \, dt = i^2R \, dt.
\]

\[
\mathcal{E} = iR.
\]

Therefore, the energy per unit charge transferred to the moving charges is equal to the energy per unit charge transferred from them.
Given the EMF devices and resistors in a circuit, we want to calculate the circulating currents. Circuit solving consists in “taking a walk” along the wires. As one “walks” through the circuit (in any direction) one needs to follow two rules:

When walking through an EMF, add $+\mathcal{E}$ if you flow with the current or $-\mathcal{E}$ against. How to remember: current “gains” potential in a battery.

When walking through a resistor, add $-iR$, if flowing with the current or $+iR$ against. How to remember: resistors are passive, current flows “potential down”.

**Example:**
Walking clockwise from $a$: $+\mathcal{E}-iR=0$.
Walking counter-clockwise from $a$: $-\mathcal{E}+iR=0$. 
**RESISTANCE RULE:** For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

**EMF RULE:** For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$.

**CHECKPOINT 1**

The figure shows the current $i$ in a single-loop circuit with a battery $B$ and a resistance $R$ (and wires of negligible resistance). (a) Should the emf arrow at $B$ be drawn pointing leftward or rightward? At points $a$, $b$, and $c$, rank (b) the magnitude of the current, (c) the electric potential, and (d) the electric potential energy of the charge carriers, greatest first.

(a) Rightward (EMF is in direction of current)
(b) All tie (no junctions so current is conserved)
(c) $b$, then $a$ and $c$ tie (Voltage is highest near battery $+$)
(d) $b$, then $a$ and $c$ tie ($U=qV$ and assume $q$ is $+$)
If one connects resistors of lower and lower value of $R$ to get higher and higher currents, eventually a real battery fails to establish the potential difference $\mathcal{E}$, and settles for a lower value.

One can represent a “real EMF device” as an ideal one attached to a resistor, called “internal resistance” of the EMF device:

\[
\mathcal{E} - ir - iR = 0 \implies i = \frac{\mathcal{E}}{r + R}
\]

The true EMF is a function of current: the more current we want, the smaller $\mathcal{E}_{\text{true}}$ we get.
**CHECKPOINT 3**

A battery has an emf of 12 V and an internal resistance of 2 Ω. Is the terminal-to-terminal potential difference greater than, less than, or equal to 12 V if the current in the battery is (a) from the negative to the positive terminal, (b) from the positive to the negative terminal, and (c) zero?

(a) $V_{\text{batt}} < 12V$ (walking with current voltage drop $-ir$

(b) $V_{\text{batt}} > 12V$ (walking against current voltage increase $+ir$

(c) $V_{\text{batt}} = 12V$ (no current and so $ir=0$)
Series: \(i\) is Constant (SERI-dQ/dt)

Two resistors are “in series” if they are connected such that the \textbf{same current} \(i\) flows in both. The “equivalent resistance” is a single imaginary resistor that can replace the resistances in series.

“Walking the loop” results in:

\[
\mathcal{E} - iR_1 - iR_2 - iR_3 = 0 \rightarrow i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}
\]

In the circuit with the equivalent resistance,

\[
\mathcal{E} - iR_{eq} = 0 \rightarrow i = \frac{\mathcal{E}}{R_{eq}}
\]

Thus,

\[
R_{eq} = \sum_{j=1}^{n} R_j
\]
CHECKPOINT 2

In Fig. 27-5a, if $R_1 > R_2 > R_3$, rank the three resistances according to (a) the current through them and (b) the potential difference across them, greatest first.

(a) all tie (current is the same in series)

(b) $V_1 > V_2 > V_3$

The voltage drop is $-iR$ proportional to $R$ since $i$ is same.
Parallel: V is Constant (PAR-V)

Two resistors are “in parallel” if they are connected such that there is the **same potential** $V$ drop through both. The “equivalent resistance” is a single imaginary resistor that can replace the resistances in parallel.

“Walking the loops” results in:

\[ E - i_1 R_1 = 0, \quad E - i_2 R_2 = 0, \quad E - i_3 R_3 = 0. \]

The total current delivered by the battery is:

\[ i = i_1 + i_2 + i_3 = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} = \frac{E}{1/R_1 + 1/R_2 + 1/R_3} \]

In the circuit with the equivalent resistor,

\[ i = \frac{E}{R_{eq}} \]

\[ \frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \]
Resistors

\[ V = iR \]

Series: \( i \) Same (SERI-\( dQ/dt \))

\[ R_{\text{ser}} = R_1 + R_2 + R_3 + \ldots \]

Parallel: \( V \) Same (PAR-V)

\[ 1/R_{\text{par}} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots \]

Capacitors

\[ Q = CV \]

Series: \( Q \) Same

\[ 1/C_{\text{ser}} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots \]

Parallel: \( V \) Same (PAR-V)

\[ C_{\text{par}} = C_1 + C_2 + C_3 + \ldots \]
A battery, with potential $V$ across it, is connected to a combination of two identical resistors and then has current $i$ through it. What are the potential difference across and the current through either resistor if the resistors are (a) in series and (b) in parallel?

(a) $V/2, \ i$

(b) $V, \ i/2$

$V = iR$

**Series: $i$ Same (SERI-dQ/dt)**

$R_{\text{ser}} = R_1 + R_2 + R_3 + ...$

$V_{\text{batt}} = iR + iR = V_{\text{batt}}/2 + V_{\text{batt}}/2$

$V/R = i$

**Parallel: $V$ Same (PAR-V)**

$1/R_{\text{par}} = 1/R_1 + 1/R_2 + 1/R_3 + ...$

$i_{\text{batt}} = i_1 + i_2 = V/R + V/R = i_{\text{batt}}/2 + i_{\text{batt}}/2$
Resistors in Series and Parallel

An electrical cable consists of 100 strands of fine wire, each having $r=2\,\Omega$ resistance. The same potential difference is applied between the ends of all the strands and results in a total current of $I=5\,A$.

(a) What is the current in each strand?
Ans: $i_p=0.05\,A$ ($i=I/100$)

(b) What is the applied potential difference?
Ans: $v_p=0.1\,V$ ($v_p=V=i_s r=\text{constant}$)

(c) What is the resistance of the cable?
Ans: $R_p=r=0.02\,\Omega$ ($1/R_p=1/r+1/r+\ldots=100/r \Rightarrow R=r/100$)

Assume now that the same 2 $\Omega$ strands in the cable are tied in series, one after the other, and the 100 times longer cable connected to the same $V=0.1\,\text{Volts}$ potential difference as before.

(d) What is the potential difference through each strand?
Ans: $v_s=0.001\,V$ ($v_s=V/100$)

(e) What is the current in each strand?
Ans: $i_s=0.0005\,A$ ($i_s=v_s/r=\text{constant}$)

(f) What is the resistance of the cable?
Ans: $200\,\Omega$ ($R_s=r+r+r+\ldots=100r$)

(g) Which cable gets hotter, the one with strands in parallel or the one with strands in series?
Ans: Each strand in parallel dissipates $P_p=iv_p=5\,\text{mW}$ (and the cable dissipates $100P_p=500\,\text{mW}$); Each strand in series dissipates $P_s=i_s v_s=50\,\mu\text{W}$ (and the cable dissipates $5\,\text{mW}$)
Example

38E. A circuit containing five resistors connected to a battery with a 12.0 V emf is shown in Fig. 28-38. What is the potential difference across the 5.0 Ω resistor?

![Circuit diagram]

Bottom loop: (all else is irrelevant) V same in parallel -- PAR-V!

\[ i = \frac{V_{\text{batt}}}{R} = \frac{12V}{8\Omega} = 1.5A \]

\[ E_5 = i_5 R_5 = (1.5A)(5.0\Omega) = 7.5V \]

IPCC: Which resistor (3 or 5) gets hotter? \( P = i^2 R \)
Example

a) Which circuit has the largest equivalent resistance?

b) Assuming that all resistors are the same, which one dissipates more power?

c) Which resistor has the smallest potential difference across it?
Example

Find the equivalent resistance between points
(a) $F$ and $H$ and
(b) $F$ and $G$.

(*Hint:* For each pair of points, imagine that a battery is connected across the pair.)
If all resistors have a resistance of 4Ω, and all batteries are ideal and have an emf of 4V, what is the current through R?

If all capacitors have a capacitance of 6µF, and all batteries are ideal and have an emf of 10V, what is the charge on capacitor C?
1. Explain Newton's First Law of Motion in your own words.


I love loopholes.