Physics 2113

Lecture 13: FRI 13 FEB

Electric Potential I

24-1 What Is Physics? 628
24-2 Electric Potential Energy 628
24-3 Electric Potential 629

Danger!
<table>
<thead>
<tr>
<th></th>
<th>Volume [m³]</th>
<th>Area [m²]</th>
<th>Circumference [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>(\frac{4\pi}{3}R^3)</td>
<td>(4\pi R^2)</td>
<td>2πR</td>
</tr>
<tr>
<td>Circle</td>
<td>(\pi R^2)</td>
<td>(\pi R^2)</td>
<td>2πR</td>
</tr>
<tr>
<td>Cylinder</td>
<td>(\pi R^2 L)</td>
<td>(2\pi RL)</td>
<td>2πR</td>
</tr>
</tbody>
</table>
Electric Potential Energy

Electric Potential Energy $U$ is Negative of the Work $W$ to Bring Charges in From Infinity:

$$U = -W_\infty$$

The Change in Potential Energy $\Delta U$ Between an Initial and Final Configuration Is Negative the Work $W$ Done by the Electrostatic Forces:

$$\Delta U = U_f - U_i = -W$$

- What is the potential energy of a single charge?
- What is the potential energy of a dipole?
- A proton moves from point $i$ to point $f$ in a uniform electric field, as shown.
  - Does the electric field do positive or negative work on the proton?
  - Does the electric potential energy of the proton increase or decrease?
Electric Potential Voltage

Electric potential voltage difference between two points = work per unit charge needed to move a charge between the two points:

$$\Delta V = V_f - V_i = -\frac{W}{q} = \Delta U/q$$

\[dW = \vec{F} \cdot d\vec{s}\]

\[dW = q_0 \vec{E} \cdot d\vec{s}\]

\[W = \int_{i}^{f} dW = \int_{i}^{f} q_0 \vec{E} \cdot d\vec{s}\]

$$\Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_{i}^{f} \vec{E} \cdot d\vec{s}$$
Electric Potential Energy, Electric Potential

**Units:**

Potential Energy = $U = [J] = \text{Joules}$

Electric Potential = $V = U/q = [J/C] = [Nm/C] = [V] = \text{Volts}$

Electric Field = $E = [N/C] = [V/m] = \text{Volts per meter}$

Electron Volt = 1eV = Work Needed to Move an Electron Through a Potential Difference of 1V:

$W = q\Delta V = e \times 1V = 1.60 \times 10^{-19} \text{ C} \times 1\text{J/C} = 1.60 \times 10^{-19} \text{ J}$
Electric Potential Voltage and Electric Potential Energy

The change in potential energy of a charge $q$ moving from point $i$ to point $f$ is equal to the work done by the applied force, which is equal to minus the work done by the electric field, which is related to the difference in electric potential voltage:

$$\Delta U = U_f - U_i = W_{\text{app}} = -W_{\text{field}} = q\Delta V$$

We move a proton from point $i$ to point $f$ in a uniform electric field, as shown.

- Does the electric field do positive or negative work on the proton?
- Does the electric potential energy of the proton increase or decrease?
- Does our force do positive or negative work?
- Does the proton move to a higher or lower potential?
Positive Work

\[ +Q \hspace{1cm} a \hspace{1cm} +Q \]

Negative Work

\[ +Q \hspace{1cm} a \hspace{1cm} -Q \]
Applied Positive Work: Potential Energy Increases

Charge Moves Uphill: I’m doing work against Field.

\[ +Q \quad a \quad +Q \]

Applied Negative Work: Potential Energy Decreases

Charge Moves Downhill: I’m Doing Work With Field

\[ +Q \quad a \quad -Q \]

Work done by field is negative of Applied work done by me.
In the figure, we move a proton from point $i$ to point $f$ in a uniform electric field. Is positive or negative work done by (a) the electric field and (b) our force? (c) Does the electric potential energy increase or decrease? (d) Does the proton move to a point of higher or lower electric potential?

(a) E-field does:
  + work? ✓
  – work? ✓

(b) Work done by me:
  + work? ✓
  – work?

(c) Potential Energy:
  increase? ✓
  decrease?

(d) Potential Voltage:
  higher? ✓
  lower?

\[ +V_{\text{high}} = + + + + + + + \]

\[ -V_{\text{low}} = - - - - - - - \]
ICPP:

Consider a positive and a negative charge, freely moving in a uniform electric field. True or false?

(a) Positive charge moves to points with lower potential.
(b) Negative charge moves to points with lower potential.
(c) Positive charge moves to a lower potential energy.
(d) Negative charge moves to a lower potential energy.

(a) True
(b) False
(c) True
(d) True
Checkpoint 1

In the figure, we move an electron from point i to point f in a uniform electric field. Is positive or negative work done by (a) the electric field and (b) our force? (c) Does the electric potential energy increase or decrease? (d) Does the proton move to a point of higher or lower electric potential?

(a) E-field does:
- work? ✔
+ work? ✔

(b) Work done by me:
- work? ✔
+ work? ✔

(c) Potential Energy:
- increase? ✔
+ decrease? ✔

(d) Potential Voltage:
- higher? ✔
- lower?

\[ -V_{\text{low}} = \]

\[ +V_{\text{high}} = \]
Electric Potential Energy, Electric Potential

**Units**

Potential Energy = $U = [J] = \text{Joules}$

Electric Potential = $V = \frac{U}{q} = [J/C] = [Nm/C] = [V] = \text{Volts}$

Electric Field = $E = [N/C] = [V/m] = \text{Volts per meter}$

\[ F = qE \quad (\text{Force is charge times Field}) \]

\[ U = qV \quad (\text{Potential Energy is charge times Potential}) \]

Electron Volt = $1eV = \text{Work Needed to Move an Electron Through a Potential Difference of 1V}$

Through a Potential Difference of 1V:

\[ W = q\Delta V = e \times 1V = 1.60 \times 10^{-19} \text{C} \times 1\text{J/C} = 1.60 \times 10^{-19} \text{J} \]
Electric Potential Energy = Joules

Electric potential energy difference $\Delta U$ between two points = work needed to move a charge between the two points:

$$\Delta U = U_f - U_i = -W$$

$$dW = \vec{F} \cdot d\vec{s}$$

$$F = q_0 \vec{E}$$

$$W = \int_i^f dW = \int_i^f \vec{F} \cdot d\vec{s} = \int_i^f q_0 \vec{E} \cdot d\vec{s}$$

$$\Delta U = U_f - U_i = -W = -q_0 \int_i^f \vec{E} \cdot d\vec{s}$$
Electric Potential Voltage = Volts = Joules/Coulomb!

Electric potential — voltage! — difference $\Delta V$ between two points = work per unit charge needed to move a charge between the two points:

$$\Delta V = V_f - V_i = -\frac{W}{q} = \frac{\Delta U}{q}$$

$$dW = \vec{F} \cdot d\vec{s}$$
$$\vec{F} = q_0 \vec{E}$$
$$W = \int dW = \int q_0 \vec{E} \cdot d\vec{s}$$
$$\Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_{i}^{f} \vec{E} \cdot d\vec{s}$$
Equal-Potential = Equipotential Surfaces

\[ \Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_{i}^{f} \vec{E} \cdot d\vec{s} \]

- The Electric Field is **Tangent** to the Field Lines
- Equipotential Surfaces are **Perpendicular** to Field Lines

- **Work Is Needed** to Move a Charge **Along a Field Line**.

- **No Work Is Needed** to Move a Charge **Along an Equipotential Surface** (Or Back to the Surface Where it Started).

- Electric Field Lines Always Point Towards Equipotential Surfaces With Lower Potential.
Electric Field Lines and Equipotential Surfaces

Why am I smiling? I’m About to Be Struck by Lightning!

http://www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html
**Equipotential Surfaces: IPCC**

For which paths is work done $W = 0$?

- I?
- II?
- III?
- IV?

For which paths is work done, $W \neq 0$?

- I?
- II?
- III?
- IV?

For which paths is work done, $W$, the same?

- I?
- II?
- III?
- IV?

**Mathematical Equations**

$\Delta V = V_f - V_i = -W / q_0$

- $\Delta V_I = V_1 - V_1 = 0$
- $\Delta V_{II} = V_3 - V_3 = 0$
- $\Delta V_{III} = V_1 - V_2 \neq 0$
- $\Delta V_{IV} = V_1 - V_2 = \Delta V_{III} \neq 0$

**Diagrams**

- Equal work is done along these paths between the same surfaces.
- No work is done along this path on an equipotential surface.
- No work is done along this path that returns to the same surface.
Conservative Forces

The potential difference between two points is independent of the path taken to calculate it: electric forces are “conservative”.

\[ \Delta V = V_f - V_i = -\frac{W}{q_0} = \frac{\Delta U}{q_0} = -\int_{i}^{f} \vec{E} \cdot d\vec{s} \]
The figure here shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an electron from one surface to another. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.

(a) →? ←? ×? ×?

(b) 1 = + 2 = + 3 = + 4 = − 5 = +

(c) 3 > 1 = 2 = 5 > 4
Summary:

- **Electric potential**: work needed to bring +1C from infinity; units \( V = \text{Volt} \)
- Electric potential uniquely defined for every point in space -- independent of path!
- Electric potential is a **scalar** — add contributions from individual point charges
- We calculated the electric potential produced by a single charge: \( V = kq/r \), and by continuous charge distributions: \( dV = kdq/r \)
- **Electric potential energy**: work used to build the system, charge by charge. Use \( W = qV \) for each charge.
"It's unified and it's a theory, but it's not the unified theory we've all been looking for."