Physics 2113

Lecture: 11 MON 09 FEB

Gauss’ Law II

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Flux Capacitor (Schematic)

Carl Friedrich Gauss 1777 – 1855
Gauss’ Law: General Case

- Consider any ARBITRARY CLOSED surface S -- NOTE: this does NOT have to be a “real” physical object!
- The TOTAL ELECTRIC FLUX through S is proportional to the TOTAL CHARGE ENCLOSED!
- The results of a complicated integral is a very simple formula: it avoids long calculations!

\[ \Phi \equiv \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{ins}}}{\varepsilon_0} \]

(One of Maxwell’s 4 equations!)
Gauss’ Law: ICPP

\[ \Phi \equiv \oint \vec{E} \cdot d\vec{A} = \frac{+q}{\varepsilon_0} ? \]

\[ \Phi \equiv \oint \vec{E} \cdot d\vec{A} = \frac{-q}{\varepsilon_0} ? \]

\[ \Phi \equiv \oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0} ? \]

\[ \Phi \equiv \oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0} ? \]

"+" = +q

"-" = -q
CHECKPOINT 2

The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicate the directions of the field lines and the magnitudes (in N · m²/C) of the flux through the six sides of each cube. (The lighter arrows are for the hidden faces.) In which situation does the cube enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?

\[
\Phi_1 = +2 + 5 + 7 - 3 - 4 - 7 \quad \Phi_2 = +3 + 5 + 10 - 3 - 4 - 6 \quad \Phi_3 = +2 + 5 + 8 - 5 - 6 - 7
\]

\[
0 \quad + \quad -
\]
Properties of Conductors

Inside a Conductor in Electrostatic Equilibrium, the Electric Field Is ZERO. Why?

Because If the Field Is Not Zero, Then Charges Inside the Conductor Would Be Moving.

SO: Charges in a Conductor Redistribute Themselves Wherever They Are Needed to Make the Field Inside the Conductor ZERO.

Excess Charges Are Always on the Surface of the Conductors.
ICPP: Conducting Sphere

• A spherical conducting shell has an excess charge of +10 C.
• A point charge of −15 C is located at center of the sphere.
• Use Gauss’ Law to calculate the charge on inner and outer surface of spherical shell

(a) Inner: +15 C; outer: 0
(b) Inner: 0; outer: +10 C
(c) Inner: +15 C; outer: −5 C

Hint: E-Field is Zero inside conductor so \[ \oint_{S_1} \vec{E} \cdot d\vec{A} = 0 = -15C / \varepsilon_0 + ? \]
Gauss’ Law: Conducting Sphere

• Inside a conductor, E = 0 under static equilibrium! Otherwise electrons would keep moving!

• Construct a Gaussian surface inside the metal as shown. (Does not have to be spherical!)

• Since E = 0 inside the metal, flux through this surface = 0

• Gauss’ Law says total charge enclosed = 0

• Charge on inner surface = +15 C

Since TOTAL charge on shell is +10 C, Charge on outer surface = +10 C - 15 C = -5 C!
Faraday’s Cage

- Given a hollow conductor of arbitrary shape. Suppose an excess charge $Q$ is placed on this conductor. Suppose the conductor is placed in an external electric field. How does the charge distribute itself on outer and inner surfaces?
  
  (a) Inner: $Q/2$; outer: $Q/2$
  (b) Inner: 0; outer: $Q$
  (c) Inner: $Q$; outer: 0

- Choose any arbitrary surface inside the metal
- Since $E = 0$, flux = 0
- Hence total charge enclosed = 0

All charge goes on outer surface!

Inside cavity is “shielded” from all external electric fields! “Faraday Cage effect”
Faraday’s Cage: Electric Field Inside Hollow Conductor is Zero

• Choose any arbitrary surface inside the metal
• Since \( E = 0 \), flux = 0
• Hence total charge enclosed = 0

All charge goes on outer surface!

Inside cavity is “shielded” from all external electric fields! “Faraday Cage effect”
Bill Downs takes a look inside the Cage of Death
Field on Conductor Perpendicular to Surface

We know the field inside the conductor is zero, and the excess charges are all on the surface. The charges produce an electric field outside the conductor.

On the surface of conductors in electrostatic equilibrium, the electric field is always perpendicular to the surface.

Why? Because if not, charges on the surface of the conductors would move with the electric field.
Charges in Conductors

• Consider a conducting shell, and a negative charge inside the shell.

• Charges will be “induced” in the conductor to make the field inside the conductor zero.

• Outside the shell, the field is the same as the field produced by a charge at the center!
Gauss’ Law: Conducting Plane

- Infinite CONDUCTING plane with uniform areal charge density $s$
- $E$ is NORMAL to plane
- Construct Gaussian box as shown.
- Note that $E = 0$ inside conductor

Applying Gauss' law, we have, $\frac{A\sigma}{\varepsilon_0} = AE$

Solving for the electric field, we get $E = \frac{\sigma}{\varepsilon_0}$
Gauss’ Law: Conducting ICPP

- Charged conductor of arbitrary shape: no symmetry; non-uniform charge density
- What is the electric field near the surface where the local charge density is $\sigma$?

(a) $\frac{\sigma}{\varepsilon_0}$  
(b) Zero  
(c) $\frac{\sigma}{2\varepsilon_0}$

Applying Gauss' law, we have, $\frac{A\sigma}{\varepsilon_0} = AE$

Solving for the electric field, we get $E = \frac{\sigma}{\varepsilon_0}$

THIS IS A GENERAL RESULT FOR CONDUCTORS!
Fig. 23-5  (a) A Gaussian cube with one edge on the x axis lies within a nonuniform electric field that depends on the value of x. (b) Each differential area element has an outward vector that is perpendicular to the area. (c) Right face: the x component of the field pierces the area and produces positive (outward) flux. The y component does not pierce the area and thus does not produce any flux. (d) Left face: the x component of the field produces negative (inward) flux. (e) Top face: the y component of the field produces positive (outward) flux.
Sample Problem

Flux through a closed cube, nonuniform field

A nonuniform electric field given by \( \vec{E} = 3.0 \hat{i} + 4.0 \hat{j} \) pierces the Gaussian cube shown in Fig. 23-5a (\( E \) is in newtons per coulomb and \( x \) is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

**KEY IDEA**

We can find the flux \( \Phi \) through the surface by integrating the scalar product \( \vec{E} \cdot d\vec{A} \) over each face.

**Right face:** An area vector \( \vec{A} \) is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector \( d\vec{A} \) for any area element (small section) on the right face of the cube must point in the positive direction of the \( x \) axis. An example of such an element is shown in Figs. 23-5b and c, but we would have an identical vector for any other choice of an area element on that face. The most convenient way to express the vector is in unit-vector notation,

\[
    d\vec{A} = dA \hat{i}.
\]

From Eq. 23-4, the flux \( \Phi \) through the right face is then

\[
\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0 \hat{i} + 4.0 \hat{j}) \cdot (dA \hat{i})
\]

\[
= \int [(3.0x)(dA) \hat{i} \cdot \hat{i} + (4.0)(dA) \hat{j} \cdot \hat{i}]
\]

\[
= \int (3.0x \, dA + 0) = 3.0 \int x \, dA.
\]

We are about to integrate over the right face, but we note that \( x \) has the same value everywhere on that face — namely, \( x = 3.0 \) m. This means we can substitute that constant value for \( x \). This can be a confusing argument. Although \( x \) is certainly a variable as we move left to right across the figure, because the right face is perpendicular to the \( x \) axis, every point on the face has the same \( x \) coordinate. (The \( y \) and \( z \) coordinates do not matter in our integral.) Thus, we have

\[
\Phi_r = 3.0 \int (3.0) \, dA = 9.0 \int dA.
\]

The integral \( \int dA \) merely gives us the area \( A = 4.0 \text{ m}^2 \) of the right face; so

\[
\Phi_r = (9.0 \text{ N/C})(4.0 \text{ m}^2) = 36 \text{ N} \cdot \text{m}^2/\text{C}. \quad \text{(Answer)}
\]

**Left face:** The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (1) The differential area vector \( d\vec{A} \) points in the negative direction of the \( x \) axis, and thus \( d\vec{A} = -dA \hat{i} \) (Fig. 23-5d). (2) The term \( x \) again appears in our integration, and it is again constant over the face being considered. However, on the left face, \( x = 1.0 \) m. With these two changes, we find that the flux \( \Phi \) through the left face is

\[
\Phi_l = -12 \text{ N} \cdot \text{m}^2/\text{C}. \quad \text{(Answer)}
\]

**Top face:** The differential area vector \( d\vec{A} \) points in the positive direction of the \( y \) axis, and thus \( d\vec{A} = dA \hat{j} \) (Fig. 23-5e). The flux \( \Phi \) through the top face is then

\[
\Phi_t = \int \vec{E} \cdot d\vec{A} = \int (3.0 \hat{i} + 4.0 \hat{j}) \cdot (dA \hat{j})
\]

\[
= \int [(3.0x)(dA) \hat{i} \cdot \hat{j} + (4.0)(dA) \hat{j} \cdot \hat{j}]
\]

\[
= \int (0 + 4.0 \, dA) = 4.0 \int dA
\]

\[
= 16 \text{ N} \cdot \text{m}^2/\text{C}. \quad \text{(Answer)}
\]