Physics 2113
Lecture 07: FRI 30 JAN
Electric Fields II

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Charles-Augustin de Coulomb (1736–1806)
Direction of Electric Field Lines

**E-Field Vectors**

Point **Away** from Positive Charge — Field **Source**!

Positives **PUSH**!

**E-Field Vectors**

Point **Towards** Negative Charge — Field **Sink**!

Negatives **Pull**!
Electric Field of a Dipole

- Electric dipole: two point charges $+q$ and $-q$ separated by a distance $d$
- Common arrangement in Nature: molecules, antennae, ...
- Note axial or cylindrical symmetry
- Define “dipole moment” vector $\mathbf{p}$: from $-q$ to $+q$, with magnitude $qd$

Cancer, Cisplatin and electric dipoles: 
http://chemcases.com/cisplat/cisplat01.htm
Electric Field On Axis of Dipole

\[ \vec{p} = qa\hat{i} \]

Superposition: \[ \vec{E} = \vec{E}_+ + \vec{E}_- \]

\[ \vec{E}_+ = \frac{kq}{(x - \frac{a}{2})^2} \]

\[ \vec{E}_- = -\frac{kq}{(x + \frac{a}{2})^2} \]

\[ \vec{E} = kq \left[ \frac{1}{(x - \frac{a}{2})^2} - \frac{1}{(x + \frac{a}{2})^2} \right] \]

\[ = kq \frac{2xa}{\left(x^2 - \frac{a^2}{4}\right)^2} \]
Electric Field On Axis of Dipole

\[ E = kq \frac{2xa}{\left(x^2 - \frac{a^2}{4}\right)^2} = \frac{2kpx}{\left(x^2 - \frac{a^2}{4}\right)^2} \]

What if \( x \gg a \) (i.e. very far away)

\[ E \approx \frac{2kpx}{x^4} = \frac{2kp}{x^3} \]

\[ |\vec{E}| \propto \frac{|\vec{p}|}{r^3} \]

\( E = \frac{p}{r^3} \) is actually true for ANY point far from a dipole (not just on axis)
Continuous Charge Distribution

- Thus Far, We Have Only Dealt With Discrete, Point Charges.
- Imagine Instead That a Charge $q$ Is Smeared Out Over A:
  - LINE
  - AREA
  - VOLUME
- How to Compute the Electric Field $E$? Calculus!!!
Charge Density

- Useful idea: charge density
- Line of charge:
  charge per unit length $= \lambda$
- Sheet of charge:
  charge per unit area $= \sigma$
- Volume of charge:
  charge per unit volume $= \rho$

\[ \lambda = \frac{q}{L} \]
\[ \sigma = \frac{q}{A} \]
\[ \rho = \frac{q}{V} \]
Computing Electric Field of Continuous Charge Distribution

- Approach: Divide the Continuous Charge Distribution Into Infinitesimally Small Differential Elements
- Treat Each Element As a POINT Charge & Compute Its Electric Field
- Sum (Integrate) Over All Elements
- Always Look for Symmetry to Simplify Calculation!

\[ dq = \lambda \, dL \]
\[ dq = \sigma \, dS \]
\[ dq = \rho \, dV \]
Differential Form of Coulomb’s Law

\[ |\vec{E}_{12}| = \frac{k |q_2|}{r_{12}^2} \]

E-Field at Point

\[ \vec{E}_{12} \]

Point \( p_1 \)

Point \( p_2 \)

Differential dE-Field at Point

\[ |d\vec{E}_{12}| = \frac{k |dq_2|}{r_{12}^2} \]

\( dq_2 \)

\( p_1 \)
Field on Bisector of Charged Rod

- Uniform line of charge $+q$ spread over length $L$
- ICPP: What is the direction of the electric field at a point $P$ on the perpendicular bisector?
  (a) Field is 0.
  (b) Along $+y$ ✓
  (c) Along $+x$
- Choose symmetrically located elements of length $dq = \lambda dx$
- $x$ components of $E$ cancel
**Line of Charge: Quantitative**

- Uniform line of charge, length L, total charge q
- Compute explicitly the magnitude of E at point P on perpendicular bisector
- Showed earlier that the net field at P is in the y direction — let’s now compute this!
Line Of Charge: Field on bisector

Distance hypotenuse: \( r = \left( a^2 + x^2 \right)^{1/2} \)

Charge per unit length: \( \lambda = \frac{q}{L} \) [C/m]

\[
dE = \frac{k(dq)}{r^2}
\]

\[
dE_y = dE \cos \theta = \frac{k(\lambda \, dx)a}{\left( a^2 + x^2 \right)^{3/2}}
\]

\[
\cos \theta = \frac{a}{r} = \frac{a}{\left( a^2 + x^2 \right)^{1/2}}
\]
Line Of Charge: Field on bisector

\[ E_y = k\lambda a \int_{-L/2}^{L/2} \frac{dx}{(a^2 + x^2)^{3/2}} = k\lambda a \left[ \frac{x}{a^2 \sqrt{x^2 + a^2}} \right]_{-L/2}^{L/2} \]

Integrate: Trig Substitution!

\[ = \frac{2k\lambda L}{a\sqrt{4a^2 + L^2}} \]

Point Charge Limit: \( L \ll a \)

\[ E_y \approx \frac{2k\lambda L}{a\sqrt{4a^2 + L^2}} = \frac{kq}{a^2} \]

Coulomb’s Law!

Line Charge Limit: \( L \gg a \)

\[ E_y \approx \frac{2k\lambda}{a} \]

Units Check!

\[ \left[ \frac{Nm^2}{C^2 \text{ m m}} \right] = \left[ \frac{N}{C} \right] \]
Binomial Approximation from Taylor Series: \( x \ll 1 \)

\[
(1 \pm x)^n \approx 1 \pm nx
\]

\[
\frac{2k\lambda L}{a\sqrt{4a^2 + L^2}} = \frac{k\lambda L}{a^2} \left[ 1 + \left( \frac{L}{2a} \right)^2 \right]^{1/2} \approx \frac{k\lambda L}{a^2} \left[ 1 - \frac{1}{2} \left( \frac{L}{2a} \right)^2 \right] \approx \frac{k\lambda L}{a^2}; \quad (L \ll a)
\]

\[
\frac{2k\lambda L}{a\sqrt{4a^2 + L^2}} = \frac{2k\lambda L}{aL} \left[ 1 + \left( \frac{2a}{L} \right)^2 \right]^{1/2} \approx \frac{2k\lambda L}{a} \left[ 1 - \frac{1}{2} \left( \frac{2a}{L} \right)^2 \right] \approx \frac{2k\lambda L}{a}; \quad (L \gg a)
\]
In the figure below, positive charge $q = 7.81$ pC is spread uniformly along a thin nonconducting rod of length $L = 14.5$ cm.

ICPP: What is the direction of the field at point $P$?

(a) Along $+x$
(b) Along $-x$
(c) Along $+y$ ✔
(d) Along $-y$
In the figure below, positive charge \( q = 7.81 \text{ pC} \) is spread uniformly along a thin nonconducting rod of length \( L = 14.5 \text{ cm} \).

(a) What is the magnitude of the electric field produced at point \( P \), at distance \( R = 6.00 \text{ cm} \) from the rod along its perpendicular bisector?

\[
E_y = \frac{2k\lambda L}{R\sqrt{4R^2 + L^2}}
\]

\[
\lambda = \frac{q}{L} = \frac{7.81 \times 10^{-12} \text{ C}}{0.145 \text{ m}} \quad R = 0.06 \text{ m}
\]
• What is the direction of the electric field at point P?

- Left (←)
- Right (→)
- Up (↑)
- Down (↓)
- Into Page (⊗)

Positives PUSH / Source of Field
Negatives PULL / Sink of Field
Problem

\[ \lambda = \frac{q}{L} \]
linear charge density

\[ dq = \lambda \, dx \]
differential charge

- Calculate the **magnitude** of the electric field at point \( P \).

\[ dE = \frac{k(dq)}{r^2} = \frac{k(dq)}{x^2} = \frac{k(\lambda \, dx)}{x^2} \]

\[ E = \int_{a}^{a+L} dE = \int_{a}^{a+L} \frac{k(\lambda \, dx)}{x^2} \]

\[ = k \lambda \int_{a}^{a+L} x^{-2} \, dx = k \lambda \left[ -x^{-1} \right]_{a}^{a+L} \]

\[ = k \lambda \left[ \frac{1}{a} - \frac{1}{a+L} \right] = k \frac{q}{L} \left[ \frac{1}{a} - \frac{1}{a+L} \right] \]